Development and Implementation of Interactive/Visual Software for Steady State and Transient Heat Conduction Problems

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Abstract

This paper describes a versatile, user-friendly, and easy to understand computer program that has been developed to teach the numerical solution of steady state and transient heat conduction. The program has been class room tested for many semesters and it was very well received by the students.

Introduction

The school of Mechanical and Aerospace Engineering at Oklahoma State University offers MAE 3233, "Heat Transfer", as a required course for the Mechanical Engineering degree and an elective course for the Aerospace Engineering degree. Student use of software to analyze 1-D and 2-D steady state and transient heat conduction problems has been an important part of this course since 1994. Use of software has been particularly important in appreciating the power of numerical methods in solving engineering heat transfer problems. The software described in this paper is based on the finite difference method and can handle three types of boundary conditions (constant temperature, specified heat flux, and convection) and two types of numerical schemes (implicit and explicit). The user has access to a built in material properties library for selection of realistic material properties. The program provides tabular output, graphical output, and shaded and animated temperature plots for steady and transient cases. The primary goal of this project was to develop MS Windows based software that is effective for teaching; easy to use, maintain and update; and freely available to all.

Motivation for the Project

Before starting this project, the authors were aware of many existing software options for computer based heat transfer analysis. Unfortunately, all of them had major drawbacks for our purposes.

There are professional level programs that can perform highly detailed heat transfer and fluid flow analysis, and most are available at a substantial educational discount. These programs have three major drawbacks: they are not designed to <u>teach</u> numerical heat transfer analysis; the time required to learn to use these programs is substantial; and the cost to individual students is still fairly high.

Many heat transfer textbooks (see for example References 1 and 2) now include software aimed at teaching the concepts of numerical heat transfer analysis, but these are only available to students who purchase the textbook. Our preferred textbook did not offer such software at the time we undertook this project. In general, we prefer to choose the textbook on the basis of the content of the book itself. Any software provided by the book is an added bonus. This paper is in no way intended as a criticism of the software available with current Heat Transfer textbooks.

We offer our software as a freely available alternative that may be adopted by anyone who finds it useful.

There is software available designed to teach numerical heat transfer analysis, that is available at low or no cost to faculty and students, including software written in 1994 by the first author. Unfortunately, those programs use old DOS based programming techniques and user interface styles. Student reaction to our own "dated" software was uniformly negative.

Our goal was to develop modern; teaching oriented software for numerical analysis of heat transfer. Our intent was to make this software freely available to our own students, and to offer it freely to other heat transfer instructors and their students.

This software development project contributed in a synergistic way to three different educational missions. It provided a high quality MS level creative component project for the MS student who developed the software. The software is now being used by undergraduates in MAE 3233. The software was written in a modular and expandable way so that it can be used for future research work by MS students.

Numerical Solution of Steady State and Transient Heat Transfer Problems

The fundamental partial differential equation for analysis of heat transfer include Laplace's equation and Poisson's equation which can be applied to steady state heat transfer, and the general heat balance equation which incorporates transient effects. These equations are derived and presented in nearly all heat transfer textbooks, including References 1-3. Analytical solutions to those equations are available for a few simple but important geometries and boundary conditions.

For most non-simple geometries or boundary conditions, where analytical solutions are not available, numerical solution techniques are used. The most common numerical techniques include finite elements, finite differences, and finite volume techniques. The finite difference method appears to be the method used most often by people working in the area of heat transfer and fluid flow, and is presented in most undergraduate textbooks on heat transfer including References 1-3. The finite difference method is usually featured prominently in most numerical methods textbooks, including Reference 4.

The finite difference method and its application to heat transfer problems will not be described in detail in this paper because it is so readily available in other references. Instead, a small sampling of the geometries and equations will be shown to give a rough idea of the technique.

The essence of the finite difference method consists of replacing the pertinent differential equation and boundary conditions with a set of algebraic equations that provide an approximate solution. Also, the continuous geometric and temporal domains are approximated with a set of discrete points in time and space. Figure 1 shows a two-dimensional sub-region of space, and a single point in time, approximated by a set of uniformly spaced points called "nodes". The uniform organization and spacing of the nodes forms what is commonly called the "grid". Because the spacing is uniform, the nodes may be identified by a pair of integers (m,n) denoting location of the node within the grid. The differential equations are approximated by algebraic equations expressing heat flow relationships between neighboring nodes within the grid.
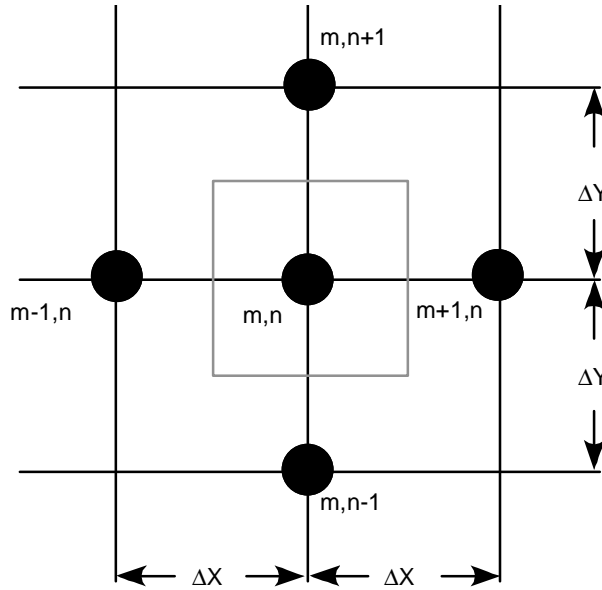
Figure 1. An interior node at location (m,n) and its neighbors.

Applying the heat balance equation around grid point (m,n) we get the following approximating algebraic equation, also known as the finite difference equation:

$$q_{m-1,n} + q_{m+1,n} + q_{m,n-1} + q_{m,n+1} + S\Delta x \Delta y = \rho c \Delta x \Delta y \frac{\partial T}{\partial t} \qquad (1)$$

Where,

$$q_{m-1,n} = k\, \Delta y \frac{(T_{m-1,n}^{p+1} - T_{m,n}^{p+1})}{\Delta x} \qquad (2)$$

k = Thermal condcutivity

ρ = Density

c = Specific heat

S = Heat generation rate per unit volume

Δ = Grid size

T = Temperature at a particular node located in space and time

m, n = Space coordinates

p = Time coordinate

Equation (1) and finite-difference equations like Equation (2) can be combined to produce:

$$k(T_{m-1,n}^{p+1} - T_{m,n}^{p+1}) + k(T_{m+1,n}^{p+1} - T_{m,n}^{p+1}) + k(T_{m,n-1}^{p+1} - T_{m,n}^{p+1}) + k(T_{m,n+1}^{p+1} - T_{m,n}^{p+1}) + S\Delta^2$$

$$= \rho c \Delta^2 \frac{T_{m,n}^{p+1} - T_{m,n}^{p}}{\Delta t} \qquad (3)$$

This finite difference equation (3) can be written at every interior point within the domain, producing a set of algebraic equations to be solved for the temperature at each node. The set of equations at the interior points must be supplemented by finite difference equations describing what is happening at the boundary of the domain. The complete set of equations generally involves the solution of N linear algebraic equations and N unknowns. The equations are solvable by any of a variety of direct or iterative techniques, presented in most numerical methods textbooks, including Reference 4.

Numerical Heat Transfer Presented at the Undergraduate Level

Undergraduate courses in heat transfer will often introduce the students to the finite difference method. It is usually assumed that students are encountering the method for the first time. It is also usually assumed that students have some experience in computer programming and some experience with methods for solving sets of linear algebraic equations.

Students are usually asked to solve a few problems by hand, over extremely simple geometric domains, having only a small number of nodes, and therefore a small number of equations. These problems give the student a good understanding of the fundamentals.

Students are then usually asked to write computer programs (or generate Excel spreadsheets) to solve larger problems, still over simple geometric domains, but having a larger number of nodes, and therefore having a larger number of simultaneous equations. A small number of these problems are usually sufficient to help the student understand the programming aspects of the finite difference method, and to explore the effects of node density on the quality of the answers produced.

With this level of understanding of finite differences, we would wish that students could use the method to gain a better understanding of heat transfer. Even with simple geometries, there are many different kinds of boundary conditions that may be applied in many different combinations. Unfortunately, each new combination of boundary conditions would require that the undergraduate students write a new computer program, and there is usually not enough time available for them to do so.

This was our fundamental motivation in starting this project: To give students who have a good undergraduate level understanding of heat transfer, and a rudimentary understanding of finite difference methods, a chance to freely explore the effects of node density and combinations of boundary conditions on heat transfer problems.

The SS-T-Conduct Program – Features and Limitations

To support the objectives given above, a program named **SS-T-Conduct** (Steady State and Transient Heat Conduction) was written. The program has the following capabilities:

a) A user friendly, education oriented program that can handle one and two-dimensional, steady state and transient heat conduction problems.

b) Users have full and easy control of the key numerical parameters (nodes and grids), material properties, and boundary conditions and parameters.

c) Users can view the effect of parameter changes on the temperature distribution instantly.

d) User can have many different ways of viewing the results, including:

- Showing the results on the screen in a tabular form.

- Printing the results to an output file.

- Showing temperature plots with time and distance for one-dimensional steady state and transient problems on the screen.

- Showing shaded temperature plots for two-dimensional steady state problems.

- Showing animation of shaded temperature plots for two-dimensional transient problems.

e) A library of material properties that is built in 'SS-T-CONDUCT.'

f) 'SS-T-CONDUCT' has undergone substantial testing and validation and is ready for use.

At this time, the program has the following limitations:

a) Only rectangular geometries, expressed in Cartesian coordinates may be modeled.

b) Only uniform grids are supported. Varying grid spacing has not yet been implemented.

c) Boundary conditions for constant temperature, constant heat flux, and constant convection are available in the program. Radiation boundary conditions have not yet been implemented. Spatially and temporally varying boundary conditions have not yet been implemented.

d) Nodal temperatures are calculated and displayed. Heat flux output is not yet available.

A Visual Tour of the SS-T-Conduct Program
     A few screen images of the program will be used to highlight the ease of use, and the capabilities and limitations of the program.
     Figure 2 shows the initial program screen. On that screen, tabs are used to allow the user to choose one of the four types of problems available: 1-D Steady State, 2-D Steady State, 1-D Transient, and 2-D Transient. On the sample screen, the 2-D Transient tab has been selected, showing the tools available for entering problem specific data. The other three problem type tabs show a similar screen layout, including an area on the left side of the screen used to set overall problem parameters, and an area to the right used to set the boundary condition type and parameters for each boundary.
     The screen also shows the menus available. The File menu allows current work to be saved and previous work to be recovered. The Properties menu allows the user to select a material from a substantial library of materials, and have those material properties entered into the appropriate locations in the program, as shown in Figure 3.

Figure 2. The initial program screen with the 2-Dimensional, Transient Tab selected.

Figure 3. The Material Selection window.

Figure 4 shows the program output screen for a 2-D transient problem. On that screen, the Tabular Output tab has been selected. The screen shows the input data summarized in the upper left, the boundary conditions summarized in the upper right, and the temperature data for every node, displayed in tabular form. Because this is a transient problem, the user may select

the point in time for displaying the temperatures. A button is available in the lower right to allow the temperature data to be saved to a text file.
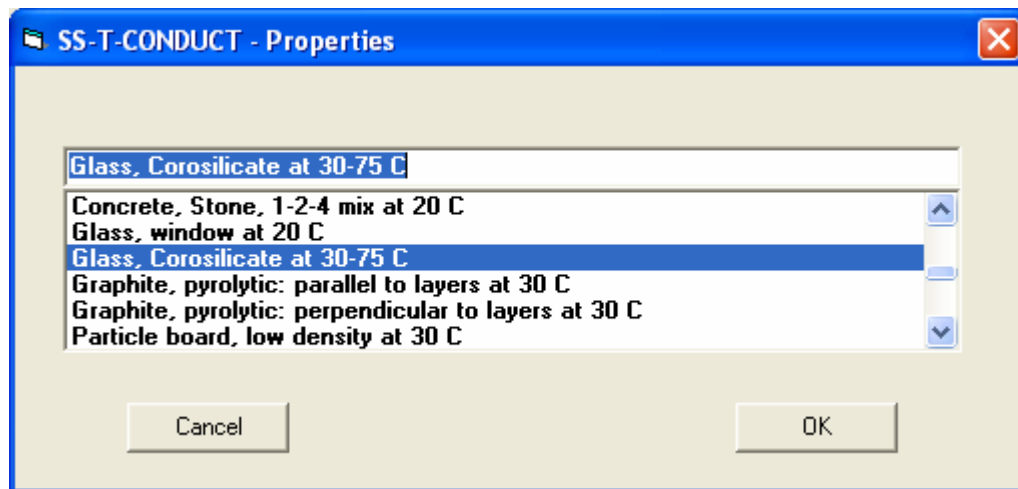
SS-T-CONDUCT OUTPUT

| Tabular Ouput | Graphical Output |

2-D Transient Problem Inputs

Grid Size (cm) = .5   Thermal Diffusivity (m^2/s) = .000002

Heat Generation (W/m^3) = 0   Time Step (s) = 2

Conductivity (W/m.K) = 3   Number Of Time Steps = 6

Initial Temperature (C) = 300

Nodes in x_Direction = 15   ○ Explicit

Nodes in y_Direction = 10   Scheme Used   ○ Implicit

Convection Environment

Constant Temperature   *Constant Temperature*

Constant Heat Flux

T (Degrees Celsuis) as a function of x (Meters) and y (Meters) at Time = 12   Sec

x (m)

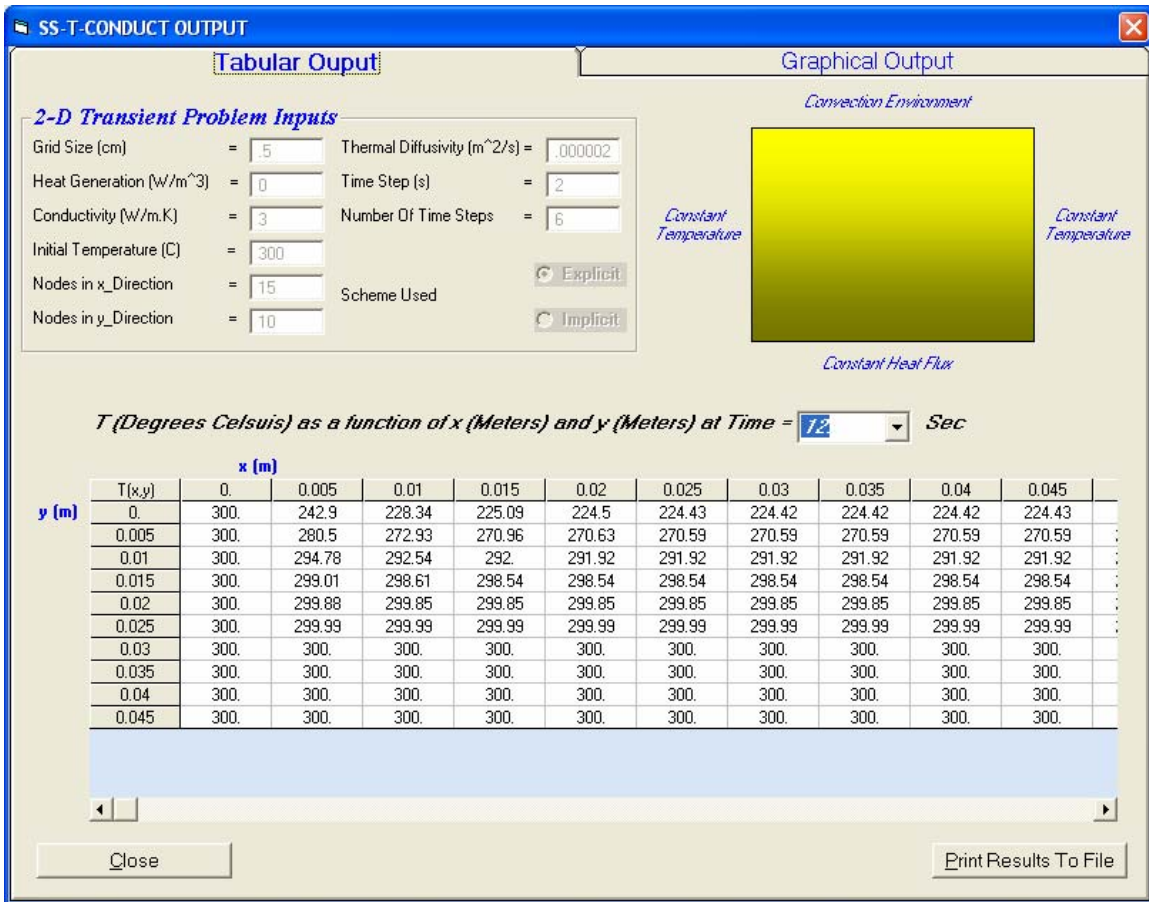| T(x,y)	0.	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04	0.045	
y (m) 0.	300.	242.9	228.34	225.09	224.5	224.43	224.42	224.42	224.42	224.43	
0.005	300.	280.5	272.93	270.96	270.63	270.59	270.59	270.59	270.59	270.59	:
0.01	300.	294.78	292.54	292.	291.92	291.92	291.92	291.92	291.92	291.92	:
0.015	300.	299.01	298.61	298.54	298.54	298.54	298.54	298.54	298.54	298.54	:
0.02	300.	299.88	299.85	299.85	299.85	299.85	299.85	299.85	299.85	299.85	:
0.025	300.	299.99	299.99	299.99	299.99	299.99	299.99	299.99	299.99	299.99	:
0.03	300.	300.	300.	300.	300.	300.	300.	300.	300.	300.	
0.035	300.	300.	300.	300.	300.	300.	300.	300.	300.	300.	
0.04	300.	300.	300.	300.	300.	300.	300.	300.	300.	300.	
0.045	300.	300.	300.	300.	300.	300.	300.	300.	300.	300.	

Close   Print Results To File

Figure 4. The output window showing tabular output.

Figure 5 shows the program output screen for a 2-D transient problem, where the Graphical Output tab has been selected. The input data is summarized on the left side of the screen. The temperature distribution is shown using a color coded temperature scale shown on the right. Because this is the output for a transient problem, the temperature output corresponds to a particular point in time. There are VCR style controls above the temperature display to allow the user to move forward and backward in time, viewing temperatures as they wish. The VCR controls also allow the entire sequence of temperature distributions to be viewed as a movie.
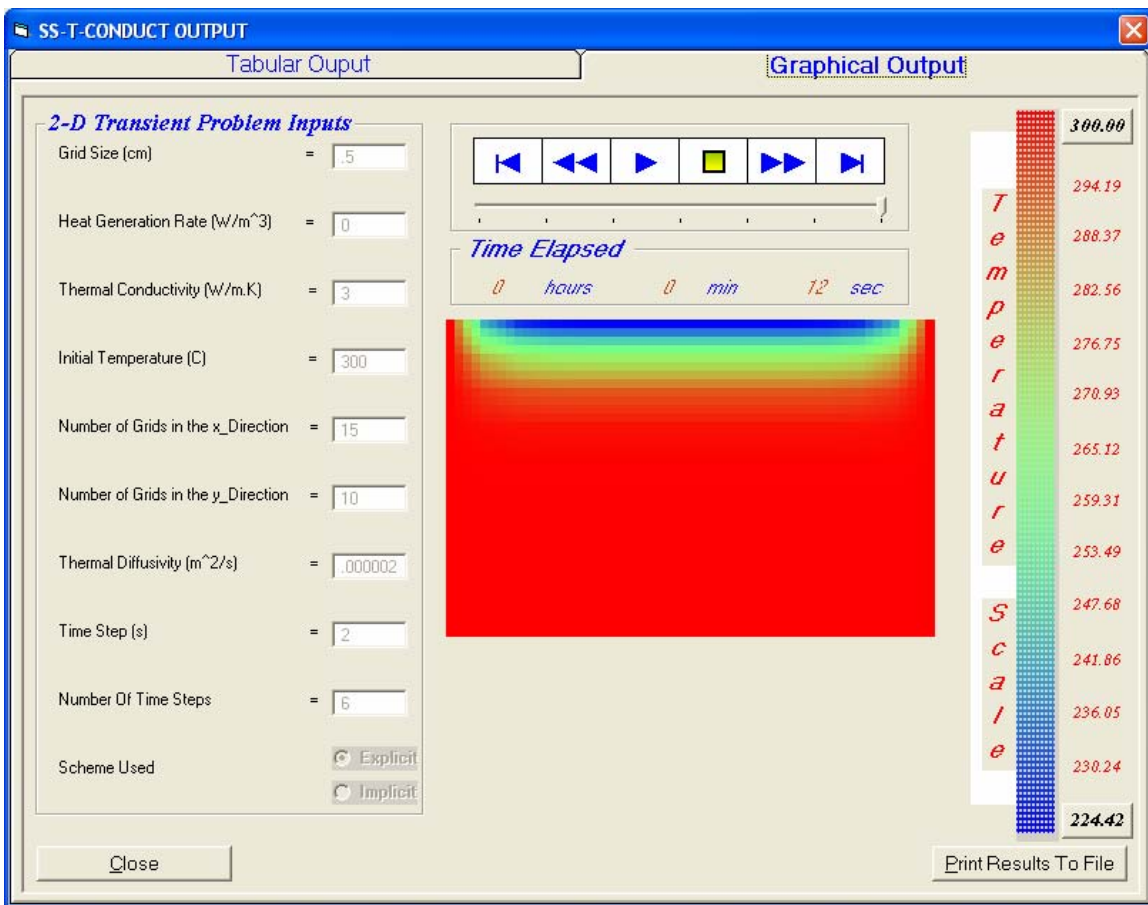

Figure 5. The output window showing graphical output.

Figures 6 and 7 show the input and output screens for a 1-Dimensional Transient problem. The output screen shows an X-Y graph of temperature vs. location for a particular point in time. The user is allowed to choose the point in time they wish to view.
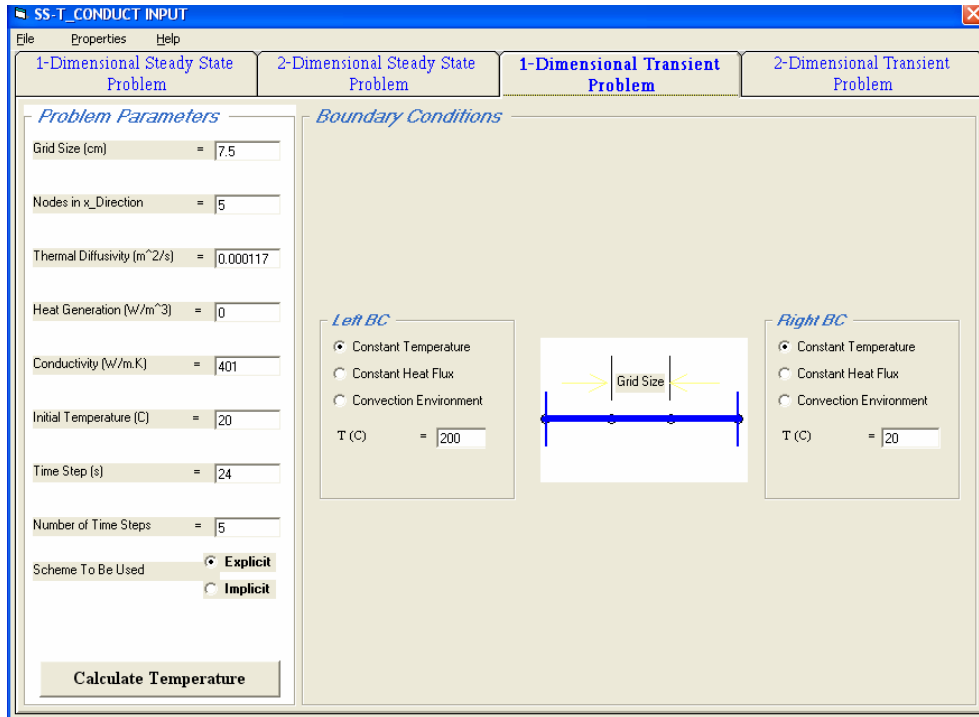
Figure 6. The input window for a 1-Dimensional Transient Problem.

Figure 7. The Graphical Output window for a 1-Dimensional Transient Problem.

Description and Evaluation of Classroom Use

The **SS-T-Conduct** (Steady State and Transient Heat Conduction) software has been used in the *Heat Transfer* course (MAE 3233) at Oklahoma State University for approximately 4 years. The MAE 3233 course is taken by both mechanical and aerospace engineering majors in their junior year. The course is required for our mechanical engineering majors and is an elective for our aerospace majors. The enrollment in the course is limited to 49 students with roughly equal distribution of mechanical and aerospace engineering majors.

In the first half of the course the software was mainly used as a homework solution tool. After first solving several problems by hand, the students then used the software to solve the same problems and additional homework problems that were calculation intensive. This gave the students a good understanding of the engineering fundamentals and equations that were used to develop the software. Many different kinds of homework problems were completed by the students with this software.

In the second half of the course, the software was used as part of a design project. The students observed the effect of changing several design variables on various heat transfer systems. Based on these results the students then explained and discussed their recommended design parameters for the set of realistic constraints they were given for the design project.

Based on the end of the semester student surveys, direct input from the students during the semester, and input from the teaching assistants assigned to the course, the software program was very useful and easy to use. The software was very well received by the students and their interest in the topic increased while using the software. The students found the software very easy to use and very helpful in greatly helping them with the understanding of the material. They were able to easily explore the effects of different design and modeling variables on heat transfer systems.

Conclusions

The **SS-T-Conduct** (Steady State and Transient Heat Conduction) software has met all the main objectives that were set initially. There are opportunities to make it even better by incorporating some of the features listed in the limitations section.

Software offer: The authors believe that this software might be of use to other universities offering similar courses. We would be glad to make the software and documentation available to other faculty, at no charge. Please contact either of the first two author's at Oklahoma State University, School of Mechanical and Aerospace Engineering.

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Biographical Information

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Hassouneh Al-Matar received his Bachelors of Mechanical Engineering from the University of Jordan, Amman, Jordan, and his M.S. Mechanical Engineering from Oklahoma State University. His interests are in the thermal sciences area.