CHAPTER 7

Void Fraction and Flow Patterns of Two-Phase Flow in Upward and Downward Vertical and Horizontal Pipes

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Abstract: A comparison of the performance of 54 void fraction correlations based on unbiased experimental data set of 3385 data points. A comprehensive literature search was undertaken for the available void fraction correlations and experimental void fraction data for upward and downward vertical and horizontal two-phase flows. The performance of the correlations in correctly predicting the diverse data set was evaluated. Comparisons between the correlations were made and appropriate recommendations were drawn. The analysis showed that most of the correlations developed are very restricted in terms of handling a wide variety of data sets. Based on this analysis void fraction correlations with the best predictive capability are highlighted.

Keywords: Two-phase flow, void fraction, flow pattern, flow visualization, vertical and horizontal pipes.

1. INTRODUCTION

Practical applications of gas-liquid flow, of two different components or a single substance, are commonly encountered in the petroleum, nuclear and process industries. The two gas and liquid phases may exist in flow of different components (*e.g.*, air and water) and/or in the event of phase change due to evaporation and condensation of a single fluid. Void fraction (α) is one of the most important parameters in characterizing two-phase flow. Void fraction is the key physical parameter for determining other two-phase parameters, namely two-phase density, gas and liquid velocities. In addition, void fraction plays an important role in the modeling of two-phase pressure drop, flow pattern transition, and heat transfer.

For industrial applications where two-phase flow is involved, the task of sizing the equipment for gathering, pumping, transporting and storing such a two-phase mixture requires the formidable task of predicting the phase distribution in the system from given operating conditions. The ability to quantify void fraction is of considerable importance in systems involving two-phase flow. For example in nuclear reactor technology, Boiling Water Reactor (BWR) uses light water as neutron moderator and coolant, and void fraction is significant in estimating the reactivity of the nuclear reactor.

The seemingly benign issue of determining the phase distribution from input conditions in a given pipe turns out to be a formidable task due to the slippage between the gas and the liquid phases. Currently, there is a plethora of void fraction correlations available in the literature. The fact that there are numerous correlations available would not be a concern had it not been for the fact that most of the correlations have some form of restrictions attached to them. For instance, one of the most common restrictions to the correlations, flow pattern dependency, is sometimes a purely subjective judgment of the investigator especially for those points on or near flow pattern boundaries. Another pitfall is that many void fraction correlations have only been validated with experimental data that is limited to specific conditions, such as pipe orientation, flow pattern, and gas-liquid combination. As a result, engineers are faced with the daunting task of choosing the appropriate correlation among the plethora of correlations available.

Further advancement in the development of better and more accurate void fraction correlations would not be proper if the tedious process of scrutinizing available correlations is disregarded or circumvented.

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Comparison of the correlations with the measured results provides a stepping stone for further developments of more accurate and improved correlations. The purpose of this work is to identify void fraction correlations that could acceptably predict most of the experimental data collected for upward and downward vertical and horizontal two-phase flows. A comprehensive search has been made in the literature to collect available void fraction correlations as well as the measured void fraction data from various sources. Experimental void fraction data was also measured rather than be solely dependent upon the data from other sources. Based on the analysis, the best performing correlations for all three pipe orientations were selected and recommendations were drawn based on their strengths and weaknesses. Since the main focus of this study is not on the development of void fraction correlations but rather in identifying the best performing correlations, detail discussions on the physics that affect void fraction are not within the scope of this study. The physics behind most of the correlations cited in this manuscript are available in the referenced sources.

2. EXPERIMENTAL SETUP FOR VOID FRACTION MEASUREMENT AND FLOW VISUALIZATION

The experimental setup is equipped for measuring heat transfer, pressure drop, void fraction, and also conducting flow visualization in air-water flow for all major flow patterns and inclination angles from 0° (horizontal) to ±90° (upward and downward vertical). The capabilities of the new experimental setup allow an undertaking that combines the study of heat transfer, flow patterns, pressure drop, void fraction, and inclination effects. The schematic of the overall experimental setup for flow pattern visualization and void fraction for the present study is given in Fig. **1**. The details of the test section are illustrated in Fig. **2**. The flow visualization section is the central portion of the void fraction section and it is constructed from a polycarbonate tube with an inner diameter of 12.7 mm. Detail discussions on the design, construction and functionality of this experimental setup are documented by Cook [1].

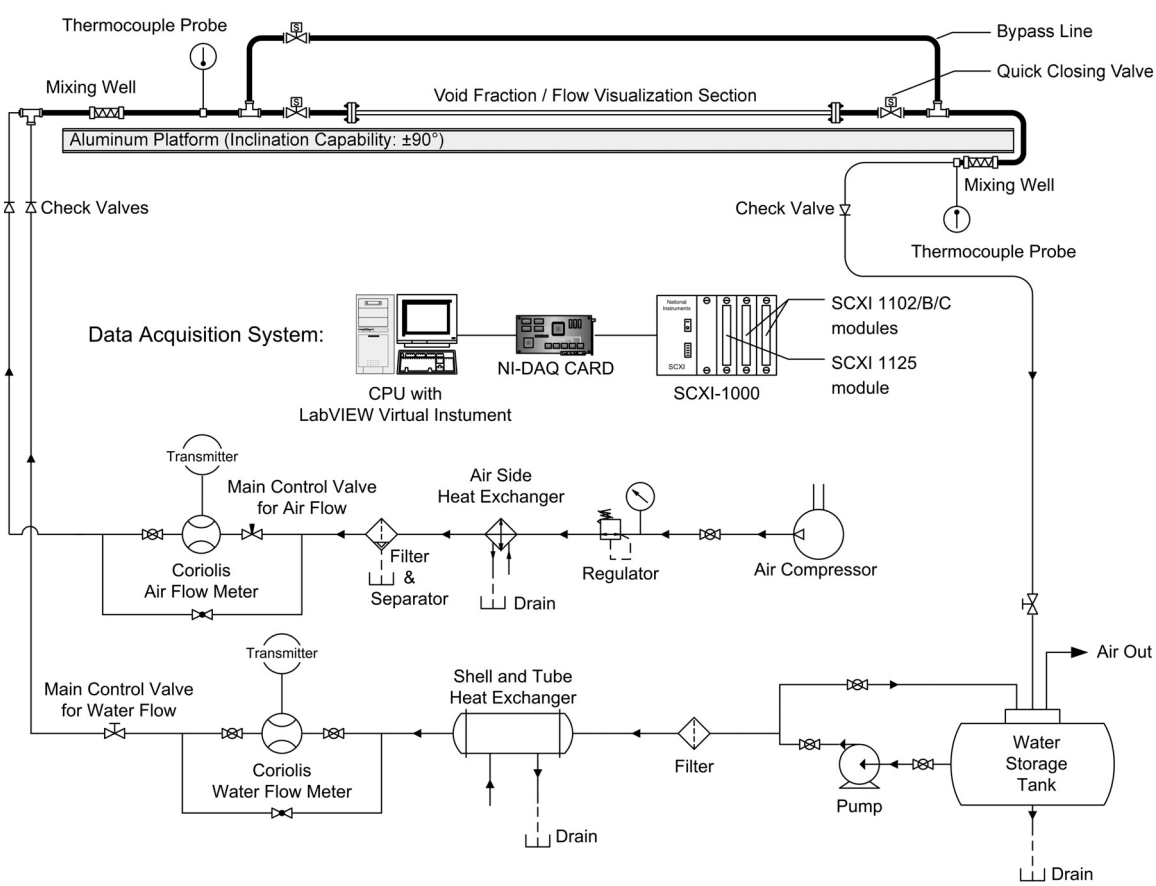


Figure 1: Schematic of experimental setup.

The fluids used in the test loop are air and water. The water is distilled and stored in a 55-gallon cylindrical polyethylene tank. A Bell & Gosset series 1535 coupled centrifugal pump was used to pump the water through an Aqua-Pure AP12T water filter. An ITT Standard model BCF 4063 one shell and two-tube pass heat exchanger removes the pump heat and the heat added during the test to maintain a constant inlet water temperature. From the heat exchanger, the water passes through an Emerson Micro Motion Coriolis flow meter (model CMF100) connected to a digital Field-Mount Transmitter (model RFT9739) that conditions the flow information for the data acquisition system. From the Coriolis flow meter it then flows into the test section.

Air is supplied *via* an Ingersoll-Rand T30 (model 2545) industrial air compressor. The air passes through a copper coil submerged in a vessel of water to lower the temperature of the air to room temperature. The air is then filtered and condensation removed in a coalescing filter. The air flow is measured by Emerson Micro Motion Coriolis flow meters (model CMF025 for high flow rates and model LMF3M for low flow rates). Both flow meters are connected to a Micro Motion 1700 transmitter. Air is regulated by a needle valve and is delivered to the test section by flexible tubing.

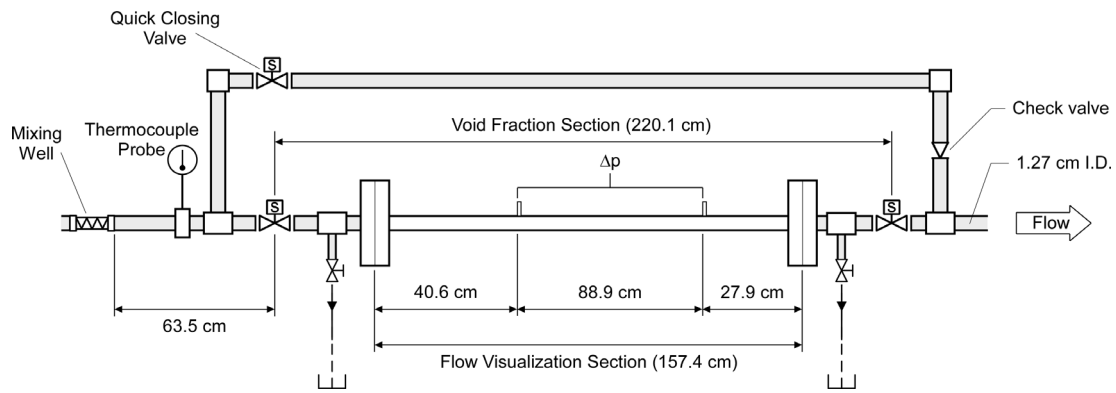


Figure 2: Test section for void fraction measurement and flow visualization.

The inlet liquid and gas temperatures and the exit bulk temperature were measured by Omega TMQSS-06U-6 thermocouple probes. Calibration of thermocouple probes showed that they were accurate within ±0.5°C. Two static mixers, one at the inlet and another at the outlet of the test section, are used to ensure that air and water are properly mixed such that accurate temperature of the mixture can be measured by the thermocouple probes. Upon exiting the test section, the water and air mixture is returned to the reservoir where it is separated and the water recycled.

The void fraction section is constructed to trap mixture of two-phase flow in order to measure the volume of the liquid portion. With the known volume of the void fraction section and the measured volume of the liquid portion, the value of the void fraction can be determined. To trap the two-phase mixture in the void fraction section, three quick closing valves are used. Two normally open valves are used for controlling fluid movement at the inlet and exit of the void fraction section, while a normally closed valve is for controlling the entry of fluid into a bypass line. The quick closing valves are W. E. Anderson Model ABV1DA101 pneumatic ball valves and they exhibit a positive seal when closed and have a closing time of 0.03 seconds.

When the valves are triggered, the two normally open valves close and the normally closed valve opens simultaneously. In this manner, a two-phase sample is trapped in the void fraction section while the air-water mixture is allowed to continue flowing through the bypass line. Air-water mixture trapped in the void fraction section was drained, and the volume of the liquid is measured. With the measured volume of the liquid (V_l), the void fraction (α) can be determined from the total volume of the test section (V_t) using $\alpha = 1 - V_l / V_t$. The uncertainty associated with the volume of the liquid is ±0.5 cm^3, and the uncertainty associated with the total volume of the test section is ±2 cm^3. Using the procedure prescribed by [2], the uncertainties associated with the measured void fraction results were estimated to be between ±1.25 and ±4.16%.

Flow patterns observations in upward and downward vertical and horizontal two-phase air-water flows were conducted as well. All observations for the flow pattern judgments were made at the flow visualization section (see Fig. **1**). By fixing the water flow rate, flow patterns were observed by varying air flow rates. Using visual observation and digital photography, distinctive flow patterns were observed.

3. EXPERIMENTAL FLOW PATTERN AND VOID FRACTION RESULTS

3.1. Flow Pattern Results

The main purpose of conducting experiment on flow patterns was to verify that the experimental setup has the capability of observing similar flow patterns as reported in the literature. By fixing the liquid flow rate, flow patterns were observed by varying gas flow rates. Using visual observation and digital photography, distinctive flow patterns were observed for upward and downward vertical and horizontal pipes. Digital images of each flow pattern were taken using a Nikon D50 digital camera with Nikkor 50 mm f/1.8D lens.

In horizontal pipe, the distinctive major flow patterns for gas-liquid two-phase flow are stratified, plug, slug, wavy and annular. Other flow patterns, such as slug/wavy, slug/bubbly, and wavy/annular, may be considered transitional flow patterns. The major flow patterns observed in this study for horizontal flow confirm the major flow patterns observed by Mandhane *et al.* [3] and Barnea *et al.* [4]. Flow patterns that are observed in horizontal gas-liquid two-phase flow are illustrated in Fig. **3**.

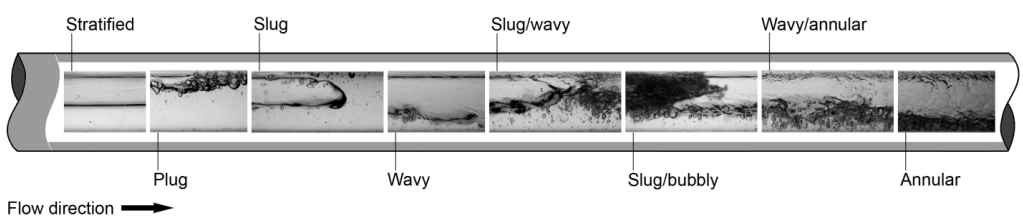


Figure 3: Flow patterns in horizontal two-phase air-water flow.

In upward vertical gas-liquid two-phase flow, the distinctive major flow patterns observed are bubble, slug, churn/froth, and annular. The major flow patterns observed in this study for upward vertical flow confirm the major flow patterns observed by Taitel *et al.* [5] and Weisman and Kang [6]. Flow patterns that are observed in upward vertical gas-liquid two-phase flow are illustrated in Fig. **4**.

In downward vertical gas-liquid two-phase flow, the distinctive major flow patterns observed are falling film, bubble, slug, froth, and annular. The major flow patterns observed in this study for downward vertical flow confirm the major flow patterns observed by Oshinowo and Charles [7]. Flow patterns that are observed in downward vertical gas-liquid two-phase flow are illustrated in Fig. **5**. For bubble flow at constant superficial gas velocity, increase in superficial liquid velocity causes the number of air bubble to increase while the bubbles decrease in size, thus allowing air bubbles to be more evenly distributed across the pipe cross-section. The observed bubble flow behavior at constant superficial gas velocity (U_{sg}) with varying superficial liquid velocity (U_{sl}) is illustrated in Fig. **6**.

The observed bubble flow behavior at constant superficial liquid velocity with varying superficial gas velocity is illustrated in Fig. **7**. For bubble flow at constant superficial liquid velocity, increase in superficial gas velocity causes the number of air bubble to increase as well as the sizes of the bubbles (see Fig. **7**). Further increase in superficial gas velocity would cause the coalescence of air bubbles that would eventually form slug flow.

3.2. Void Fraction Results

The void fraction experiments were conducted to meet several objectives. First, the measured void fraction results were to be used for validation of experimental data from other sources. Since the uncertainties associated with the measured void fractions from this study are determinable, they can be used to validate

experimental data compiled from other sources. This validation is especially informative, since not all the sources reported detailed experimental uncertainties. Second, the measured void fraction results were to be used to fill in the gaps in regions that were not comprehensively measured by other sources. For example, in upward vertical flow, the experimental results from this study have provided 26% of data points to the data compiled from other sources for $0.25 \leq \alpha \leq 0.5$. Also, experimental results from this study for upward vertical flow have added 67 data points to the 58 data points compiled from other sources for churn/froth flow. Third, the measured void fraction results were to be used to evaluate variation of void fraction with superficial velocities of gas and liquid and groupings of various flow patterns for $0 < \alpha < 1$.

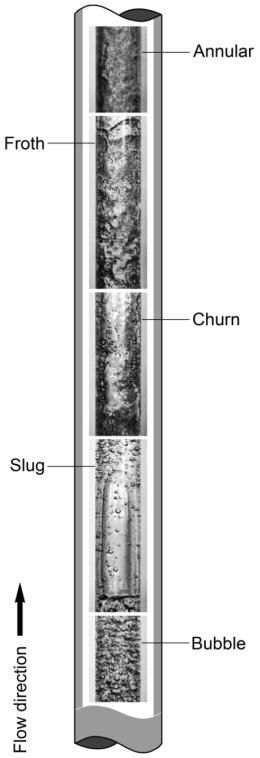


Figure 4: Flow patterns in upward vertical two-phase air-water flow.

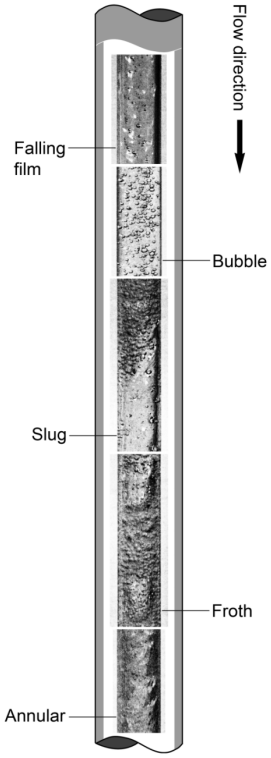
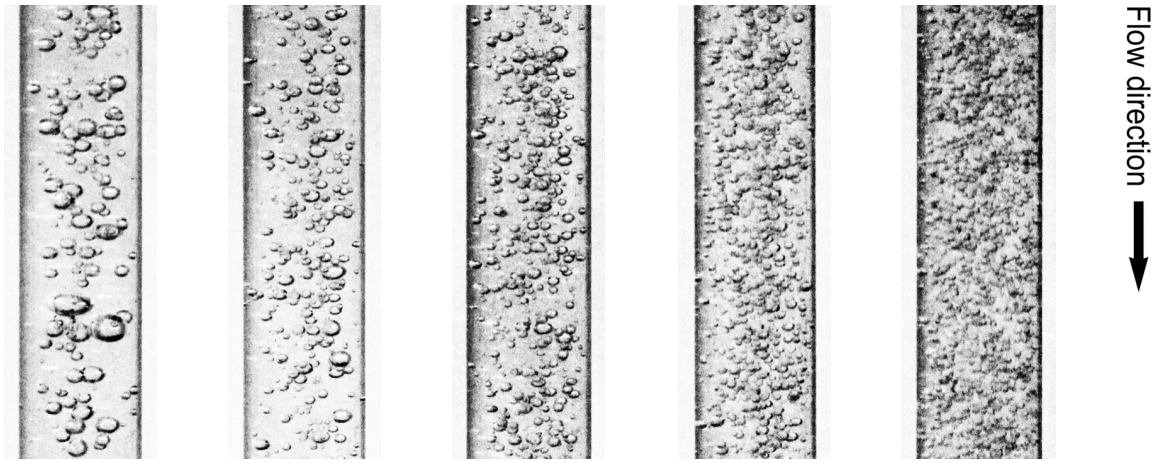
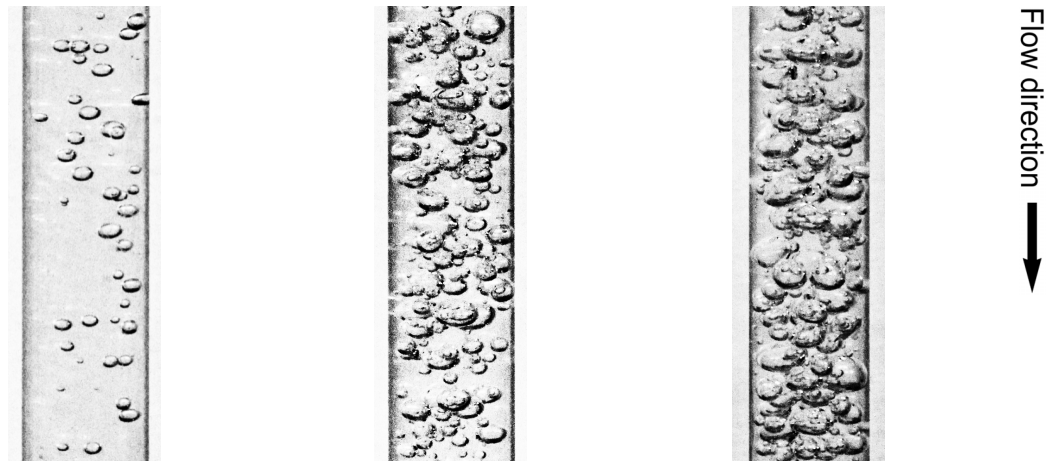


Figure 5: Flow patterns in downward vertical two-phase air-water flow.



U_{sl} = 0.36 m/s, U_{sg} = 0.04 m/s | U_{sl} = 0.48 m/s, U_{sg} = 0.04 m/s | U_{sl} = 0.60 m/s, U_{sg} = 0.04 m/s | U_{sl} = 0.84 m/s, U_{sg} = 0.04 m/s | U_{sl} = 1.08 m/s, U_{sg} = 0.04 m/s

Figure 6: Downward vertical air-water bubble flow at U_{sg} = 0.04 m/s and varying U_{sl}.



U_{sl} = 0.24 m/s U_{sl} = 0.24 m/s U_{sl} = 0.24 m/s

U_{sg} = 0.008 m/s U_{sg} = 0.04 m/s U_{sg} = 0.08 m/s

Figure 7: Downward vertical air-water bubble flow at U_{sl} = 0.24 m/s and varying U_{sg}.

The experimental results of void fraction in upward and downward vertical and horizontal flows were measured from the test section for void fraction measurement and flow visualization, as illustrated in Fig. **2**. The variations of void fraction with superficial gas and liquid velocities for upward and downward vertical and horizontal flows were established from the experimental results (see Figs. **8-10**, respectively). As superficial liquid velocity increases, the increase in liquid holdup causes the void fraction versus superficial gas velocity curves to shift lower. At low superficial gas velocities ($U_{sg} < 2$ m/s) small increase in superficial gas velocity caused rapid increase in the void fraction. The rapid increase in the void fraction suggests difficulty in correlating void fraction accurately in this low superficial gas velocity region. For $U_{sg} > 2$ m/s, void fraction increases gradually with increasing superficial gas velocity.

The aforementioned characteristics of void fraction variation with superficial gas and liquid velocities are consistent for upward and downward vertical and horizontal flows, as shown in Figs. **8-10**, respectively. For upward vertical flow, the void fraction trends observed in the experimental results are similar to the observations of Jiang and Rezkallah [8], Mukherjee [9], Sujumnong [10], and Woods *et al.* [11]. In Figs. **8** and **9**, the groupings of various flow patterns on the variation of void fraction with superficial gas velocity curves are shown. For upward vertical flow, bubble and slug flows are confined to low-range superficial gas velocity with void fraction ranges of $0.05 < \alpha < 0.5$ and $0.2 < \alpha < 0.7$, respectively. Churn/froth flow is found in mid-range superficial gas velocity with void fraction range of $0.3 < \alpha < 0.8$. At a given superficial gas velocity, churn flow has higher void fraction than froth flow. Annular flow is in the high superficial gas velocity region with void fraction range of $0.7 < \alpha < 0.9$. For downward vertical flow, bubble and slug flows are confined to low-range superficial gas velocity with void fraction ranges of $0.02 < \alpha < 0.3$ and $0.2 < \alpha < 0.7$, respectively. Froth and falling film flows are found in mid-range superficial gas velocity with void fraction ranges of $0.4 < \alpha < 0.7$ and $0.7 < \alpha < 0.9$, respectively. Annular flow is in the high superficial gas velocity region with void fraction range of $0.7 < \alpha < 0.9$.

4. EXPERIMENTAL DATABASE

In an effort to make the evaluation of void fraction correlations unbiased, experimental data from various sources were compiled. Including the experimental data measured for present study, a total of 3385 experimental data points (1208 for upward vertical, 909 for downward vertical, and 1268 for horizontal) from 22 different sources were compiled. As summarized in Table **1**, the experimental data include void fraction measurements in pipe diameters ranging from 9.53 to 102 mm, and operating pressures ranging from 101 to 5990 kPa, with different gas-liquid combinations of upward and downward vertical and horizontal two-phase flows. Based on sources with reported experimental uncertainties, the experimental data have associated uncertainties of less than 10%.

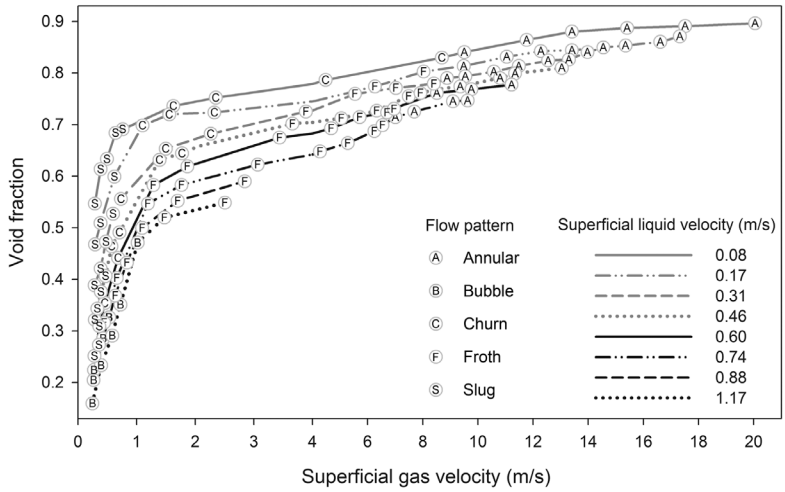


Figure 8: Variation of void fraction with superficial gas and liquid velocities for upward vertical flow.

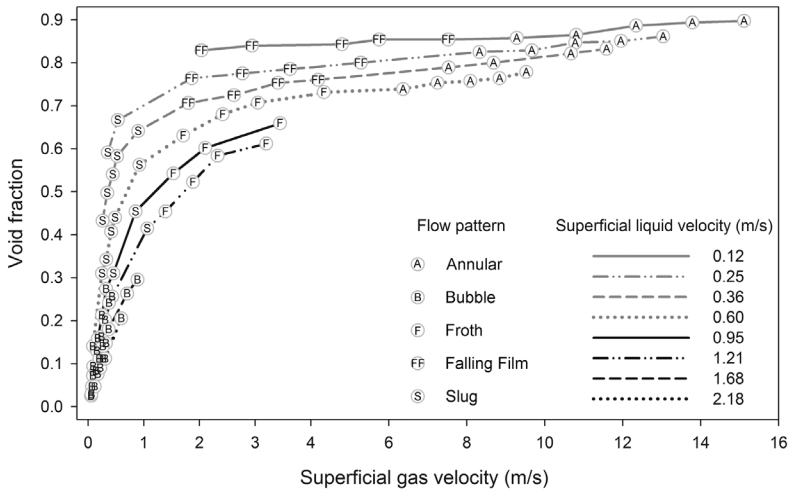


Figure 9: Variation of void fraction with superficial gas and liquid velocities for downward vertical flow.

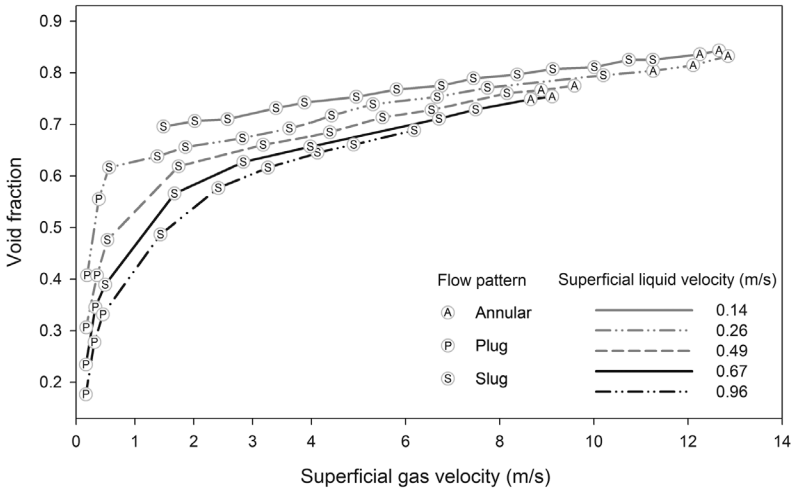


Figure 10: Variation of void fraction with superficial gas and liquid velocities for horizontal flow.

The compiled experimental data were measured at different experimental facilities with various measurement techniques: quick closing valve, gamma-ray absorption, capacitance sensor, manometer, neutron scattering, and conductance probe. The measurement technique of using manometer, also referred to as manometric method, to measure void fraction is only applicable for vertical flow and limited flow condition. In this measurement technique, pressure drop between pressure taps along the test section is measured. The measured pressure drop, where frictional and accelerational contributions are assumed negligible, is then used for determining the void fraction from $\Delta p = [\rho_g \alpha + \rho_l (1-\alpha)]\, g H$. Thus, the void fraction determined from this method relies heavily on the assumption that frictional and accelerational contributions are negligible. The feasibility of this method is therefore primarily limited to vertical flow and flow condition in which the contribution of gravity significantly dominates over those of friction and acceleration. Miyahara *et al.* [12] indicated that void fraction results measured from the manometric method agree satisfactorily with results measured from conductance probe for void fraction below 0.5.

Experimental data measured for the present study were compared with experimental data from other sources at similar superficial velocities of gas and liquid for upward and downward vertical and horizontal flows. A total of 557 data points (210 for upward vertical, 252 for downward vertical, and 115 for horizontal) from various independent sources were used in this comparison. The experimental data from the various sources were measured at different experimental facilities for different pipe diameters (9.53 to 102 mm) and different gas-liquid flows.

For upward vertical flow, 210 experimental data points from other sources with similar superficial gas and liquid velocities were identified and compared with experimental data measured for the present study. The 210 experimental data points from eight different sources were measured for various pipe diameters (12.7 to 76 mm) and various operating pressure (101 to 3410 kPa). All 210 experimental data points are within ±10% agreement with the experimental data from present study. The result of this comparison is shown in Fig. **11**. The comparison of the experimental data from present study and other sources, for this flow orientation, has a Root-Mean-Square (RMS) error of 4.2%, and 77% of the 210 experimental data points are within ±5% agreement.

For downward vertical flow, 252 experimental data points from other sources with similar superficial gas and liquid velocities were identified and compared with experimental data measured for the present study. The 252 experimental data points from eight different sources were measured for various pipe diameters (9.53 to 90 mm) and various operating pressure (101 to 501 kPa). All 252 experimental data points are within ±10% agreement with the experimental data from present study (see Fig. **12**), and 67% of the 252 experimental data points are within ±5% agreement. The comparison of the experimental data from present study and other sources, for this flow orientation, has a Root-Mean-Square (RMS) error of 4.8%.

For horizontal flow, 115 experimental data points from other sources with similar superficial gas and liquid velocities were identified and compared with experimental data measured for the present study. The 115 experimental data points from six different sources were measured for various pipe diameters (19 to 102 mm) and various operating pressure (101 to 2850 kPa). All 115 experimental data points are within ±10% agreement with the experimental data from present study, and 72% of the 115 experimental data points are within ±7% agreement. The result of this comparison is shown in Fig. **13**. The comparison of the experimental data from present study and other sources, for this flow orientation, has a Root-Mean-Square (RMS) error of 6.2%.

The RMS errors found from the comparisons of experimental data from other sources and present study, for upward and downward vertical and horizontal flow, are between 4.2 to 6.2%. The RMS errors of these comparisons are comparable with the uncertainties associated with the experimental data of present study, which is within ±4.16%. The agreement between the experimental data from present study and other sources validated the goodness of the data compiled for this study.

Table 1: Experimental database used for comparison with results from present study and void fraction correlations.

Source	Pipe diameter (mm)	Fluids	No. of data points	Operating pressure (kPa)
Upward vertical flow (1208 data points)				
Present study [1]	12.7	Air-water	153	114–260
Beggs [13] [1]	25.4 & 38.1	Air-water	27	527–677
Chokshi [14] [2]	76	Air-water	103	1220–3410
Fernandes [15] [1]	50.7	Air-water	88	
Isbin *et al.* [16] [2]	22.1	Steam-water	22	101
Mukherjee [9] [3]	38.1	Air-kerosene	65	267–609
Oshinowo [17] [1]	25.4	Air-water	153	134–206
	25.4	Air-glycerin	172	141–199
Schmidt *et al.* [18] [2]	54.5	Nitrogen-water	20	
Nguyen [19] [1]	45.5	Air-water	224	107–125
Sujumnong [10] [1]	12.7	Air-glycerin	77	102–307
	12.7	Air-water	104	101–343
Downward vertical flow (909 data points)				
Present study [1]	12.7	Air-water	193	113–237
Beggs [13] [1]	25.4 & 38.1	Air-water	26	437–662
Hernandez *et al.* [20] [1]	50.8	Air-water	39	
Jiang and Rezkallah [8] [2]	9.53	Air-water	81	
Lorenzi and Sotgia [21] [4]	32, 44 & 90	Air-water	141	
Mukherjee [9] [3]	38.1	Air-kerosene	54	253–567
	38.1	Air-lube oil	48	282–456
Nguyen [19] [1]	45.5	Air-water	79	101–105
Oshinowo [17] [1]	25.4	Air-glycerin	78	145–203
	25.4	Air-water	112	134–205
Paras [22] [5]	19.1	Air-water	35	
Usui and Sato [23] [6]		Air-water	25	
Horizontal flow (1268 data points)				
Present study [1]	12.7	Air-water	184	110–242
Abdul-Majeed [24] [1]	50.8	Air-kerosene	88	197–919
Badie *et al.* [25] [2]	78	Air-oil*	30	
	78	Air-water	36	
Beggs [13] [1]	25.4 & 38.1	Air-water	58	358–680
Chen *et al.* [26] [3]	77.9	Air-kerosene	48	175–233
Eaton [27] [1]	54.5 & 102	Natural gas-water	238	2110–5990
Franca and Lahey [28] [1]	19	Air-water	88	101–116
Minami and Brill [29] [1]	77.9	Air-kerosene	57	301–667
	77.9	Air-water	54	320–589
Mukherjee [9] [3]	38.1	Air-kerosene	75	194–634
Nguyen [19] [1]	45.5	Air-water	270	101–115
Ottens [30]	51	Air-water	42	

[1] Quick closing valve, [2] Gamma-ray absorption, [3] Capacitance sensor, [4] Manometer, [5] Neutron scattering, [6] Conductance probe. * Shell Tellus 22 oil.

The results from the comparisons of experimental data from other sources and present study (see Figs. **11**, **12**, and **13**) also suggest, at least within the confines of these comparisons, that void fraction is not affected by pipe diameter (9.53 to 102 mm), operating pressure (101 to 3410 kPa), and type of gas-liquid flow. In addition, referring to the information listed in Table **1**, results shown in Figs. **11-13** indicate that experimental void fraction results measured *via* different techniques are generally in good agreement. This observation confirms with the findings of Jiang and Rezkallah [8] and Schmidt *et al.* [18], which had compared void fractions measured by gamma-ray absorption technique and quick closing valve technique.

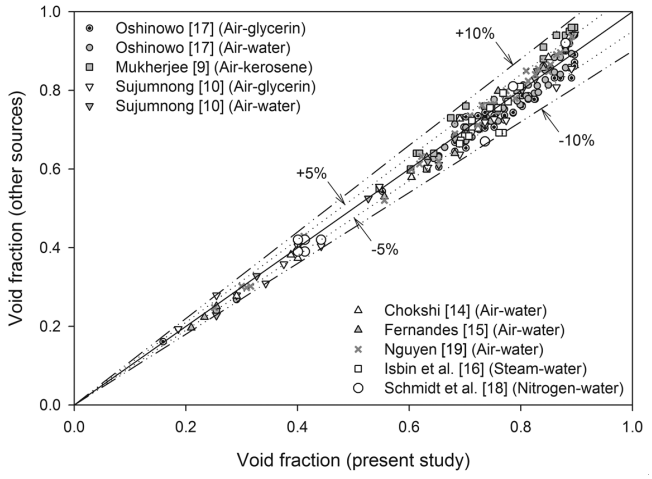


Figure 11: Comparison between experimental data from present study and other sources (see Table **1**) for upward vertical flow.

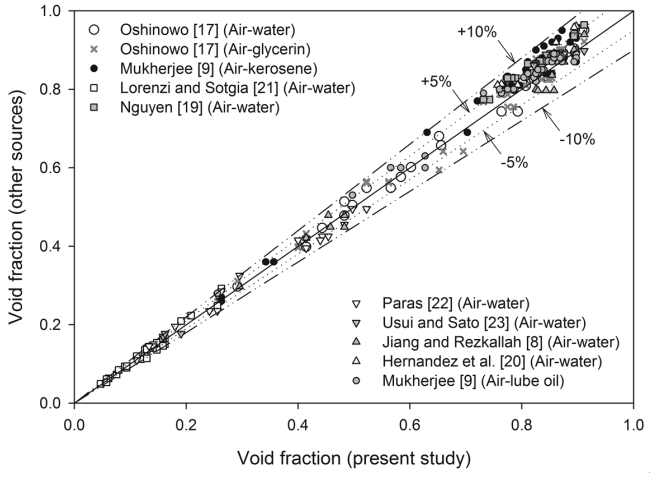


Figure 12: Comparison between experimental data from present study and other sources (see Table **1**) for downward vertical flow.

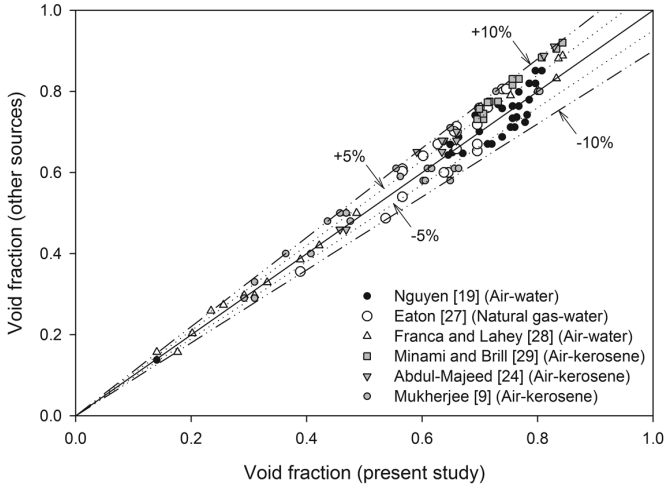


Figure 13: Comparison between experimental data from present study and other sources (see Table **1**) for horizontal flow.

5. EVALUATION OF VOID FRACTION CORRELATIONS WITH EXPERIMENTAL DATA

Fifty four void fraction correlations have been identified from the literature and considered in this analysis. These correlations were considered due to the reason that complete information on input parameters is available from the compiled experimental database; such that these correlations can be used to calculate void fractions and compared with measured results. Very few of the void fraction correlations identified for this study have been compared with experimental data that is as extensive as such that is summarized in Table **1**. Many of the correlations are "specialized" correlations and were originally proposed for specific conditions such as pipe orientation and flow pattern. Thus, their performances outside their specified conditions have not been scrutinized. In the interest of determining the "best" correlation for each pipe orientation (upward and downward vertical and horizontal), the correlations were subjected to two levels of assessment.

The first level is the evaluation of the correlations with experimental data on the basis of the overall performance, where the results predicted by the correlations for an entire database ($0 < \alpha < 1$) of each pipe orientation were compared with the measured results. The first level of assessment by itself is perfunctory, since it overlooks the strengths and weaknesses of the correlations in specific ranges of void fraction. The number of experimental data points is not uniformly distributed throughout the entire void fraction range of $0 < \alpha < 1$ (majority of the data points are in $0.75 < \alpha < 1$), hence relying solely on overall performances of the correlations could lead to biased interpretations. Subsequently, the second level is the analysis of the correlations in smaller void fraction ranges, by dividing the entire void fraction range into four categories: 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75, and 0.75 to 1. Dividing the analysis into specific void fraction ranges would also reveal the most accurate correlations for each specific range; thus allowing the access to correlations with higher accuracies in specific void fraction range of interest.

Criteria for satisfactory performance were specified to aid the evaluation of the void fraction correlations with the experimental data (see Table **4**). The criteria summarized in Table **4** were not set arbitrarily, but rather based on how the correlations responded to the comparison with experimental data. For instance, in $0 < \alpha \leq 0.25$ the predictions by the correlations for upward vertical and horizontal flows have greater disparity than that of downward vertical flow. Hence, the criteria set for downward vertical flow are more restrictive than the criteria set for upward vertical and horizontal flows in $0 < \alpha \leq 0.25$ (see Table **4**). Similarly, the criteria set for $0.75 < \alpha < 1$ are more restrictive than the criteria set for $0 < \alpha \leq 0.25$ (see Table **4**), since the correlations performed with higher accuracies in $0.75 < \alpha < 1$ than in $0 < \alpha \leq 0.25$ for all three pipe orientations.

Out of the 54 void fraction correlations considered, 23 of them are mentioned in this article because they were found to compare satisfactorily with the compiled experimental results. Table **2** lists the 23 void fraction correlations to be discussed in this article and the reason in which these correlations were selected will be explained in this section. The sources for the other correlations that were analyzed and evaluated but not discussed here, because of unsatisfactory comparison with experimental results, are listed in Table **3**.

5.1. Performance of Correlations for the Entire Void Fraction Range

The 54 void fraction correlations considered in this analysis were compared with the data points in the experimental database summarized in Table **1** for upward and downward vertical and horizontal pipe orientations. Out of 54 correlations compared with the experimental data for $0 < \alpha < 1$, the correlations that compared satisfactorily with the experimental data and the comparison results are listed in Table **5**. The equations for the correlations listed in Table **5** are given in Table **2**.

For upward vertical flow, eight correlations were determined to be satisfactory (see Table **5**). For the number of data points predicted within the ±10% error band of the experimental data, the correlation of Rouhani and Axelsson [49] performed the best, with 856 (70.9%) data points predicted within the error band. A noticeable observation was made that the correlations of Rouhani and Axelsson [49] and Ishii [42] are the only correlations, among the 54 correlations considered in this study, that predicted at least 85, 75, and 65% of the data points within ±20, ±15, and ±10% error bands of the experimental data, respectively. Based on the evaluation thus far, the choice of which correlation to use lies with the user's interest of the

accuracy in the correlation. If the interest of accuracy is between ±15 and ±20% error bands of the experimental data, then the correlation of Nicklin *et al.* [47] is preferable. If the interest is for higher accuracy with error bands of the experimental data between ±10 and ±15%, then the correlation of Rouhani and Axelsson [49] is preferable.

The similarity of the Nicklin *et al.* [47], Bonnecaze *et al.* [33], and Kokal and Stanislav [44] correlations is noteworthy. The correlations of Bonnecaze *et al.* [33] and Kokal and Stanislav [44] are slight variant of Nicklin *et al.* [47] correlation, hence all three correlations produced very similar results when compared with the experimental data (see Table **5**). The only variation between these three correlations is the relation for gas phase drift velocity (u_{gu}). Comparing the correlations of Nicklin *et al.* [47] and Bonnecaze *et al.* [33], there is an added term of $(1-\rho_g/\rho_l)$ in the gas phase drift velocity (u_{gu}) relation for Bonnecaze *et al.* [33] correlation. In a similar manner, Kokal and Stanislav [44] correlation has an added term of $(1-\rho_g/\rho_l)^{0.5}$ in the gas phase drift velocity (u_{gu}) relation of their correlation. Both additional terms in the gas phase drift velocity (u_{gu}) relations of Bonnecaze *et al.* [33] and Kokal and Stanislav [44] correlations do not impact the results significantly, since the density ratio (ρ_g/ρ_l) is very small for most gas-liquid combinations.

For downward vertical flow, seven correlations were determined to be satisfactory (see Table **5**). The correlation of Gomez *et al.* [37] predicted the most number of experimental data points within the ±20, ±15, and ±10% error bands as well as with the lowest RMS error. Following Gomez *et al.* [37], both correlations of Nicklin *et al.* [47] and Bonnecaze *et al.* [33] predicted 859 (94.5%) and 796 (87.6%) within ±20 and ±15%, respectively. In downward vertical flow, the gas phase drift velocity (u_{gu}) is in the downward direction, thus a negative sign applied to the u_{gu} for the correlations of [33, 44, 47, 49, 51], as indicated in Table **2**. Although in this first level of assessment, the Gomez *et al.* [37] correlation appears to be the best correlation for downward vertical flow, the second level of assessment is necessary to confirm its satisfactory performances in different void fraction ranges.

For horizontal flow, eight correlations were determined to be satisfactory (see Table **5**). The correlation of Woldesemayat and Ghajar [52] predicted the most number of experimental data points within the ±20, ±15, and ±10% error bands with RMS error of 16%. Following Woldesemayat and Ghajar [52], the Minami and Brill [29] correlation predicted 1173 (92.5%) and 1115 (87.9%) within ±20 and ±15%, respectively. Based on the evaluation thus far, the Woldesemayat and Ghajar [52] correlation is the best correlation, and further scrutiny in different void fraction ranges would evaluate its performances for these categories.

Table 2: List of void fraction correlations that performed satisfactorily and discussed in this study.

Source	Correlation	
Armand [31] – Massena [32]	$\alpha = (0.833 + 0.167x)\dfrac{U_{sg}}{U_{sg}+U_{sl}}$	(1)
Bonnecaze *et al.* [33]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg}+U_{sl})+u_{gu}}$ where $C_0 = 1.2$	(2)
	$u_{gu} = 0.35\sqrt{gD}\,(1-\rho_g/\rho_l)$	(2*a*)*
Chisholm [34]	$\alpha = \left[1+\sqrt{1-x(1-\rho_l/\rho_g)}\;\dfrac{1-x}{x}\dfrac{\rho_g}{\rho_l}\right]^{-1}$	(3)
Dix [35]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg}+U_{sl})+u_{gu}}$	(4)
	$C_0 = \dfrac{U_{sg}}{U_{sg}+U_{sl}}[1+(U_{sl}/U_{sg})^b]$ where $b = (\rho_g/\rho_l)^{0.1}$	(4*a*)

Table 2: ***cont….***

	$u_{gu} = 2.9\left(g\sigma\dfrac{\rho_l - \rho_g}{\rho_l^2}\right)^{0.25}$	(4*b*)
El-Boher *et al.* [36]	$\alpha = \left[1 + 0.27\left(\dfrac{x}{1-x}\dfrac{\rho_l}{\rho_g}\right)^{-0.69}(Fr_{sl})^{-0.177}\left(\dfrac{\mu_l}{\mu_g}\right)^{0.378}\left(\dfrac{Re_{sl}}{We_{sl}}\right)^{0.067}\right]^{-1}$	(5)
	where $Fr_{sl} = \dfrac{U_{sl}^2}{gD}$, $Re_{sl} = \dfrac{\rho_l U_{sl} D}{\mu_l}$, $We_{sl} = \dfrac{\rho_l U_{sl}^2 D}{\sigma}$	
Gomez *et al.* [37]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$ where $C_0 = 1.15$	(6)
	$u_{gu} = 1.53\left(g\sigma\dfrac{\rho_l - \rho_g}{\rho_l^2}\right)^{0.25}(1-\alpha)^{0.5}\sin\theta$	(6*a*)
Greskovich and Cooper [38]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$ where $C_0 = 1.0$	(7)
	$u_{gu} = 0.671\sqrt{gD}\,(\sin\theta)^{0.263}$	(7*a*)
Guzhov *et al.* [39]	$\alpha = 0.81\dfrac{U_{sg}}{U_{sg} + U_{sl}}\left(1 - e^{-2.2\sqrt{Fr}}\right)$ where $Fr = \dfrac{(U_{sg} + U_{sl})^2}{gD}$	(8)
Hibiki and Ishii [40]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$	(9)
	$C_0 = (1.2 - 0.2\sqrt{\rho_g/\rho_l})\,(1 - e^{-18\alpha})$	(9*a*)
	$u_{gu} = \left(4g\sigma\dfrac{\rho_l - \rho_g}{\rho_l^2}\right)^{0.25}(1-\alpha)^{1.75}$	(9*b*)
Huq and Loth [41]	$\alpha = 1 - \dfrac{2\,(1-x)^2}{1 - 2x + \sqrt{1 + 4x\,(1-x)\,(\rho_l/\rho_g - 1)}}$	(10)
Ishii [42] [1]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$	(11)
	$C_0 = \min(C_{01}, C_{02})$	(11*a*)
	$u_{gu} = \min(u_{gu1}, u_{gu2})$	(11*b*)
	See Ohkawa and Lahey [43] for C_{01}, C_{02}, u_{gu1}, and u_{gu2}	
Kokal and Stanislav [44]	$\alpha = \dfrac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$ where $C_0 = 1.2$	(12)
	$u_{gu} = 0.345\,[gD(1 - \rho_g/\rho_l)]^{0.5}$	(12*a*)*
Lockhart and Martinelli [45] [2]	$\alpha = \left[1 + 0.28\left(\dfrac{1-x}{x}\right)^{0.64}\left(\dfrac{\rho_g}{\rho_l}\right)^{0.36}\left(\dfrac{\mu_l}{\mu_g}\right)^{0.07}\right]^{-1}$	(13)

Table 2: *cont….*

Minami and Brill [29]	$\alpha = \exp\left[-\left(\frac{\ln Z + 9.21}{8.7115}\right)^{4.3374}\right]$	(14)
	Z is a combination of dimensionless numbers, see [29]	
Morooka *et al.* [46]	$\alpha = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$ where $C_0 = 1.08$, $u_{gu} = 0.45$	(15)
Mukherjee [9]	$\alpha = 1 - \exp\left[(C_1 + C_2 \sin\theta + C_3 \sin^2\theta + C_4 N_{l\mu}) \frac{N_{gu}^{C_5}}{N_{lu}^{C_6}}\right]$	(16)
	$N_{gu} = U_{sg}[\rho_l / (g\sigma_l)]^{1/4}$	(16*a*)
	$N_{lu} = U_{sl}[\rho_l / (g\sigma_l)]^{1/4}$	(16*b*)
	$N_{l\mu} = \mu_l(\rho_l \sigma_l^3)^{-1/4}$	(16*c*)
	$C_1, C_2, \ldots, C_6$ are constants given by Mukherjee [9]	
Nicklin *et al.* [47]	$\alpha = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$ where $C_0 = 1.2$	(17)
	$u_{gu} = 0.35\sqrt{gD}$	(17*a*)*
Premoli *et al.* [48] [3]	$\alpha = \left\{1 + \frac{\rho_g}{\rho_l}\left(\frac{1-x}{x}\right)\left[1 + K\left(\frac{Y}{1+CY} - CY\right)^{0.5}\right]\right\}^{-1}$	(18)
	$K = 1.578 Re^{-0.19}(\rho_l / \rho_g)^{0.22}$	(18*a*)
	$C = 0.0273 We Re^{-0.51}(\rho_l / \rho_g)^{-0.08}$	(18*b*)
	where $y = \left(\frac{1-x}{x}\frac{\rho_g}{\rho_l}\right)^{-1}$, $Re = \frac{GD}{\mu_l}$, $We = \frac{G^2 D}{\rho_l \sigma}$	
Rouhani and Axelsson [49]	$\alpha = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$	(19)
	$C_0 = 1 + 0.2(1-x)(gD\rho_l^2 / G^2)^{0.25}$	(19*a*) [4]
	$C_0 = 1 + 0.2(1-x)$	(19*b*) [5]
	$u_{gu} = 1.18\left(g\sigma\frac{\rho_l - \rho_g}{\rho_l^2}\right)^{0.25}$	(19*c*)*
Smith [50]	$\alpha = \left\{1 + \frac{\rho_g}{\rho_l}\left(\frac{1-x}{x}\right)\left[0.4 + 0.6\left(\frac{\rho_l / \rho_g + 0.4(1/x - 1)}{1 + 0.4(1/x - 1)}\right)^{0.5}\right]\right\}^{-1}$	(20)
Sun *et al.* [51]	$\alpha = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$	(21)
	$C_0 = [0.82 + 0.18(p_{sys} / p_{cr})]^{-1}$	(21*a*)

Table 2: ***cont….***

	$u_{gu} = 1.41\left(g\sigma\frac{\rho_l - \rho_g}{\rho_l^2}\right)^{0.25}$	(21*b*)*
Woldesemayat and Ghajar [52]	$\alpha = \frac{U_{sg}}{C_0(U_{sg} + U_{sl}) + u_{gu}}$	(22)
	$C_0 = \frac{U_{sg}}{U_{sg} + U_{sl}}\left[1 + \left(\frac{U_{sl}}{U_{sg}}\right)^{(\rho_g/\rho_l)^{0.1}}\right]$	(22*a*)
	$u_{gu} = 2.9(1.22 + 1.22\sin\theta)^{p_{atm}/p_{sys}}\left[\frac{gD\sigma(1+\cos\theta)(\rho_l - \rho_g)}{\rho_l^2}\right]^{0.25}$	(22*b*)
	The leading constant of 2.9 in Eq. (22*b*) carries a unit of $m^{-0.25}$	
Yamazaki and Yamaguchi [53]	$\frac{\alpha}{(1-\alpha)(1-K\alpha)} = \frac{\alpha_h}{1-\alpha_h}$ where $\alpha_h = \frac{U_{sg}}{U_{sg} + U_{sl}}$	(23)
	$K = 2.0 - 0.4/\alpha_h$ for $\alpha_h \le 0.2$	(23*a*)
	$K = -0.25 + 1.25\alpha_h$ for $\alpha_h \ge 0.2$	(23*b*)

[1] As given by Ohkawa and Lahey [43], [2] As given by Butterworth [54], [3] As given by Woldesemayat and Ghajar [52], [4] As given by Rouhani [55], in this analysis, it is used for $0 < \alpha \le 0.25$, [5] As given by Woldesemayat and Ghajar [52], in this analysis it is used for $\alpha > 0.25$.
* For downward vertical flow, a negative sign is applied on these equations for u_{gu}.

Table 3. Sources for void fraction correlations that were evaluated but did not perform satisfactorily.

Source	Source	Source
Armand [31] – Chisholm [56] [1]	Bankoff [57] [1]	Baroczy [58] [1, 2]
Beggs [13] [1]	Bestion [59] [1, 3]	Czop *et al.* [60] [1]
Dimentiev *et al.* [61] [1, 4]	Filimonov *et al.* [62]	Gardner [63] [1]
Hughmark [64] [1]	Inoue *et al.* [65] [1, 3]	Jowitt *et al.* [66] [1, 3]
Kowalczewski [1, 5]	Kütüçüğlu [1, 5]	Lahey and Moody [67] [6, 7]
Madsen [68] [1]	Maier and Coddington [69] [1, 3]	Mattar and Gregory [70] [1]
Moussali [1, 5]	Neal and Bankoff [71] [1]	Ohkawa and Lahey [43] [7]
Orell and Rembrand [72] [7]	Shvarts *et al.* [73] [7]	Sonnenburg [74] [3, 7]
Spedding and Chen [75] [1]	Spedding *et al.* [76] [1]	Sterman [77] [1]
Takeuchi *et al.* [78] [7]	Thom [79] [1, 2]	Wilson *et al.* [80] [1]
Yeh and Hochreiter [81] [7]		

[1] Also given by Woldesemayat and Ghajar [52], [2] Also given by Butterworth [54], [3] Also given by Coddington and Macian [82], [4] Also given by Kataoka and Ishii [83], [5] Also given by Isbin and Biddle [84], [6] Also given by Ohkawa and Lahey [43], [7] Also given by Godbole [85].

5.2. Performance of Correlations for $0 < \alpha \le 0.25$

Due to lower values of void fraction, the percentage error in prediction with respect to measured data was expected to be more prominent than for the case with higher void fraction values. The high percentage error from the correlations in this region is in part a result of the rapid increase in void fraction at low superficial gas velocity (see Figs. **8-10**). The rapid increase in void fraction makes correlating void fraction accurately in this region difficult. Due to the relatively higher percentage error in prediction with respect to experimental data in this low range of $0 < \alpha \le 0.25$, larger error bands and RMS error were used for the criteria to determine a satisfactory correlation (see Table **4**).

For upward vertical flow, 199 experimental data points from the entire database were categorized in the void fraction range of $0 < \alpha \leq 0.25$. Six correlations that are listed in Table **5** performed satisfactorily in the $0 < \alpha \leq 0.25$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Greskovich and Cooper [38], Nicklin *et al.* [47], and Rouhani and Axelsson [49] are among the three best correlations in the $0 < \alpha \leq 0.25$ range. The results of the comparison of the three aforementioned correlations with experimental data are listed in Table **7**. Although the performance of Greskovich and Cooper [38] correlation is not listed in Table **5**, it is among the three best correlations in the void fraction range of $0 < \alpha \leq 0.25$ and it predicted the most number of experimental data points within ±10% error band. The equations for all three correlations are listed in Table **2**.

For downward vertical flow, 237 experimental data points from the entire database were categorized in the void fraction range of $0 < \alpha \leq 0.25$. Four correlations that are listed in Table **5** performed satisfactorily in the $0 < \alpha \leq 0.25$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Bonnecaze *et al.* [33], Gomez *et al.* [37], and Nicklin *et al.* [47] are among the three best correlations in the $0 < \alpha \leq 0.25$ range. The results of the comparison of the three aforementioned correlations with experimental data are listed in Table **8**. The equations for all three correlations are listed in Table **2**.

Table 4: Criteria for Determining Satisfactory Correlation.

Category	Criteria for satisfactory correlation in error band and RMS error
Upward vertical flow	
Entire range	at least 85 & 75% of data points predicted within ±20 & ±15%, RMS error ≤ 30%
$0 < \alpha \leq 0.25$	at least 80% of data points predicted within ±30%, RMS error ≤ 60%
$0.25 < \alpha \leq 0.5$	at least 80% of data points predicted within ±20%, RMS error ≤ 20%
$0.5 < \alpha \leq 0.75$	at least 80% of data points predicted within ±15%, RMS error ≤ 15%
$0.75 < \alpha < 1$	at least 80% of data points predicted within ±10%, RMS error ≤ 10%
Downward vertical flow	
Entire range	at least 90 & 80% of data points predicted within ±20 & ±15%, RMS error ≤ 15%
$0 < \alpha \leq 0.25$	at least 80% of data points predicted within ±20%, RMS error ≤ 15%
$0.25 < \alpha \leq 0.5$	at least 90 & 80% of data points predicted within ±20 & ±15%, RMS error ≤ 15%
$0.5 < \alpha \leq 0.75$	at least 90 & 80% of data points predicted within ±20 & ±15%, RMS error ≤ 15%
$0.75 < \alpha < 1$	at least 95 & 80% of data points predicted within ±15 & ±10%, RMS error ≤ 10%
Horizontal flow	
Entire range	at least 90 & 70% of data points predicted within ±20 & ±10%, RMS error ≤ 30%
$0 < \alpha \leq 0.25$	at least 60% of data points predicted within ±20%, RMS error ≤ 40%
$0.25 < \alpha \leq 0.5$	at least 70% of data points predicted within ±20%, RMS error ≤ 20%
$0.5 < \alpha \leq 0.75$	at least 90 & 80% of data points predicted within ±20 & ±15%, RMS error ≤ 15%
$0.75 < \alpha < 1$	at least 95 & 90% of data points predicted within ±15 & ±10%, RMS error ≤ 10%

For horizontal flow, there are only 40 experimental data points from the entire database were categorized in the void fraction range of $0 < \alpha \leq 0.25$. Only three correlations that are listed in Table **5** performed satisfactorily in the $0 < \alpha \leq 0.25$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Huq and Loth [41], Premoli *et al.* [48], and Rouhani and Axelsson [49] are among the three best correlations in the $0 < \alpha \leq 0.25$ range. Premoli *et al.* [48] correlation is among the three best correlations in the void fraction range of $0 < \alpha \leq 0.25$, but it is not among the correlations that performed satisfactorily for $0 < \alpha < 1$. The results of the comparison of the three aforementioned correlations with experimental data are listed in Table **9**. The equations for all three correlations are listed in Table **2**. The limited number of experimental data points in this void fraction range is noteworthy, and a more conclusive assessment can be obtained had there been more data points available for comparison.

5.3. Performance of Correlations for $0.25 < \alpha \leq 0.5$

For upward vertical flow, 190 experimental data points from the entire database were categorized in the void fraction range of $0.25 < \alpha \leq 0.5$. Five correlations that are listed in Table **5** performed satisfactorily in

the $0.25 < \alpha \leq 0.5$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Bonnecaze *et al.* [33], Kokal and Stanislav [44], and Nicklin *et al.* [47] are among the three best correlations in the $0.25 < \alpha \leq 0.5$ range, and their performances for $0 < \alpha < 1$ are also listed in Table **5**. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **7**, respectively.

Table 5: Results of correlations that compared satisfactorily with the entire experimental database ($0 < \alpha < 1$).

Correlations	Percentage of data points predicted within ±10%	±15%	±20%	RMS error (%)
Upward vertical flow (1208 data points)				
Bonnecaze *et al.* [33]	62.4	84.4	91.7	23.9
Guzhov *et al.* [39]	55.0	77.6	88.7	24.6
Ishii [42]	66.7	80.5	87.3	28.0
Kokal and Stanislav [44]	62.3	84.4	91.6	24.0
Morooka *et al.* [46]	62.3	79.1	87.8	23.6
Nicklin *et al.* [47]	62.4	84.4	91.7	23.9
Rouhani and Axelsson [49]	70.9	86.8	91.1	25.2
Sun *et al.* [51]	58.4	78.1	91.1	23.9
Downward vertical flow (909 data points)				
Bonnecaze *et al.* [33]	68.0	87.6	94.5	10.3
Gomez *et al.* [37]	79.2	90.4	96.1	9.2
Kokal and Stanislav [44]	68.0	87.3	94.1	10.4
Nicklin *et al.* [47]	68.0	87.6	94.5	10.3
Rouhani and Axelsson [49]	76.3	86.8	91.0	11.2
Sun *et al.* [51]	65.6	84.0	91.7	11.2
Yamazaki and Yamaguchi [53]	70.5	82.2	89.0	13.7
Horizontal flow (1268 data points)				
Armand [31] – Massena [32]	78.7	86.4	90.5	13.6
Chisholm [34]	75.0	86.2	90.7	13.6
Huq and Loth [41]	71.8	83.2	89.9	13.3
Minami and Brill [29]	74.1	87.9	92.5	14.1
Mukherjee [9]	74.4	85.8	92.0	17.2
Rouhani and Axelsson [49]	78.2	87.2	91.6	12.8
Smith [50]	72.5	84.5	90.5	13.5
Woldesemayat and Ghajar [52]	82.5	89.1	93.0	16.0

$$\text{RMS error} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left[\frac{(\alpha_{\text{calc}})_i - (\alpha_{\text{meas}})_i}{(\alpha_{\text{meas}})_i}\right]^2} \times 100\%$$

where N is the number of experimental data points

For downward vertical flow, 105 experimental data points from the entire database were categorized in the void fraction range of $0.25 < \alpha \leq 0.5$. All seven correlations that are listed in Table **5** performed satisfactorily in the $0.25 < \alpha \leq 0.5$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Bonnecaze *et al.* [33], Nicklin *et al.* [47], and Rouhani and Axelsson [49] are among the three best correlations in the $0.25 < \alpha \leq 0.5$ range, and their performances for $0 < \alpha < 1$ are also listed in Table **5**. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **8**, respectively.

For horizontal flow, 145 experimental data points from the entire database were categorized in the void fraction range of $0.25 < \alpha \leq 0.5$. Only three correlations that are listed in Table **5** performed satisfactorily in

the $0.25 < \alpha \leq 0.5$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Minami and Brill [29], Mukherjee [9], and Woldesemayat and Ghajar [52] are among the three best correlations in the $0.25 < \alpha \leq 0.5$ range, and their performances for $0 < \alpha < 1$ are also listed in Table **5**. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **9**, respectively.

Table 6: Qualitative performance of satisfactory correlations (see Table **5**) in four void fraction ranges.

	Void fraction range			
Correlations	**0.00 to 0.25**	**0.25 to 0.50**	**0.50 to 0.75**	**0.75 to 1.00**
Upward vertical flow (1208 data points)				
Bonnecaze *et al.* [33]	S	S	S	NS
Guzhov *et al.* [39]	NS	NS	S	NS
Ishii [42]	NS	NS	S	S
Kokal and Stanislav [44]	S	S	S	NS
Morooka *et al.* [46]	S	NS	NS	S
Nicklin *et al.* [47]	S	S	S	NS
Rouhani and Axelsson [49]	S	S	S	S
Sun *et al.* [51]	S	S	S	NS
Downward vertical flow (909 data points)				
Bonnecaze *et al.* [33]	S	S	S	NS
Gomez *et al.* [37]	S	S	S	S
Kokal and Stanislav [44]	S	S	S	NS
Nicklin *et al.* [47]	S	S	S	NS
Rouhani and Axelsson [49]	NS	S	S	S
Sun *et al.* [51]	NS	S	S	NS
Yamazaki and Yamaguchi [53]	NS	S	S	S
Horizontal flow (1268 data points)				
Armand [31] – Massena [32]	S	NS	NS	S
Chisholm [34]	NS	NS	NS	S
Huq and Loth [41]	S	NS	NS	S
Minami and Brill [29]	NS	S	NS	NS
Mukherjee [9]	NS	S	NS	NS
Rouhani and Axelsson [49]	S	NS	S	S
Smith [50]	NS	NS	NS	S
Woldesemayat and Ghajar [52]	NS	S	S	S

NS = not satisfactory, S = satisfactory

5.4. Performance of Correlations for $0.5 < \alpha \leq 0.75$

For upward vertical flow, 351 experimental data points from the entire database were categorized in the void fraction range of $0.5 < \alpha \leq 0.75$. Seven correlations that are listed in Table **5** performed satisfactorily in the $0.5 < \alpha \leq 0.75$ range (see Table **6**). Out of the 54 correlations considered in this study, correlations of Bonnecaze *et al.* [33], Kokal and Stanislav [44], and Nicklin *et al.* [47] are among the three best correlations in the $0.5 < \alpha \leq 0.75$ range, and their performances for $0 < \alpha < 1$ are also listed in Table **5**. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **7**, respectively.

For downward vertical flow, 126 experimental data points from the entire database were categorized in the void fraction range of $0.5 < \alpha \leq 0.75$. All seven correlations that are listed in Table **5** performed satisfactorily in the $0.5 < \alpha \leq 0.75$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Nicklin *et al.* [47], Rouhani and Axelsson [49], and Woldesemayat and Ghajar [52] are

among the three best correlations in the $0.5 < \alpha \leq 0.75$ range. Woldesemayat and Ghajar [52] correlation is the best correlations in the void fraction range of $0.5 < \alpha \leq 0.75$, but it is not among the correlations that performed satisfactorily for $0 < \alpha < 1$. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **8**, respectively.

Table 7: Results of three best correlations from comparison with experimental data in specific void fraction ranges for upward vertical flow.

Correlations	Percentage of data points predicted within ±10%	±15%	±20%	RMS error (%)
$0 < \alpha \leq 0.25$ (199 data points)				
Greskovich and Cooper [38]	45.2	57.8	67.8	55.1
Nicklin *et al.* [47]	37.2	54.3	70.9	53.4
Rouhani and Axelsson [49]	40.2	62.8	71.9	56.5
$0.25 < \alpha \leq 0.5$ (190 data points)				
Bonnecaze *et al.* [33]	53.2	80.5	85.3	16.9
Kokal and Stanislav [44]	53.2	81.6	84.7	16.9
Nicklin *et al.* [47]	53.2	80.5	85.3	16.8
$0.5 < \alpha \leq 0.75$ (351 data points)				
Bonnecaze *et al.* [33]	80.3	93.4	96.6	9.7
Kokal and Stanislav [44]	80.1	93.2	96.6	9.7
Nicklin *et al.* [47]	80.3	93.4	96.6	9.7
$0.75 < \alpha < 1$ (468 data points)				
Armand [31] – Massena [32]	94.7	99.4	100	5.3
Dix [35]	93.8	98.7	99.8	5.2
Rouhani and Axelsson [49]	94.0	99.4	99.6	5.6

Table 8: Results of three best correlations from comparison with experimental data in specific void fraction ranges for downward vertical flow.

Correlations	Percentage of data points predicted within ±10%	±15%	±20%	RMS error (%)
$0 < \alpha \leq 0.25$ (237 data points)				
Bonnecaze *et al.* [33]	48.5	73.4	88.2	13.5
Gomez *et al.* [37]	62.9	79.3	93.7	11.4
Nicklin *et al.* [47]	48.5	73.4	88.2	13.5
$0.25 < \alpha \leq 0.5$ (105 data points)				
Bonnecaze *et al.* [33]	75.2	91.4	96.2	10.2
Nicklin *et al.* [47]	75.2	91.4	96.2	10.2
Rouhani and Axelsson [49]	77.1	91.4	94.3	9.9
$0.5 < \alpha \leq 0.75$ (126 data points)				
Nicklin *et al.* [47]	77.0	85.7	92.1	9.3
Rouhani and Axelsson [49]	75.4	86.5	95.2	9.7
Woldesemayat and Ghajar [52]	80.2	92.1	97.6	8.2
$0.75 < \alpha < 1$ (441 data points)				
Gomez *et al.* [37]	89.3	97.7	99.3	7.0
Rouhani and Axelsson [49]	91.2	97.5	98.2	6.8
Woldesemayat and Ghajar [52]	91.4	95.2	97.3	7.4

For horizontal flow, 289 experimental data points from the entire database were categorized in the void fraction range of $0.5 < \alpha \leq 0.75$. Only two correlations that are listed in Table **5** performed satisfactorily in the $0.5 < \alpha \leq 0.75$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Rouhani and Axelsson [49], Sun *et al.* [51], and Woldesemayat and Ghajar [52] are among the three best correlations in the $0.5 < \alpha \leq 0.75$ range. Correlation of Sun *et al.* [51] is not among the correlations that performed satisfactorily for $0 < \alpha < 1$, but it is among the three best correlations in the void fraction range of $0.5 < \alpha \leq 0.75$. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **9**, respectively.

5.5. Performance of Correlations for $0.75 < \alpha < 1$

For upward vertical flow, 468 experimental data points from the entire database were categorized in the void fraction range of $0.75 < \alpha < 1$. Out of the eight correlations listed in Table **5**, only three correlations performed satisfactorily in the $0.75 < \alpha < 1$ range (see Table **6**). Out of all the correlations considered in this study, correlations of Armand [31] – Massena [32], Dix [35], and Rouhani and Axelsson [49] are among the three best correlations in the $0.75 < \alpha < 1$ range. Correlations of Armand [31] – Massena [32] and Dix [35] are among the three best correlations in the void fraction range of $0.75 < \alpha < 1$, but they are not among the correlations that performed satisfactorily for $0 < \alpha < 1$. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **7**, respectively.

For downward vertical flow, 441 experimental data points from the entire database were categorized in the void fraction range of $0.75 < \alpha < 1$. Three correlations that are listed in Table **5** performed satisfactorily in the $0.75 < \alpha < 1$ range (see Table **6**). Among the 54 correlations considered in this study, correlations of Gomez *et al.* [37], Rouhani and Axelsson [49], and Woldesemayat and Ghajar [52] are among the three best correlations in the $0.75 < \alpha < 1$ range. Woldesemayat and Ghajar [52] correlation is not among the correlations that performed satisfactorily for $0 < \alpha < 1$, but it is among the three best correlations in the void fraction range of $0.75 < \alpha < 1$. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **8**, respectively.

Table 9: Results of three best correlations from comparison with experimental data in specific void fraction ranges for horizontal flow.

Correlations	Percentage of data points predicted within ±10%	±15%	±20%	RMS error (%)
$0 < \alpha \leq 0.25$ (40 data points)				
Huq and Loth [41]	27.5	47.5	65.0	33.9
Premoli *et al.* [48]	35.0	50.0	62.5	37.4
Rouhani and Axelsson [49]	32.5	50.0	62.5	34.2
$0.25 < \alpha \leq 0.5$ (145 data points)				
Minami and Brill [29]	43.4	61.4	75.9	17.6
Mukherjee [9]	54.4	65.5	82.1	14.6
Woldesemayat and Ghajar [52]	47.6	67.6	80.7	16.2
$0.5 < \alpha \leq 0.75$ (289 data points)				
Rouhani and Axelsson [49]	70.6	84.8	91.3	11.7
Sun *et al.* [51]	69.2	87.2	90.3	12.3
Woldesemayat and Ghajar [52]	73.7	86.2	92.7	12.0
$0.75 < \alpha < 1$ (794 data points)				
Armand [31] – Massena [32]	97.0	99.9	100	4.9
Lockhart and Martinelli [45]	95.0	98.6	99.6	4.6
Premoli *et al.* [48]	97.2	98.9	99.1	4.1

For horizontal flow, 794 experimental data points from the entire database were categorized in the void fraction range of $0.75 < \alpha < 1$. Six correlations that are listed in Table **5** performed satisfactorily in the $0.75 < \alpha < 1$

range (see Table **6**). Among the 54 correlations considered in this study, correlations of Armand [31] – Massena [32], Lockhart and Martinelli [45], and Premoli *et al.* [48] are among the three best correlations in the $0.75 < \alpha < 1$ range. Correlations of Lockhart and Martinelli [45] and Premoli *et al.* [48] are among the three best correlations in the void fraction range of $0.75 < \alpha < 1$, but they are not among the correlations that performed satisfactorily for $0 < \alpha < 1$. The equations for the three aforementioned correlations and the results of their comparison with experimental data are listed in Table **2** and Table **9**, respectively.

5.6. Summary of Correlations Performances

The correlations listed in Table **5** were selected on the basis of overall performance, which overlooks the strengths and weaknesses in specific ranges of void fraction. For upward vertical flow, this overall comparison with the experimental data yielded 2 correlations with best prediction of experimental data, namely correlations of Nicklin *et al.* [47] and Rouhani and Axelsson [49]. Also, the overall comparisons showed that correlations of Gomez *et al.* [37] and Woldesemayat and Ghajar [52] have the best predictions for downward vertical and horizontal flows, respectively. The subsequent logical approach was to analyze the selected correlations in ranges, by dividing the entire void fraction range into four ranges: 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75, and 0.75 to 1. The qualitative outcomes of the correlations listed in Table **5** and their performances, whether satisfactory or not satisfactory, were summarized in Table **6**.

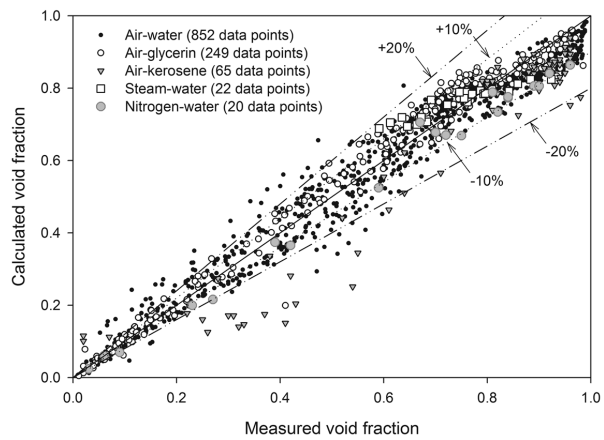


Figure 14: Comparison of the Rouhani and Axelsson [49] correlation with 1208 experimental data points for upward vertical flow.

For upward vertical flow, the comparison of the void fraction correlations with experimental data in each of the four specific ranges confirmed that the correlation of Rouhani and Axelsson [49] is the best correlation. This finding is based on the correlation's performance as one of the best for $0 < \alpha < 1$ (see Table **5**) and also the only correlation that is satisfactory for each of the four void fraction ranges (see Table 6). It is worthwhile to note that the Nicklin *et al.* [47] correlation performed best for predicting void fraction within $0.25 \leq \alpha \leq 0.75$. The correlation of Nicklin *et al.* [47] performed unsatisfactorily in $0.75 < \alpha < 1$, and this is because the correlation uses a constant distribution parameter ($C_0 = 1.2$). It is worth noting that the Nicklin *et al.* [47] correlation was originally developed for slug flow, and this analysis showed that it is the best performing correlation for $0.25 \leq \alpha \leq 0.75$, which is the void fraction range for slug and churn/froth flows (see Fig. **8**). Comparison of the correlation of Rouhani and Axelsson [49] with the experimental data is shown in Fig. **14**. The results in Fig. **14** show that the Rouhani and Axelsson [49] correlation predicted the experimental data generally well for different pipe diameters (12.7 to 76 mm), operating pressures (101 to 3413 kPa), and gas-liquid combinations.

For downward vertical flow, the comparison of the void fraction correlations with experimental data in each of the four specific ranges confirmed that the Gomez *et al.* [37] correlation is the best correlation. The correlation of Gomez *et al.* [37] has the least disparity when compared with experimental data for $0 < \alpha < 1$ (see Table **5**), and it is also the only correlation that performed satisfactorily in each four void fraction

ranges (see Table **6**). The comparison of the Gomez *et al.* [37] correlation with the experimental data is shown in Fig. **15**. The results in Fig. **15** show that the Gomez *et al.* [37] correlation predicted the experimental data generally well for different pipe diameters (9.53 to 90 mm), operating pressures (101 to 501 kPa), and gas-liquid combinations.

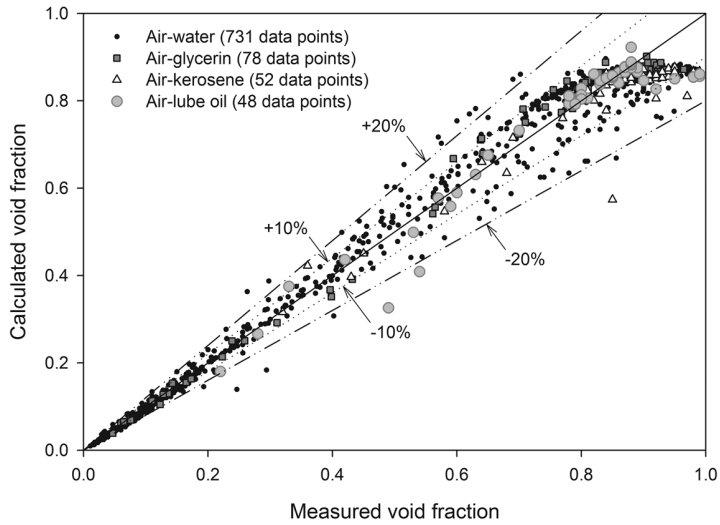


Figure 15: Comparison of the Gomez *et al.* [37] correlation with 909 experimental data points for downward vertical flow.

For horizontal flow, the correlation of Woldesemayat and Ghajar [52] has the best predictions for $0 < \alpha < 1$. The comparison of the void fraction correlations with experimental data in each of the four specific ranges showed that the Woldesemayat and Ghajar [52] correlation is the best correlation. It is the only correlation that performed satisfactorily for three different ranges of void fraction (see Table **6**), and it performed unsatisfactorily for $0 < \alpha \leq 0.25$. The number of experimental data points available for $0 < \alpha \leq 0.25$ is very limited, and a more conclusive assessment can be obtained if more number of data points are available for comparison. The comparison of the Woldesemayat and Ghajar [52] correlation with the experimental data is shown in Fig. **16**. The results in Fig. **16** show that the Woldesemayat and Ghajar [52] correlation performed satisfactorily for different pipe diameters (19 to 102 mm), operating pressures (101 to 2850 kPa), and gas-liquid combinations.

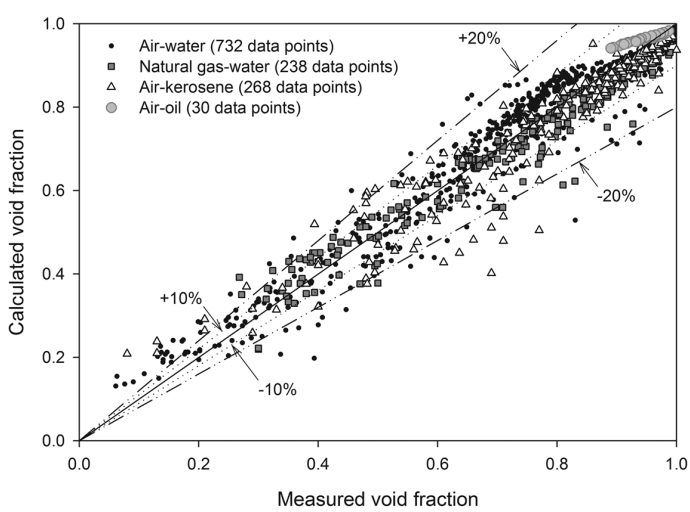


Figure 16: Comparison of the Woldesemayat and Ghajar [52] correlation with 1268 experimental data points for downward vertical flow.

6. CONCLUDING REMARKS

A very extensive comparison of most of the void fraction correlations available in the open literature was made against 3385 experimental data points for upward and downward vertical and horizontal two-phase flows. The best performing correlations considering the total number of data points predicted as well as their relative consistency in performance were highlighted. Furthermore the best correlations for four different void fraction ranges, namely 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75, and 0.75 to 1, were highlighted as well as the best flow pattern specific correlations. Results of these categorical comparisons would allow the access to correlations with higher accuracies for specific void fraction range of interest.

In addition to identifying correlations with minimum disparity when compared to experimental data, the effort of this undertaking also resulted in the compilation of an extensive experimental void fraction database. The experimental database is not only useful for validating void fraction correlations, but also for extending the understanding of the physics for two-phase flow. For instance, experimental void fraction results can be used to aid in the development of the theory-based closure relations for the wall shear and interfacial shear stresses in terms of liquid holdup and liquid and gas velocities [86]. Another example of the usefulness of experimental void fraction results is in aiding the development of two-phase flow models; such as a recently developed unified annular flow model by Thome and Cioncolini [87], which integrated together models for predicting the entrained liquid fraction, the wall shear stress and the velocity profile in the annular liquid film. Experimental void fraction results are also useful in the analysis of two-phase pressure drop.

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NOMENCLATURE

C_0	two-phase distribution parameter, dimensionless
D	pipe inside diameter, m
Fr	Froude number, dimensionless
g	gravitational acceleration, m s^{-2}
G	mass flux or mass velocity, kg m^{-2} s^{-1}
H	distance between pressure taps, m
p	pressure, Pa
Re	Reynolds number, dimensionless
u_{gu}	gas phase drift velocity, m s^{-1}
U	velocity, m s^{-1}
V	volume, m^3
We	Weber number, dimensionless
x	flow quality, dimensionless

Greek Symbols

α	void fraction, dimensionless
θ	inclination angle, rad.
μ	dynamic viscosity, kg m^{-1} s^{-1}
ρ	density, kg m^{-3}

σ	surface tension, N m^{-1}

Subscripts

atm	atmosphere
cr	critical
g	gas
h	homogeneous
l	liquid
sg	superficial gas
sl	superficial liquid
sys	system
t	total

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