

# Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

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## Abstract

A comparison of the performance of 68 void fraction correlations based on unbiased data set (2845 data points) covering wide range of parameters than previous assessments was made. A comprehensive literature search was undertaken for the available void fraction correlations and experimental void fraction data. After systematically refining the data, the performance of the correlations in correctly predicting the diverse data sets was evaluated. Comparisons between the correlations were made and appropriate recommendations drawn. The analysis showed that most of the correlations developed are very restricted in terms of handling a wide variety of data sets. Based on the observations made, an improved void fraction correlation which could acceptably handle all data sets regardless of flow patterns and inclination angles was suggested. It was shown that this correlation has the best predictive capability than all the correlations considered in this study.

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*Keywords:* Void fraction; Two-phase; Comparison; Correlations; Horizontal flow; Upward inclined flow

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## 1. Introduction

Two-phase flow, theoretically, is the simultaneous flow of two of any of the three discrete phases (solid, liquid or gas) of any substance or combination of substances. Practical applications of a gas–liquid flow, of a single substance or two different components, are commonly encountered in the petroleum, nuclear and process industries. The two phases may be of different components and/or there could be a phase change due to evaporation and condensation of a single fluid.

In the industrial applications where two-phase flow exists, the task of sizing the equipment for gathering, pumping, transporting and storing such a two-phase mixture requires the formidable task of predicting the phase distribution in the system from given operating conditions. Once this phase distribution is known, the problem may be simplified in such a way that it could be approached and tackled in a similar fashion analogous to single-phase flow.

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One of the critical unknown parameters involved in predicting the pressure loss and heat transfer (see for example Kim and Ghajar, 2006) in any gas–liquid system is the void fraction ( $\varepsilon$ ) or liquid holdup ( $1 - \varepsilon$ ) which is the volume of space occupied by the gas or liquid, respectively. The seemingly benign issue of determining the phase distribution from input conditions in a given pipe is complicated as a result of the slippage between the gas and the liquid phases. Owing to the complexity and lack of understanding of the basic underlying physics of the problem, the majority of the analyses were more inclined towards empirical correlations. For the sake of simplicity both the theoretical models as well as the empirical correlations that are collected from the literature and discussed here would be simply referred to as void fraction correlations.

The design engineer is faced with the difficult task of choosing the “right” correlation among the plethora of correlations available. The fact that there are plenty of correlations available would not be a concern had it not been for the fact that most of the correlations have some form of restrictions attached to them. The idea of combining different correlations which claim to do well for the specific flow characteristics for which they are developed in trying to take care of all the design need would entail its own difficulty. For instance, one of the most common restrictions to the correlations, flow pattern dependency, is sometimes a purely subjective judgment of the investigator especially for those points on or near flow pattern boundaries. Moreover, one is bound to run into some discontinuity when switching from one correlation to the other for different practical operating conditions. The other pitfall is that the majority of the void fraction data and also the correlations are primarily developed for mostly horizontal orientation though some vertical flows and inclined cases have been investigated.

The purpose of this work is to find a void fraction correlation that could acceptably predict most of the experimental data collected for all inclination angles as well as the different flow patterns without resorting back to very complex expressions that require iterative schemes with multiple solutions. A comprehensive search is made in the open literature to collect all the available void fraction correlations as well as the measured void fraction data from various sources. Comparisons of predictive performance and promise of the capability of the collected void fraction correlations was made using the collected data from the literature.

Based on the analysis, the best performing correlations were selected and recommendations were drawn on their strengths and weaknesses. A detailed study of the performance of several of the top performing correlations together with the data points which they could not handle well was made. Introducing correction factors which take into account the physical nature of the data sets, an improved version to one of these correlations was suggested that could handle data points which failed to be captured by the original correlation.

## 2. Correlations

Sixty eight correlations have been considered in this analysis for which we have complete information to calculate (predict) void fraction based on input parameters in our database, see Table 1. In presenting the correlations, different categorizing criteria were sought into which the developed correlations might conveniently fall. Rather than going after physical parameters which most of the time are too narrow, we have followed the work of Vijayan et al. (2000) to classify the correlations into four categories as shown in Table 1. These are:

*Slip ratio correlations:* A general expression for this type of correlation was put forward by Butterworth (1975) as a function of the ratios between wetness fraction ( $1 - x$ ) and  $x$  the “quality” or “dryness fraction”, where  $x$  is defined as the ratio of gas flow rate to the total flow rate; the ratios of densities of the gas and liquid phase ( $\rho_G$  and  $\rho_L$ ); and the ratios of the viscosities of the liquid and gas phase ( $\mu_L$  and  $\mu_G$ ). Most of the correlations in this category have a fixed constant term in the expression for the void fraction. However, the constant term in the expressions of Smith (1969),  $A_{SM}$ , and Premoli et al. (1970),  $A_{PRM}$ , are much more complex as shown in Table 1. For example the constant term  $A_{PRM}$  in Premoli et al.’s correlation is a complex function of the total mass flux ( $G$ ), surface tension ( $\sigma$ ), and acceleration of gravity ( $g$ ).

*$K\varepsilon_H$  correlations:* These correlations are a constant or some functional multiple of the no-slip or homogeneous void fraction,  $\varepsilon_H$ . Bankoff (1960) named the constant,  $K$ , a flow parameter which was shown to be a function of the pressure ( $P$ ). Hughmark (1962) improved this parameter introducing the Froude number ( $Fr$ ), based on the two-phase mixture velocity ( $U_M$ ). The liquid input content ( $\lambda$ ) and Reynolds number ( $Re$ ) were also incorporated into the flow parameter expression. Kowalczewski (Isbin and Biddle, 1979)

Table 1  
Void fraction correlations considered for this study

Author/source	Void fraction correlation
<i>Slip ratio correlations</i>	
Homogeneous	
Lockhart and Martinelli (1949)	$\varepsilon = \left[ 1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right) \left( \frac{\mu_L}{\mu_G} \right) \right]^{-1}$
Fauske (1961)	$\varepsilon = \left[ 1 + 0.28 \left( \frac{1-x}{x} \right)^{0.64} \left( \frac{\rho_G}{\rho_L} \right)^{0.36} \left( \frac{\mu_L}{\mu_G} \right)^{0.07} \right]^{-1}$
Fujie (1964)	$\varepsilon = \left[ 1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right)^{0.5} \right]^{-1}$
Thom (1964)	$\varepsilon = \left[ 1 + \left( \sqrt{(68947.57/P)\varepsilon + 1} \right) \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right) \right]^{-1}$
Zivi (1964)	$\varepsilon = \left[ 1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right)^{0.89} \left( \frac{\mu_L}{\mu_G} \right)^{0.18} \right]^{-1}$
Turner and Wallis (1965)	$\varepsilon = \left[ 1 + \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right)^{0.72} \left( \frac{\rho_G}{\rho_L} \right)^{0.4} \left( \frac{\mu_L}{\mu_G} \right)^{0.08} \right]^{-1}$
Baroczy (1966)	$\varepsilon = \left[ 1 + \left( \frac{1-x}{x} \right)^{0.74} \left( \frac{\rho_G}{\rho_L} \right)^{0.65} \left( \frac{\mu_L}{\mu_G} \right)^{0.13} \right]^{-1}$
Smith (1969)	$\varepsilon = \left[ 1 + A_{SM} \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right) \right]^{-1}$ where $A_{SM} = 0.4 + 0.6 \sqrt{\frac{\rho_L}{\rho_G} + 0.4 \left( \frac{1-x}{x} \right)} / \left[ 1 + 0.4 \left( \frac{1-x}{x} \right) \right]$
Premoli et al. (1970)	$\varepsilon = \left[ 1 + A_{PRM} \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right) \right]^{-1}$ where $A_{PRM} = 1 + F_1 \left\{ \frac{y}{1+yF_2} - yF_2 \right\}$ , $F_1 = 1.578 Re_L^{-0.19} \left( \frac{\rho_L}{\rho_G} \right)^{0.22}$ $F_2 = 0.0273 We_L Re_L^{-0.51} \left( \frac{\rho_L}{\rho_G} \right)^{-0.08}$ , $y = \left[ \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right) \right]^{-1}$ , $We_L = \frac{G^2 D}{\sigma \rho_L}$ , $Re_L = \frac{GD}{\mu_L}$
Chisholm (1973)	$\varepsilon = \left[ 1 + \sqrt{1 - x \left( 1 - \frac{\rho_L}{\rho_G} \right) \left( \frac{1-x}{x} \right) \left( \frac{\rho_G}{\rho_L} \right)} \right]^{-1}$
Madsen (1975)	$\varepsilon = \left[ 1 + \left( \frac{1-x}{x} \right)^b \left( \frac{\rho_G}{\rho_L} \right)^{-0.5} \right]^{-1}$ where $b = 1 + \log \left( \frac{\rho_L}{\rho_G} \right) \left( \log \left( \frac{1-x}{x} \right) \right)^{-1}$
Spedding and Chen (1984)	$\varepsilon = \left[ 1 + 2.22 \left( \frac{1-x}{x} \right)^{0.65} \left( \frac{\rho_G}{\rho_L} \right)^{0.65} \right]^{-1}$
Chen (1986)	$\varepsilon = \left[ 1 + 0.18 \left( \frac{1-x}{x} \right)^{0.6} \left( \frac{\rho_G}{\rho_L} \right)^{0.33} \left( \frac{\mu_L}{\mu_G} \right)^{0.07} \right]^{-1}$
Hamersma and Hart (1987)	$\varepsilon = \left[ 1 + 0.26 \left( \frac{1-x}{x} \right)^{0.67} \left( \frac{\rho_G}{\rho_L} \right)^{0.33} \right]^{-1}$
Petalaz and Aziz (1997)	$\varepsilon = \left[ 1 + 0.735 \left( \mu_L^2 U_{SG}^2 / \sigma^2 \right)^{0.074} \left( \frac{1-x}{x} \right)^{-0.2} \left( \frac{\rho_G}{\rho_L} \right)^{-0.126} \right]^{-1}$
Zhao et al. (2000)	$\varepsilon = \left[ 1 + \varepsilon^{-0.125} \left( \frac{1-x}{x} \right)^{0.875} \left( \frac{\rho_G}{\rho_L} \right)^{0.875} \left( \frac{\mu_L}{\mu_G} \right)^{0.875} \right]^{-1}$
<i>K<sub>εH</sub> correlations</i>	
Armand (1946)	$\varepsilon = 0.833 \varepsilon_H$
Chisholm (1983), Armand (1946)	$\varepsilon = \frac{1}{\varepsilon_H + (1 - \varepsilon_H)^{0.5}} \varepsilon_H$
Bankoff (1960)	$\varepsilon = [0.71 + (1.45 \times 10^{-2})P] \varepsilon_H$ where $P$ in MPa
Hughmark (1962)	$\varepsilon = \left[ \frac{Re^{1/6} Fr^{1/8}}{\lambda^{1/4}} \right] \varepsilon_H$ where $Re = \frac{GD}{(1-\varepsilon)\mu_L + \varepsilon\mu_G}$ , $Fr = \frac{U_M^2}{gD}$ , $\lambda = \frac{U_{SL} - U_{SG}}{U_{SL} + U_{SG}}$
Nishino and Yamazaki (1963)	$\varepsilon = 1 - \left[ \frac{1-x}{x} \frac{\rho_G}{\rho_L} \right]^{0.5} (\varepsilon)^{0.5}$
Kowalczewski (1964) <sup>a</sup>	$\varepsilon = \varepsilon_H - 0.7(1 - \varepsilon_H)^{0.5} Fr^{-0.045} \left( 1 - \frac{P}{P_c} \right)$
Guzhov et al. (1967)	$\varepsilon = 0.81 \varepsilon_H (1 - \exp(-2.2\sqrt{Fr}))$

(continued on next page)

Table 1 (continued)

Author/source	Void fraction correlation
Loscher and Reinhardt (1973) <sup>b</sup>	$\varepsilon = \varepsilon_H - 0.7(1 - \varepsilon_H)^{0.5} Fr^{-0.045} \left(1 - \frac{P}{P_c}\right)$
Greskovich and Cooper (1975)	$\varepsilon = \left[1 + 0.671 \left(\frac{\sin \theta}{Fr^{0.5}}\right)^{0.263}\right]^{-1} \varepsilon_H$
Moussali <sup>a</sup>	$\varepsilon = 1 - \frac{(30.4U_{SG}/U_{SL}) + 11}{60(1 + 1.6U_{SG}/U_{SL})(1 + 3.2U_{SG}/U_{SL})}$
Kutucuglu <sup>a</sup>	$\varepsilon = \varepsilon_H - (1 - \varepsilon_H)^{0.5} Fr^{0.2} \left(1 - \frac{P}{P_c}\right)^2$
Czop et al. (1994)	$\varepsilon = -0.285 + 1.097\varepsilon_H$
Armand–Massina <sup>c</sup>	$\varepsilon = (0.833 + 0.167x)\varepsilon_H$
<i>Drift flux correlations</i>	
Filimonov et al. (1957)	$\varepsilon = U_{SG}/(U_M + U_{GM})$ where $U_{GM} = (0.65 - 0.0385P) \left(\frac{D_H}{0.063}\right)^{0.25}$ For $P < 12.7$ $U_{GM} = (0.33 - 0.00133P) \left(\frac{D_H}{0.063}\right)^{0.25}$ For $P \geq 12.7$
Dimentiev et al. <sup>d</sup>	$\varepsilon = 1.07j_g^{0.8} D_H^{*-0.25} \left(\frac{\rho_G}{\rho_L - \rho_G}\right)^{-0.23}$ For $j_g \left(\frac{\rho_G}{\rho_L - \rho_G}\right)^{-0.5} \leq 3.7$ or $\varepsilon = 1.9j_g^{0.34} D_H^{*-0.25} \left(\frac{\rho_G}{\rho_L - \rho_G}\right)^{0.09}$ For $j_g \left(\frac{\rho_G}{\rho_L - \rho_G}\right)^{-0.5} > 3.7$
Wilson et al. (1961)	$\varepsilon = 0.56157F^{0.62086} \left(\frac{\rho_G}{\rho_L - \rho_G}\right)^{0.0917} \left(\frac{L}{D}\right)^{0.11033}$ For $F < 2$ $\varepsilon = 0.68728F^{0.41541} \left(\frac{\rho_G}{\rho_L - \rho_G}\right)^{0.10737} \left(\frac{L}{D}\right)^{0.11033}$ For $F \geq 2$
Nicklin et al. (1962)	$\varepsilon = U_{SG}/(1.2U_M + 0.35\sqrt{gD})$
Hughmark (1965)	$\varepsilon = U_{SG}/1.2U_M$
Gregory and Scott (1969)	$\varepsilon = U_{SG}/1.19U_M$
Rouhani and Axelsson (1970)	$\varepsilon = x/\rho_G \left[ C_0 \left( \frac{x}{\rho_G} + \frac{1-x}{\rho_L} \right) + \frac{U_{GM}}{G} \right]^{-1}$ where $U_{GM} = \left( \frac{1.18}{\sqrt{\rho_L}} \right) (g\sigma(\rho_L - \rho_G))^{0.25}$ $C_0 = 1 + 0.2(1 - x)$ $C_0 = 1 + 0.2(1 - x)(gD)^{0.25} \left(\frac{\rho_L}{G}\right)^{0.5}$
Rouhani I	
Rouhani II	
Bonnecaze et al. (1971)	$\varepsilon = U_{SG} / \left( 1.2U_M + 0.35\sqrt{gD} \left( 1 - \frac{\rho_G}{\rho_L} \right) \right)$
Mattar and Gregory (1974)	$\varepsilon = U_{SG}/(1.3U_M + 0.7)$
Kokal and Stanislav (1989)	$\varepsilon = U_{SG} / \left( 1.2U_M + 0.345 \sqrt{\frac{gD(\rho_L - \rho_G)}{\rho_L}} \right)$
Dix <sup>c</sup>	$\varepsilon = U_{SG} \left\{ U_{SG} \left( 1 + \left( \frac{U_{SL}}{U_{SG}} \right) \left( \frac{\rho_G}{\rho_L} \right)^{0.1} \right) + 2.9 \left( \frac{g\sigma[\rho_L - \rho_G]}{\rho_L^2} \right)^{0.25} \right\}^{-1}$
Toshiba <sup>c</sup>	$\varepsilon = U_{SG}/(1.08U_M + 0.45)$
Sun et al. (1980)	$\varepsilon = U_{SG} \left\{ \left[ 0.82 + 0.18 \frac{P}{P_c} \right]^{-1} U_M + 1.41 \left( \frac{g\sigma[\rho_L - \rho_G]}{\rho_L^2} \right)^{0.25} \right\}^{-1}$
Jowitt <sup>c</sup>	$\varepsilon = U_{SG} / \left\{ \left[ 1 + 0.796 \exp \left( -0.061 \sqrt{\frac{\rho_L}{\rho_G}} \right) \right] U_M + 0.034 \left( \sqrt{\frac{\rho_L}{\rho_G}} - 1 \right) \right\}$
Bestion <sup>c</sup>	$\varepsilon = U_{SG} / \left[ U_M + 0.188 \left( \frac{gD[\rho_L - \rho_G]}{\rho_G} \right)^{0.5} \right]$
Inoue et al. (1993)	$\varepsilon = U_{SG}/[(0.00676P + 1.026)U_M + (0.0051m + 0.0691)(0.0942P^2 - 1.99P + 12.6)]$
Maier and Coddington (1997)	$\varepsilon = U_{SG}/[(C_{MC1} P + C_{MC2})U_M + \{(v_1 P^2 + v_2 P + v_3)G + (v_4 P^2 + v_5 P + v_6)\}]$ where $C_{MC1} = 0.00257$ , $C_{MC2} = 1.0062$ , $v_1 = 6.73 \times 10^{-7}$ , $v_2 = -8.81 \times 10^{-5}$ , $v_3 = 0.00105$ , $v_4 = 0.00563$ , $v_5 = 0.123$ , $v_6 = 0.8$
<i>General correlations</i>	
Sterman (1956)	$\varepsilon = 0.2 \left( \frac{U_{SG}^4 \rho_G^{0.6} (\rho_L - \rho_G)^{0.4}}{g\sigma} \right)^{0.2} \left( \frac{d_1}{D} \right)^{0.25}$ If $\frac{d_1}{D} > 1 \Rightarrow \frac{d_1}{D} = 1$ where $d_1 = 260 \frac{\rho_G^{0.2}}{(\rho_L - \rho_G)^{0.7}} \left( \frac{\sigma}{g} \right)^{0.5}$
Flanigan (1958)	$\varepsilon = [1 + 3.063U_{SG}^{-1.006}]^{-1}$
Chisholm and Laird (1958)	$\varepsilon = 1 + \left[ 0.8 / \left( 1 + \frac{21}{X_u} + \frac{1}{X_u^2} \right) \right]^{1.75}$

Table 1 (continued)

Author/source	Void fraction correlation
Hoogendoorn (1959)	$\frac{\varepsilon}{1-\varepsilon} = 0.6 \left[ U_{SG} \left( 1 - \frac{\varepsilon}{1-\varepsilon} \frac{U_{SL}}{U_{SG}} \right) \right]^{0.85}$
Wallis (1969)	$\varepsilon = [1 + X_H^{0.8}]^{-0.38}$
Neal and Bankoff (1965)	$\varepsilon = 1.25 \left( \frac{U_{SG}}{U_M} \right)^{1.88} \left( \frac{U_{SL}^2}{gD} \right)^{0.2}$
Beggs (1972)	$\frac{\varepsilon(\theta)}{\varepsilon(0)} = 1 + C \left[ \sin(1.8\theta) - \frac{1}{3} \sin^3(1.8\theta) \right]$
Mukherjee (1979)	$\varepsilon = 1 - \exp \left( C_1 + C_2 \sin\theta + C_3 \sin^2\theta + C_4 \frac{\mu_L}{(\rho_L \sigma^3)^{0.25}} \right) \left[ U_{SG} \left( \frac{\rho_L}{g\sigma} \right)^{0.25} \right]^{C_5} \left[ U_{SL} \left( \frac{\rho_L}{g\sigma} \right)^{0.25} \right]^{-C_6}$
Gardner (1980) – 1	$\frac{\varepsilon}{(1-\varepsilon)^{0.5}} = 1.7 \left\{ \frac{U_{SG} \rho_L^{0.5}}{[(\rho_L - \rho_G) g \sigma]^{0.25}} P_y^{0.16} \right\}^{2/3}$
Gardner (1980) – 2	$\frac{\varepsilon}{(1-\varepsilon)^{0.5}} = 11.2 \left\{ \frac{U_{SG} \rho_L^{0.5}}{[(\rho_L - \rho_G) g \sigma]^{0.25}} P_y^{0.3} \right\}^{2/3}$
Tandon et al. (1985)	For $50 < Re_L < 1125$ $\varepsilon = 1 - 1.928 Re_L^{-0.315} \left( 0.15 \left( \frac{1}{X_H} + \frac{2.85}{X_H^{0.476}} \right) \right)^{-1} + 0.9293 Re_L^{-0.63} \left( 0.15 \left( \frac{1}{X_H} + \frac{2.85}{X_H^{0.476}} \right) \right)^{-2}$ For $Re_L > 1125$ $\varepsilon = 1 - 38 Re_L^{-0.088} \left( 0.15 \left( \frac{1}{X_H} + \frac{2.85}{X_H^{0.476}} \right) \right)^{-1} + 0.0361 Re_L^{-0.176} \left( 0.15 \left( \frac{1}{X_H} + \frac{2.85}{X_H^{0.476}} \right) \right)^{-2}$
El-Boher et al. (1988)	$\varepsilon = \left[ 1 + 0.27 \varepsilon_H^{-0.69} (Fr)^{-0.177} \left( \frac{\mu_L}{\mu_G} \right)^{0.378} \left( \frac{Re}{We_L} \right)^{0.067} \right]^{-1}$ where $Fr = \frac{U_{SL}^2}{gD}$ , $Re = \frac{\rho_G U_{SL} D}{\mu_L}$ , $We = \frac{\rho_L U_{SL}^2 D}{\sigma}$
Minami and Brill (1987)	$\varepsilon = \exp \left\{ - \left[ \frac{\ln Z_1 + 9.21}{8.7115} \right]^{4.3374} \right\}$ where $Z_1 = \frac{1.84 L^{0.575}}{U_{SG} D^{0.0277}} \left( \frac{\rho_L^{0.5804}}{g^{0.3696} \sigma^{0.1804}} \right)^{-0.25} \left( \frac{p}{101325} \right)^{0.05} \mu_L^{0.1}$
Kawaji et al. (1987)	$\varepsilon = 1.05 \left( \frac{\rho_L^{0.5} U_{SL}}{[gD(\rho_L - \rho_G)]^{0.5}} \right)^{0.5}$
Spedding and Spence (1989)	$\frac{\varepsilon}{1-\varepsilon} = [0.45 + 0.08 \exp(-100(0.25 - U_{SL}^2))] \left( \frac{U_{SG}}{U_{SL}} \right)^{0.65}$
Hart et al. (1989)	$\varepsilon = \left\{ 1 + \frac{U_{SL}}{U_{SG}} \left[ 1 + \left( 108 \frac{\rho_L}{\rho_G} Re_{SL}^{-0.726} \right)^{0.5} \right] \right\}^{-1}$
Huq and Loth (1992)	$\varepsilon = 1 - \frac{2(1-x)^2}{1-2x + \left[ 1+4x(1-x) \left( \frac{\rho_L}{\rho_G} - 1 \right) \right]^{0.5}}$
Abdulmajeed (1996)	$\varepsilon = 1 - 0.528 (U_{SG} U_{SL})^{-0.216121} (E_L)_{theo}$ For turbulent flow $(E_L)_{theo} = \exp(-0.9304919 + 0.5285852R - 9.219634 \times 10^{-2} R^2 + 9.02418 \times 10^{-4} R^4)$ For laminar flow $(E_L)_{theo} = \exp(-1.1 + 0.6788495R - 0.1232191 \times 10^{-2} R^2 - 1.778653 \times 10^{-3} R^3 + 1.626819 \times 10^{-3} R^4)$ where $R = \ln X$ and $X = \left[ \frac{U_{SG} \rho_G \mu_L}{U_{SL} \rho_L \mu_G} \right]^L \frac{\rho_L U_{SL}^2}{\rho_G U_{SG}^2}$ with $L = 0.2$ for turbulent flow and $L = 1$ for laminar flow
Gomez et al. (2000)	$\varepsilon = 1 - \exp - (0.45\theta + 2.48 \times 10^{-6} Re_M)$
Graham et al. (2001)	$\varepsilon = \left( 1 + \frac{1}{Fr} + \frac{1}{X_H} \right)^{-0.321}$ where $Fr = \left( \frac{G^2 x^3}{(1-x) \rho_G g D} \right)^{0.5}$

<sup>a</sup> Isbin and Biddle (1979).

<sup>b</sup> Friedel (1977).

<sup>c</sup> Leung (2005).

<sup>d</sup> Kataoka and Ishii (1987).

<sup>e</sup> Coddington and Macian (2002).

and Loscher and Reinhardt (Friedel, 1977) introduced a pressure ratio correction based on thermodynamic critical pressure ( $P_c$ ) for single component flows.

*Drift flux correlations:* Zuber and Findlay (1965) developed an expression for predicting void fraction taking into consideration the non uniformity in the flow captured by a distribution parameter ( $C_0$ ), and the drift velocity ( $U_{GM}$ ), defined as the difference between the gas phase velocity ( $U_G$ ) and the two-phase mixture velocity ( $U_M$ ). The superficial gas velocity ( $U_{SG}$ ) has an equally important part in the expression. All the correlations that fall into this category differ in their terms for the distribution coefficient and drift velocity or both. Filimonov et al. (1957) considered a hydraulic diameter ( $D_H$ ), Dimentiev et al. (Kataoka and Ishii, 1987) defined dimensionless hydraulic diameter ( $D_H^*$ ) and gas flux ( $j_g$ ) as  $D_H^* = D_H \left( \frac{\sigma}{g(\rho_L - \rho_G)} \right)^{-0.5}$  and  $j_g = U_{SG} / \left[ \frac{\rho_L - \rho_G}{g\sigma} \right]^{0.25}$ , respectively while Inoue et al. (1993) added total mass dependency ( $m$ ) in their drift velocity terms. Wilson et al. (1961) defined a modified Froude number,  $F = U_{SG} \left[ \frac{\rho_L - \rho_G}{g\sigma} \right]^{0.25}$ , a Laplace length parameter,  $L = \left[ \frac{\sigma}{g(\rho_L - \rho_G)} \right]^{0.5}$  and the pipe diameter,  $D$ , in their correlation. Similarly, Maier and Coddington (1997) curve fitted their data with eight constants,  $C_{MC1}$ ,  $C_{MC2}$ ,  $v_1$  to  $v_6$  which were introduced in their drift flux type correlation.

*General void fraction correlations:* These are mostly empirical in nature with the basic underlying physical principles incorporated into the different physical parameters when developing them. Chisholm and Laird (1958), Wallis (1969), Tandon et al. (1985) and Graham et al. (2001) used the Lockhart and Martinelli (1949) parameter,  $X_{tt} = \left( \frac{\mu_L}{\mu_G} \right)^{0.1} \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{\rho_G}{\rho_L} \right)^{0.5}$ , in their correlations. El-Boher et al. (1988) defined the Froude number ( $Fr$ ) and Weber number ( $We$ ) based on the superficial liquid velocity ( $U_{SL}$ ), while Graham et al. (2001) included a Froude rate parameter ( $Fr$ ). Beggs (1972) introduced an inclination correction factor ( $C$ ) to predict void fraction at any pipe inclination angle ( $\theta$ ) as a function of the horizontal void fraction. Similarly, Mukherjee (1979) curve fitted his data with six constants ( $C_1$  to  $C_6$ ) for horizontal and upward inclinations while Minami and Brill (1987) correlated their data with a parameter,  $Z_1$ , for predicting void fraction in horizontal pipes. Abdulmajeed (1996) defined a parameter,  $L$ , with different values for laminar and turbulent flow to modify the Lockhart and Martinelli (1949) parameter. Then, a variable  $R$  was introduced as a logarithmic function of the Lockhart and Martinelli (1949) parameter. The data was fitted with this variable to calculate the theoretical liquid holdup,  $(E_L)_{theo}$ , first and subsequently corrected to reach at the true liquid holdup. All the correlations in the four categories have been presented in Table 1.

### 3. Previous comparison work

It is to be noted that different investigators in the different industries in which the void fraction is an important factor to be determined have done various comparisons with selected void fraction correlations commonly used in that specific industry while some other have tried to consider a broader range of correlations that are used in different applications.

#### 3.1. Dukler et al. (1964) horizontal pipe comparison

The first void fraction correlation comparison work was done by Dukler et al. (1964). It consists of 706 refined void fraction data points of Hoogendoorn (1959) obtained from tests run in 1, 2, 3.5 and 5.5 inch diameter horizontal pipes with liquid viscosities of 3 and 20 cp. The void fraction correlations considered were Hoogendoorn (1959), Hughmark (1962) and Lockhart and Martinelli (1949). Using statistical tools such as arithmetic mean deviation, standard deviation and defining a new variable to account for the fractional deviation which includes 68% of the population to measure the spread of data, it was shown that the Hughmark (1962) correlation was able to perform better than the other two.

#### 3.2. Marcano (1973) horizontal pipe comparison

Marcano compared five correlations; Lockhart and Martinelli (1949), Hughmark (1962), Dukler et al. (1969), Eaton et al. (1967), Guzhov et al. (1967) and Beggs (1972) correlations using the data of Eaton (1966) and Beggs (1972). The total data points consisted of 238 natural gas–water (Eaton) and 58 air–water

(Beggs). The statistical parameters considered in the Dukler et al. (1969) comparison were calculated here to determine the relative accuracy of the correlations. It was shown that the Eaton et al. (1967) and Beggs (1972) correlations performed well owing to the fact that the data used for comparison is the data from which these correlations were developed. The correlations of Dukler et al. (1969) and Lockhart and Martinelli (1949) were reported to have acceptable results while the rest were totally unsatisfactory.

Dropping out the “unreliable” low liquid holdup data (below 0.1), the correlations were further compared by Marcano (1973). It was noted that the performance of the correlations was better. The performance of the correlations for specific liquid holdup (void fraction) ranges were also analyzed where Eaton et al. (1967), Guzhov et al. (1967) and Beggs (1972) correlations were found to do better for void fractions less than 0.65 (or liquid hold-up above 0.35) while on the lower range of liquid holdup only the Eaton et al. (1967) correlation gave acceptable results. For liquid holdup below 0.1 none of the correlations gave reasonable accuracy though the Dukler et al. (1969) correlation and the no slip model gave best estimates. One of the many reasons for this could be the unreliability of the measurement of the data for this range of liquid holdup.

### 3.3. Palmer (1975) inclined pipe comparison

Using 174 liquid holdup (void fraction) water–natural gas experimental data from a 51 mm diameter pipe line with three uphill (at 4.2°, 7.1° and 7.5° from the horizontal) and three downhill (at 4.3°, 3.8° and 6.3° from the horizontal) test sections, Palmer (1975) compared the correlations of Beggs (1972), Flanigan (1958) and Guzhov et al. (1967). The percent error, average percent error and the standard deviation were calculated to make the comparison. It was concluded that the Beggs (1972) correlation gave good predictions of the void fraction for uphill flow. The Flanigan (1958) correlation was the least accurate which may be expected as it did not consider downhill flow and also the fact that it was given as an approximate estimate by Flanigan.

### 3.4. Mandhane et al. (1975) horizontal pipe comparison

Mandhane et al. used 2700 void fraction (liquid holdup) data contained in the University of Calgary Multiphase Pipe Flow Data Bank. A two step procedure for calculating the void fraction was used in that the flow pattern is first predicted and then a correlation developed for that flow regime is used to calculate the liquid holdup. The flow pattern map of Mandhane et al. (1974) was used for the analysis. Twelve void fraction correlations were considered. They were, Lockhart and Martinelli (1949), Hoogendoorn (1959), Eaton et al. (1967), Hughmark (1962), Guzhov et al. (1967), Chawla (1969), Beggs (1972), Dukler et al. (1969), Scott (1962), Agrawal et al. (1973), Hughmark (1965) and Levy (1960).

Five different measures of error, namely the root mean-square error, mean absolute error, simple mean error, mean-percentage absolute error and the mean-percentage error were used. Arbitrary designation of the void fraction ranges to the flow pattern was made to see the predictive capability of the correlations within each range of liquid holdup (void fraction) values or flow regime. Using the flow pattern map of Mandhane et al. (1974), the Hughmark (1962) correlation is recommended for the bubble, elongated bubble and slug flow regimes, the flow pattern specific correlation of Agrawal et al. (1973) for the stratified, Lockhart and Martinelli (1949) correlation for annular, annular-mist and the Beggs (1972) correlation for the dispersed-bubble flow regimes. No correlation was recommended for the total data points even though the recommended error values for all the parameters were given.

Comparing the correlation using the flow maps of Hoogendoorn (1959), Govier and Aziz (1972) and Baker (1954), it was shown that the Hughmark (1962) correlation predicts the bubble, elongated bubble regimes in all the four maps and the slug regime in all but the Baker (1954) map where the Chawla (1969) is seen to out-perform it. The stratified regime is predicted by the Agrawal et al. (1973) correlation in the Mandhane et al. (1974) and Baker (1954) maps while the Dukler et al. (1964) correlation predicts that of Hoogendoorn (1959) and Govier and Aziz (1972). This is easily explainable as it has an advantage over the others due to the bias it has towards these two data sets from which it was developed. The annular, annular-mist regime is handled by the Lockhart and Martinelli (1949) in all the flow pattern maps. In the dispersed-bubble regime the Beggs (1972) correlation has predicted the data in Mandhane et al. (1975) while Hughmark (1962) was able to handle that of Hoogendoorn (1959) and Govier and Aziz (1972). The Hoogendoorn (1959) correlation is shown to

predict the data in the Baker (1954) map quite well. All in all it was concluded that none of the correlations gave satisfactory results in the annular and annular mist flow regimes.

### 3.5. Papathanassiou (1983) horizontal pipe comparison

The physically realistic range within which two-phase flow exists for a set of operating conditions and specified fluid was deduced from consideration of equal wall shear stresses between the phases which gives minimum void fraction and a minimum slip ratio of one which gives the maximum possible void fraction. Using these a void fraction spectrum graph is constructed which is used to compare the correlation of Lockhart and Martinelli (1949), Hoogendoorn (1959), Bankoff (1960) and Hughmark (1962). All but the Bankoff (1960) correlation have similar trend in predicting void fraction. The Bankoff (1960) correlation is seen to under predict the void fraction mainly due to the fact that it was developed for vertical flow. It was shown that the Lockhart and Martinelli (1949) correlation falls out of the physically realistic range at a void fraction of about 0.4. At very high void fraction ranges of 0.7 and above the Bankoff (1960) correlation is seen to fall in the unrealistic region. The disagreement between the void fraction correlation predictions in the lower and upper extreme values of void fraction ranges is explained by the fact that few experimental results at this range exist when the correlations were developed.

### 3.6. Spedding et al. (1990) inclined (2.75°) pipe comparison

Spedding et al. considered 60 correlations and the data of Spedding and Nguyen (1976) to make a comparison of the void fraction correlations at an upward angle of 2.75° from the horizontal. Criteria that the average predicted value to be within  $\pm 15\%$  with a 30% spread was used to consider the predictive performance of a correlation as satisfactory. Focusing our attention on the conclusions given for all upward inclinations, it was given that the Nicklin et al. (1962) correlation gave acceptable prediction for bubble and slug flows for all inclination angles. Moreover, the Bonnecaze et al. (1971), Premoli et al. (1970) and Lockhart and Martinelli (1949) correlations were reported to handle some of the flow regimes regardless of inclination angle. Lockhart and Martinelli (1949) was also reported to handle the blow through slug flow pattern satisfactorily while the Spedding and Chen (1984) correlation was able to predict the annular plus roll wave flow pattern. It was concluded that the droplet flow pattern could not be handled by any of the correlations.

### 3.7. Abdulmajeed (1996) horizontal pipe comparison

In an effort to simplify the mechanistic model of Taitel and Dukler (1976) and developing a new correlation, Abdulmajeed (1996) collected 88 air-kerosene void fraction data in a horizontal 51 mm diameter pipe and compared 12 correlations. Namely: Armand (1946), Hughmark and Pressburgh (1961), Hughmark (1962), Eaton et al. (1967), Guzhov et al. (1967), Beggs (1972), Gregory et al. (1978), Brill et al. (1981), Chen and Spedding (1981, 1983), Mukherjee and Brill (1983), Minami and Brill (1987) and Abdulmajeed (1996). The comparison parameters used were average percent error, absolute average percent error and the standard deviation. It was shown that the correlation of Abdulmajeed (1996) is able to predict void fraction in stratified, slug and annular flow regime which covers a range of flow regimes in contrast to the implicit Taitel and Dukler (1976) model which was specifically developed for stratified flow.

### 3.8. Spedding (1997) general ( $-90^\circ$ to $+90^\circ$ ) comparison

Spedding undertook an extensive comparison on more than 100 void fraction correlations using the air-water data of Spedding and his coworkers (1976, 1979, 1989, 1991, 1993). The pipe diameters ranged from 26 mm to 95.3 mm. Prediction of a correlation was considered satisfactory for average predicted values that fall within  $\pm 15\%$  of the data with spread of individual values within  $\pm 30\%$ . Eighteen flow regimes were identified for which the predictive capability of the correlations assessed. The comparison ranged from  $-90^\circ$  to  $+90^\circ$  with fair representation of major inclination angles. It was noted that no single model

could handle all flow regimes and angle of inclination satisfactorily. Different void fraction correlations were recommended for different flow regimes for horizontal/upward inclined and downward flow separately.

### 3.9. Friedel and Diener (1998) horizontal/vertical upward comparison

Based on 24,000 experimental data points in single component water and R12, and two component air–water, a comparison of 13 (refined selection of the originally 26 correlations) void fraction correlations was undertaken. Some of the correlations are proprietary of which no information was given while the others were collected from the open literature. Some of the correlations considered in the analysis that passed the limiting void fraction criteria, for  $x = 0$ ,  $\varepsilon = 0$  and  $x = 1$ ,  $\varepsilon = 1$ , and were considered in our comparison are Huq and Loth (1992), Kowalczewski (Isbin and Biddle, 1979), Smith (1969), the Loscher and Reinhardt correlation reported by Friedel (1977), Rouhani and Axelsson (1970) and Premoli et al. (1970).

The average predictive accuracy of the correlations was established by calculating the statistical parameters; the scatter of logarithmic ratios, scatter of absolute deviations and average of logarithmic ratios for the mean void fraction and mean density. The Rouhani and Axelsson's (1970) first correlation (Rouhani I) has been recommended as having an accurate predictive capability.

Based on the literature search done and presented above, the few numbers of void fraction correlation comparisons made and the recommendations put forward by the investigators are very narrow in nature complicating the task of reaching at objective conclusions very difficult. This is due to the fact that, the correlations selected for comparison are those that are popular to a particular industry (like the petroleum, nuclear industries and so on), limited to a certain physical restriction which may not be always practical (horizontal pipe, air–water mixture) or biased towards one experimental set up for which complete and detailed description of and/or the data that was obtained thereby are not clearly indicated. Hence, an objective assessment of all the available correlations in the open literature with all the available data from different sources covering wide range of operating characteristics including any recent data which was not included in the previous comparisons is justified. This analysis has considered a total of 2845 data points from eight sources, four different fluid types, and three inclination angle categories with 68 void fraction correlations which makes it a thorough and unbiased assessment than any of the previous comparisons.

## 4. Experimental database and fluid properties

In the experimental data collection process, effort has been made to make the database as unbiased, wide range covering and having an acceptable quality as was practically possible. After the initial screening process that primarily considered the completeness of the data set, the data of Eaton (1966), Beggs (1972), Spedding and Nguyen (1976), Mukherjee (1979), Minami and Brill (1987), Franca and Lahey (1992), Abdulmajeed (1996) and Sujumnong (1997) were selected with which the performance of the void fraction correlations are to be compared. The range of diameters, inclination angles, number of data points and fluids covered by the data is summarized in Table 2.

Each set of data was tested against simple but essential requirements to determine if all the data points are within the realistic range of values. One screening criterion was to identify the measured void fraction in any two-phase flow that exceeded the value calculated by the homogeneous model. This yielded 13 of Beggs (1972), 20 of Spedding and Nguyen (1976), 61 of Mukherjee (1979), 1 of Eaton (1966), 5 of Abdulmajeed (1996), 3 of Franca and Lahey (1992) and 3 of Sujumnong (1997) data to be out of the realistic range. These data points were therefore excluded from the data set.

In addition, following the work of Zuber and Findlay (1965), plotting any two-phase flow experimental data on the weighted mean velocity ( $U_{SG}/\varepsilon$ ) versus mixture velocity ( $U_M$ ), a few abnormally isolated data points that do not fall into any data clusters (accounting for flow pattern change) were suspect to be unreliable and hence had been taken out. These are 1 of Spedding and Nguyen (1976), 14 of Mukherjee (1979) and 4 of Franca and Lahey (1992). Twelve data points from Franca and Lahey (1992) which were identical in all aspects (repeated runs) but reported as separate points were also taken out.

Table 2  
Characteristics of database sources

Source	Physical flow configuration/characteristics	Mixture considered	Measurement technique	No. of data points used
Eaton (1966)	Horizontal, ID 52.5 mm and 102.26 mm	Natural gas–water (NW)	Quick-closing valves	237
Beggs (1972)	Horizontal, Uphill (5°, 10°, 15°, 20°, 35°, 55°) and vertical ID 25.4 mm and 38.1 mm	Air–water (AW)	Quick-closing valves	291
Spedding and Nguyen (1976)	Horizontal, Uphill (2.75°, 20.75°, 45°, 70°) and vertical ID 45.5 mm	Air–water (AW)	Quick-closing valves	1383
Mukherjee (1979)	Horizontal, Uphill (5°, 20°, 30°, 50°, 70°, 80°) and vertical ID 38.1 mm	Air–kerosene (AK)	Capacitance probes	558
Minami and Brill (1987)	Horizontal, ID 77.93 mm	Air–water (AW) and air–kerosene (AK)	Quick-closing valves	54 and 57
Franca and Lahey (1992)	Horizontal, ID 19 mm	Air–water (AW)	Quick-closing valves	80
Abdulmajeed (1996)	Horizontal, ID 50.8 mm	Air–kerosene (AK)	Quick-closing valves	83
Sujumong (1997)	Vertical, ID 12.7 mm	Air–water (AW)	Quick-closing valves	101

## 5. Evaluation of void fraction correlations

For ease of reference and quick read-through between some of the similarities and/or disparity of the conclusions drawn from this work and the previous assessments, the current comparison work is divided into three sections namely horizontal, upward inclined, and vertical. These sections will be separately analyzed and a recommendation drawn for each section before the final summary is presented.

In line with our ambitious goal of recommending a single or a combination of correlations that could handle all angles regardless of flow regimes, inclination angles, fluid types and so on, we have removed all the restrictions on all the correlations in trying to see their promise to handle most if not all of the diverse data points in the database. The fact that this is an over stretch on all of the correlations should be well noted. Hence, all subsequent analyses and comparisons made which may dictate unfavorable conclusions on some or all correlations should be taken from this perspective only and should not be an undermining factor against the correlations in any way.

In making the relative comparisons, we have used the simple error percentage and a cutoff percentage, the latter one being the total number (percentage) of the data points that have to be correctly predicted within the specified index to deem the correlation good or not. In order to be able to cater for three different scenarios to make comparisons between the correlations that one could pick from; a decision was made to go for a progressively increasing cutoff percentage as the percentage error index is relaxed out. By this we mean that a lower cutoff percentage of 75% was chosen for the more restrictive 5% error index while cutoff percentages of 80% and 85% were used for the 10% and 15% error indices, respectively. For a very detailed comparison of the performance of all the correlations with all the data sets, the interested reader may refer to Woldesemayat (2006).

### 5.1. Horizontal flow comparison

A total of 900 data points from seven sources with eight datasets was used for the comparison in this section. The data set of Eaton (1966) could not be satisfactorily predicted by any of the correlations for the more restrictive 5% error index. The closest best prediction is given by Premoli et al. (1970) that could only predict 68.4% of the total data points. The same trend is observed for the data of Abdulmajeed (1996) and Mukherjee (1979). The only exception are the correlations of Filimonov et al. (1957) and Premoli et al. (1970) that were able to predict 77.1% and 74.7% of Abdulmajeed (1996) data, respectively. The next best prediction comes from Mukherjee (1979) and Minami and Brill (1987) were 72.3% and 73.5% of the data points of Abdulmajeed

(1996) were predicted correctly. The performance of all the rest of the correlations was far from being satisfactory. Even those of Mukherjee (1979) and Minami and Brill (1987) failed to give acceptable prediction of Mukherjee (1979) data.

This clearly shows the lack of capability of all of the correlations to satisfactorily predict void fraction for fluids different from air–water mixtures from which most of them were originally developed. A point worth mentioning here is that the correlations of Mukherjee (1979) and Minami and Brill (1987) have an edge over the others as they were developed from the same data set that was used here for the comparison. The other notable observation here is that the correlations developed from the respective data sets by Mukherjee (1979) and Abdulmajeed (1996) could only capture 51.6% and 57.8% of their own data within the 5% error index. The exception in this regard is the correlation by Minami and Brill (1987) that was able to predict 84.2% of their own air–kerosene data within the 5% error index. However, it captured a fair 73.5% of Abdulmajeed (1996) data, with only 53.2% of Mukherjee (1979) and 55.3% of Eaton (1966) data. Hence, even for the same or similar fluid types, correlations developed from specific experimental data sets with fitted constants fail to adequately predict data sets from other test setups and operating conditions. The trend in predictive capability for the air–water data is almost similar for the 5% error index with a few exceptions. The Beggs (1972) correlation was able to predict 82.1% of his own data while Lockhart and Martinelli (1949), Wallis (1969) and Spedding and Chen (1984) predicted 74.1%, 74.4% and 75.6% of Spedding and Nguyen (1976) data, respectively.

The performance of all correlations improves as the percentage error index is relaxed out to 10% and 15%. With the 10% error index 9, 7, and 9 correlations were able to predict the data of Abdulmajeed (1996), Eaton (1966) and the air–kerosene data of Minami and Brill (1987), respectively. It must be noted here that only the correlation of Premoli et al. (1970), Mukherjee (1979), Minami and Brill (1987) and Graham et al. (2001) were able to predict two data sets within this index whereas all the rest only predicted a single data set. Considering all the data sets, the correlations of Armand–Massina (Leung, 2005), Premoli et al. (1970) and Minami and Brill (1987) are the ones which came close to the set criteria and are worth a general recommendation for the 10% error index.

The performance of most of the correlations further improves with an extended error percentage index of 15%. Here a fair number of correlations were seen to perform rather well as compared to the lower error percentage indices. The performance of the best performing correlations for the different data sources that met our criteria for the  $\pm 15\%$  error index on a cumulative basis are presented in Table 3. The ones worth

Table 3

Percentage of horizontal data points correctly predicted within  $\pm 15\%$  by the best performing correlations for the different data sources

Correlation/data source	Eaton (1966)	Beggs (1972)	Spedding and Nguyen (1976)	Mukherjee (1979)	Minami and Brill (1987)	Minami and Brill (1987)	Franca and Lahey (1992)	Abdulmajeed (1996)
Fluids considered	NW	AW	AW	AK	AW	AK	AW	AK
Total data points	237	56	270	62	54	57	81	83
Armand–Massina <sup>a</sup>	86.5	96.4	87.4	88.7	83.3	87.7	88.9	81.9
Hughmark (1962)	91.1	92.9	86.7	79.0	96.3	98.2	80.2	88.0
Smith (1969)	87.3	98.2	87.0	80.6	75.9	78.9	77.8	80.7
Premoli et al. (1970)	87.8	89.3	93.0	75.8	83.3	93.0	50.6	83.1
Rouhani I <sup>b</sup>	93.2	89.3	88.9	77.4	100.0	100.0	75.3	86.7
Dix (1971) <sup>c</sup>	88.6	78.6	81.5	75.8	85.2	89.5	81.5	94.0
Beggs (1972)	92.4	92.9	77.4	83.9	85.2	91.2	75.3	83.1
Chisholm (1973)	87.3	100.0	84.8	88.7	77.8	80.7	85.2	80.7
Mukherjee (1979)	93.7	82.1	80.7	75.8	100.0	100.0	67.9	89.2
Chisholm (1983), Armand (1946)	85.7	100.0	84.8	88.7	81.5	86.0	85.2	80.7
Minami and Brill (1987)	94.9	94.6	79.6	77.4	100.0	100.0	87.7	89.2

<sup>a</sup> Leung (2005).

<sup>b</sup> Rouhani and Axelsson (1970).

<sup>c</sup> Coddington and Macian (2002).

consideration here are due to Armand–Massina (Leung, 2005), Hughmark (1962), Smith (1969), Premoli et al. (1970), Rouhani and Axelsson (1970) first correlation (Rouhani I), Dix (Coddington and Macian, 2002), Beggs (1972), Chisholm (1973, 1983), Mukherjee (1979), Armand (1946), Toshiba (Coddington and Macian, 2002), Huq and Loth (1992), Minami and Brill (1987) and Graham et al. (2001).

On a cumulative analysis, all the issues we raised when discussing the individual data sets would have their own effect on the overall performance of each correlation. This has been observed as the performance of all the correlations have deteriorated heavily especially within the 5% error index. The only correlation that was able to give a fair result is that of Premoli et al. (1970) with 65.2% which is much lower than the “acceptable” cutoff index we set out earlier. The performance is seen to improve for the wider error indices where the correlation of Armand–Massina (Leung, 2005), Minami and Brill (1987), Premoli et al. (1970) and Rouhani I (1970) were able to predict 79.1%, 78.3%, 78.2% and 77% of the whole data set within 10% error index, respectively. On the other hand correlations that gave an acceptably good prediction within the 15% error index are Rouhani I (1970), Hughmark (1962), Armand–Massina (Leung, 2005), Mukherjee (1979), Minami and Brill (1987), Chisholm (1973, 1983) and Armand (1946) in order of decreasing accuracy. Another set of correlations worth a general recommendation with very close performance to the ones listed above are due to Smith (1969), Premoli et al. (1970), Dix (Coddington and Macian, 2002), Beggs (1972), Toshiba (Coddington and Macian, 2002), Huq and Loth (1992) and Graham et al. (2001).

A simple scatter plot of the measured versus calculated void fractions for the Rouhani I (1970) correlation that predicted 89.2% of the data points in the horizontal flow database within the 15% error index is presented in Fig. 1.

It is observed that the correlation of Hughmark (1962), which is the second best correlation in terms of the total percentage of data points predicted, shows a superior consistent performance over the others. The Rouhani I (1970) correlation is seen to under predict the data most of the time while all the other correlations tend to over predict the data on the whole range of values.

For air–water mixture, 16 correlations are observed to give a prediction of 85% and above for the 15% error index with the best predictions coming from Armand–Massina (Leung, 2005), Rouhani I (1970) and Hughmark (1962) with predictions of 87% and above. A very close performance to these correlations were also given by Chisholm (1973, 1983) and Armand (1946) which predicted 86% of the data within the 15% error

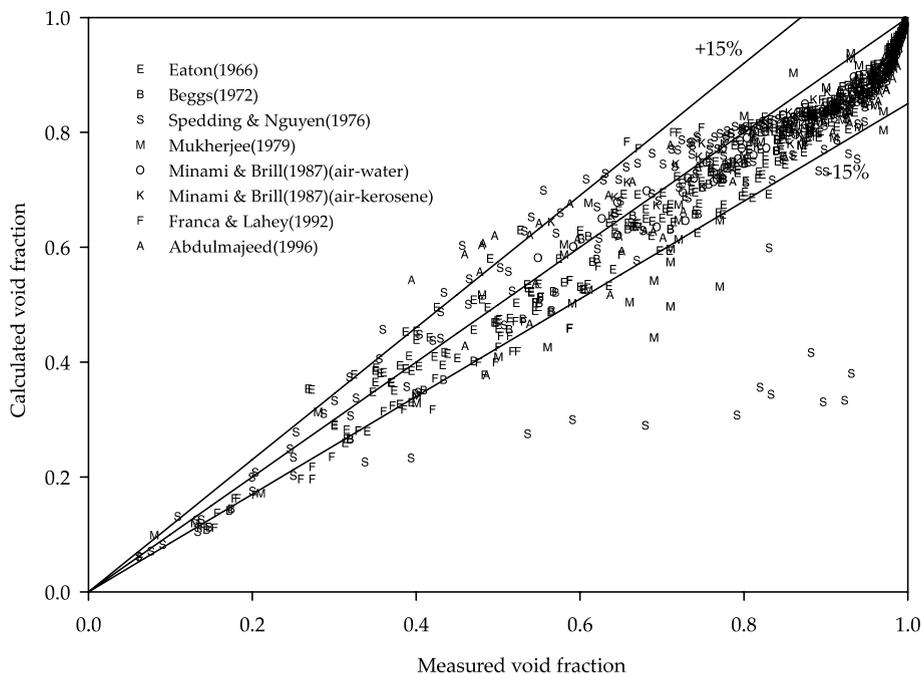


Fig. 1. Comparison of Rouhani I (1970) correlation with measured horizontal experimental data.

index while Smith (1969), Minami and Brill (1987), El-Boher et al. (1988) and Huq and Loth (1992) captured 85% of the data within the 15% error index.

For the non air–water data, the correlations of Beggs (1972) and Graham et al. (2001) gave the best prediction of 89% and 88%, respectively. This is quite unexpected as these correlations were developed from air–water and refrigerant data sets that have quite different physical properties from kerosene and natural gas. Rouhani I (1970) and Toshiba (Coddington and Macian, 2002) predicted 86% of the data set for the 15% error index with close predictions of 85% coming from Armand–Massina (Leung, 2005), Filimonov et al. (1957), Chisholm (1973, 1983), Premoli et al. (1970) and Armand (1946).

## 5.2. Inclined flow comparison

In this section data sets from three different sources Beggs (1972), Spedding and Nguyen (1976) and Mukherjee (1979) were used for comparisons. All the inclination angles within the data set would then be considered separately first before making a consolidated summary for all.

### 5.2.1. Inclined flow comparison with the data of Beggs (1972)

Beggs reported 209 void fraction measurements for seven different pipe inclination angles between 5° and 75° from the horizontal. Owing to the fact that there are only a small number of data points and the inclination is very close to the horizontal, four correlations were seen to perform acceptably with the tightest 5% error index for the 5° inclination data set. These are the simpler models by Armand–Massina (Leung, 2005), Chisholm (1973, 1983), Armand (1946) and the correlation by Beggs (1972). As was observed in the horizontal comparison, the performance of the correlations improve for the wider error bands of 10% and 15% where 12 and 21 correlations were found to predict at least 80% and 85% of the data sets, respectively. The same trend is observed for all inclination angles except for a few exceptions. The Minami and Brill (1987) correlation fairly predicted the 10° and 20° data within the 5% error index while Chisholm (1973, 1983), Armand (1946) and Smith (1969) also gave similar results for the 15° data. A consistent acceptable prediction for the high inclination angle ranges (35–75°) comes from Dix (Coddington and Macian, 2002) correlation which predicted the data for all error indices quite well.

On a combined assessment for all the inclination angles, excellent prediction of the data was given by Chisholm (1973, 1983), Armand (1946), Gregory and Scott (1969), Hughmark (1962) and Armand–Massina (Leung, 2005). The correlations by Filimonov et al. (1957), Premoli et al. (1970), Rouhani I (1970), Dix and Toshiba (Coddington and Macian, 2002) gave comparable results to the other set of correlations mentioned above. Among these correlations the Hughmark (1962) correlation gave the best prediction of 96.7% of the data within the 15% error index while Premoli et al. (1970), Rouhani I (1970), Filimonov et al. (1957), Toshiba and Dix (Coddington and Macian, 2002) gave 96.2%, 95.7%, 95.2%, 94.3% and 91.4%, respectively.

### 5.2.2. Inclined flow comparison with the data of Spedding and Nguyen (1976)

An extensive void fraction data restricted to only four inclination angles was given by Spedding and Nguyen (1976). The angles considered were 2.75°, 20.75°, 45° and 75°. A total of 889 void fraction measurements were reported. This data set is where almost all the predictions by the correlations fall short of being acceptable in all the error indices across all inclination angles.

For near horizontal data (2.75°) the simple correlation of Lockhart and Martinelli (1949) and Wallis (1969) were the only ones to give a fairly acceptable prediction for the 5% error index. However, the Lockhart and Martinelli (1949) correlation could not predict the wider error indices acceptably which was not logically expected. Wallis (1969) and Rouhani I (1970) are two correlations which gave a close to acceptable results for the 10% error index for the near horizontal inclination and Toshiba (Coddington and Macian, 2002) for 20.75° with none of the correlations acceptably predicting any of the data set at all. Filimonov et al. (1957) were able to predict within the 15% error index for 45° and 75°, Rouhani I (1970) for 2.75° and Dix (Coddington and Macian, 2002) for 20.75° and 75° inclination angles, respectively.

The correlation of Toshiba (Coddington and Macian, 2002) was the only one that fairly predicted the data set across all inclination angles for error indices of 10% and 15%. The next best performance comes from Dix (Coddington and Macian, 2002) followed by Filimonov et al. (1957) and Rouhani I (1970).

### 5.2.3. Inclined flow comparison with the data of Mukherjee (1979)

Mukherjee reported air–kerosene void fraction data for six pipe inclination angles. The pipe inclinations researched were 5°, 20°, 30°, 50°, 70° and 80°. A total of 444 data points were considered here for making the comparison between the void fraction correlations. Here again the capability of the correlations to predict the data sets within the 5% error index could only average 57.3% of the data points across all inclination angles which is far below what we have set out as an acceptable cutoff percentage and far from being applicable to any use by any standard.

The prediction performance of the correlations within the 10% error index has seen a general improvement with the only exception of the 20° set for which none of the correlations were able to predict the data on and above the specified 80% cutoff percentage. Referring to the combined inclined data set of Mukherjee (1979), the correlation of Gregory and Scott (1969), Armand–Massina (Leung, 2005), Rouhani I (1970) and Hughmark (1962) have performed well on the 15% error index and in aggregate across the whole range of inclination angles considered and are worth a recommendation for this data set. The correlations that fall into the next best category are that of Beggs (1972), Chisholm (1973, 1983), Greskovich and Cooper (1975), Armand (1946), Sun et al. (1980) and Minami and Brill (1987). The correlations of Premoli et al. (1970), Toshiba and Dix (Coddington and Macian, 2002) and Filimonov et al. (1957) which will be shown to have an acceptable performance for the combined inclined database only gave a reasonable average prediction of 75% on the 15% error index for this data set.

The predictive capability of seven correlations by data source and inclination angle which gave acceptable overall results for the inclined data set is presented in Table 4. For the range of inclination angles considered from all sources, none of the correlations gave acceptable results within the 5% error index. The only correlation worth consideration being that of Minami and Brill (1987) which predicted 67% of the inclined data set of Beggs (1972). The results of the wider error bands are a little bit encouraging where quite a number of correlations were able to predict the data of Beggs (1972) and Mukherjee (1979). The data of Spedding and Nguyen (1976) yielded only to the correlations of Toshiba and Dix (Coddington and Macian, 2002) where the latter correlation falls short of the set cutoff percentage by a slight margin.

Table 4  
Percentage of inclined data points correctly predicted within  $\pm 15\%$  by the best performing correlations for the different data sources

Correlation/ data source	Beggs (1972) (air–water)							Spedding and Nguyen (1976) (air–water)				Mukherjee (1979) (air–kerosene)						
	5	10	15	20	35	55	75	2.75	20.75	45	70	5	20	30	50	70	80	
Inclination angle (degrees)																		
Total data points	31	29	31	30	30	29	29	226	188	196	279	57	74	74	67	80	92	
Armand– Massina <sup>a</sup>	100.0	100.0	100.0	100.0	93.3	86.2	96.6	81.4	72.3	52.6	54.1	94.7	87.8	91.9	76.1	87.5	85.9	
Filimonov et al. (1957)	96.8	96.6	96.8	93.3	93.3	93.1	96.6	67.3	71.3	86.7	86.0	82.5	86.5	81.1	70.1	68.8	77.2	
Hughmark (1962)	96.8	96.6	100.0	100.0	96.7	93.1	93.1	83.2	79.3	55.6	61.6	93.0	81.1	89.2	80.6	77.5	88.0	
Premoli et al. (1970)	96.8	93.1	100.0	100.0	93.3	93.1	96.6	83.6	77.7	63.8	69.5	82.5	81.1	82.4	80.6	77.5	78.3	
Rouhani I <sup>b</sup>	93.5	96.6	100.0	100.0	96.7	89.7	93.1	87.2	81.4	61.2	69.5	91.2	82.4	89.2	83.6	76.3	90.2	
Dix (1971) <sup>c</sup>	83.9	89.7	87.1	93.3	96.7	96.6	93.1	77.4	85.1	78.6	87.1	77.2	85.1	85.1	76.1	70.0	78.3	
Toshiba (1989) <sup>c</sup>	93.5	96.6	93.5	96.7	93.3	93.1	93.1	77.0	92.6	79.1	93.2	82.5	86.5	82.4	74.6	68.8	82.6	

<sup>a</sup> Leung (2005).

<sup>b</sup> Rouhani and Axelsson (1970).

<sup>c</sup> Coddington and Macian (2002).

It is our conclusion that the poor performance of the correlations within the narrow error index (5%) may not be due to the weaknesses of the correlations alone but the accuracy of the data sets which for most of the cases was not explicitly reported and should also be questioned. This observation is not confined to the current data set alone but equally applies to all the data sets used in the horizontal, inclined and the subsequent vertical comparisons.

As was in the horizontal comparison, a simple scatter plot of the measured versus calculated void fraction for the top performing correlation that fairly predicted the whole inclined set is given in Fig. 2. The correlations worth recommendation for this data set are that of Toshiba and Dix (Coddington and Macian, 2002), Rouhani I (1970), Filimonov et al. (1957), Premoli et al. (1970) and Hughmark (1962) in order of decreasing accuracy. The correlations of Toshiba and Dix (Coddington and Macian, 2002) were seen not only to give a very good prediction but also the most consistent performance across the whole void fraction range. The slight under prediction by the correlations of Dix and Toshiba (Coddington and Macian, 2002) for void fractions less than 0.8 is quite acceptable. Rouhani I (1970) has a moderate over prediction for void fractions less than 0.6 while the correlations of Premoli et al. (1970) and Hughmark (1962) have an excessive over prediction for void fractions less than 0.8. The range involved in this case is very wide making a general recommendation of these two correlations a bit controversial despite their high percentage of data points correctly predicted within the 15% error index especially for the correlation of Premoli et al. (1970).

The fifth ranked Filimonov et al. (1957) correlation has a fairly acceptable scatter across the whole range which is much better than the performance by Premoli et al. (1970) which is ranked fourth in terms of the aggregate data points correctly predicted for the inclined data set. Analyzing the performance of the correlations for the air–water mixture separately and taking the wider 15% error index, the correlation of Toshiba (Coddington and Macian, 2002) predicted 87.4% of the combined data (1098 points) of Beggs (1972) and Spedding and Nguyen (1976) while Dix (Coddington and Macian, 2002) and Filimonov et al. (1957) gave 84.1% and 81.5%, respectively. On the other hand, for the total inclined air–kerosene data of Mukherjee (1979), the best prediction was given by Gregory and Scott (1969), Armand–Massina (Leung, 2005), Rouhani I (1970) and Hughmark (1962) predicting 87.6%, 87.2%, 85.4% and 84.7% of the data within 15% error margin, respectively.

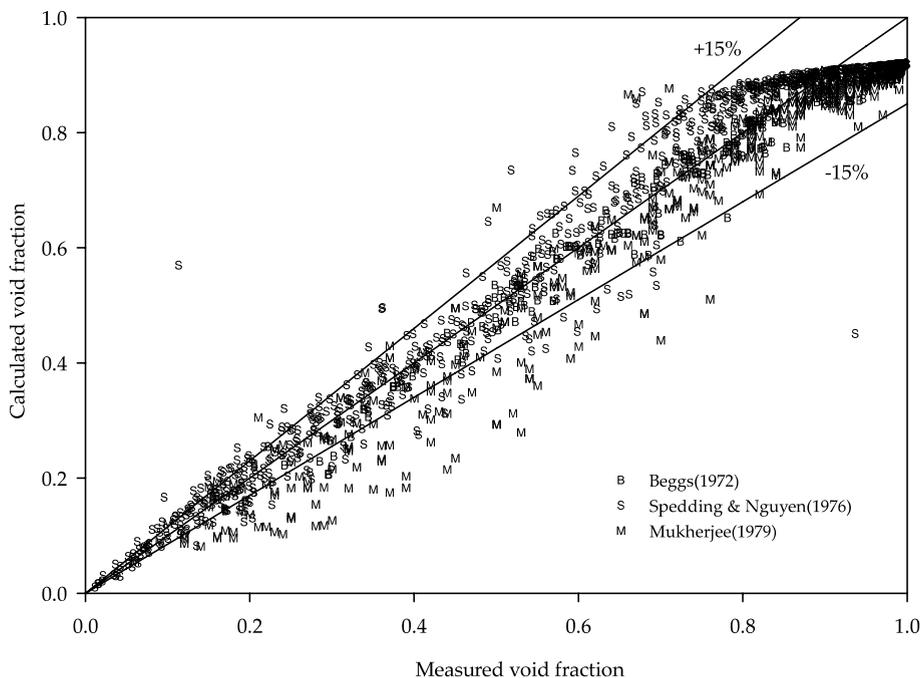


Fig. 2. Comparison of Toshiba (Coddington and Macian, 2002) correlation with measured combined inclined experimental data.

## 6. Vertical flow comparison

A total of 403 data points are included for these comparisons from four sources namely, [Beggs \(1972\)](#), [Spedding and Nguyen \(1976\)](#), [Mukherjee \(1979\)](#) and [Sujumnong \(1997\)](#). Over half of the data points in this data set are from [Spedding and Nguyen \(1976\)](#) which would influence most of the final conclusions and recommendations.

The correlation by [Beggs \(1972\)](#) and [Minami and Brill \(1987\)](#) were the only ones that made an acceptably close prediction within the 5% error index for the data of [Beggs \(1972\)](#). None of the correlations were able to predict a sufficient percentage of the data points in any of the data sets within this range. The performance shows improvement for the wider error bands for all the data sets except the data of [Spedding and Nguyen \(1976\)](#). This has been the case for all the previous comparisons. The only new correlation that came into the picture is that of [El-Boher et al. \(1988\)](#) which gave an 83.5% prediction of the [Spedding and Nguyen \(1976\)](#) data within the 15% error index even though it was not able to satisfactorily predict the data of [Sujumnong \(1997\)](#).

Considering their general performance across the three other data sets, the correlation by [Armand–Massina \(Leung, 2005\)](#), [Filimonov et al. \(1957\)](#), [Hughmark \(1962\)](#), [Smith \(1969\)](#), [Gregory and Scott \(1969\)](#), [Premoli et al. \(1970\)](#), [Rouhani I \(1970\)](#), [Chisholm \(1973, 1983\)](#), [Armand \(1946\)](#), [Toshiba \(Coddington and Macian, 2002\)](#) and [Huq and Loth \(1992\)](#) have acceptable prediction and are worth consideration.

Consolidating our observation of the separate data sets into one, we have presented the predictive capability of the correlations for the whole vertical database in [Table 5](#). The drift flux correlations by [Toshiba \(Coddington and Macian, 2002\)](#), [Rouhani I \(1970\)](#) and [Filimonov et al. \(1957\)](#) once again have shown an excellent performance in predicting the vertical database within the 15% error index. Six other correlations have given a fairly good result within this range. These are, [Hughmark \(1962\)](#), [Nicklin et al. \(1962\)](#), [Premoli et al. \(1970\)](#), [Bonnecaze et al. \(1971\)](#), [Dix \(Coddington and Macian, 2002\)](#) and [Kokal and Stanislav \(1989\)](#). The correlations of [Hughmark \(1962\)](#), [Premoli et al. \(1970\)](#) and [Dix \(Coddington and Macian, 2002\)](#) are worth a general recommendation while the other three being discarded as unsatisfactory as could be seen from their poor performance within the 10% error index.

Implementing our simple scatter check to see the consistency of the top performing correlation, the correlation of [Toshiba \(Coddington and Macian, 2002\)](#) which has once again showed a very good consistency with only a few “stray” under predictions is presented in [Fig. 3](#).

The [Rouhani I \(1970\)](#) correlation gave an excellent prediction all in all and has a noticeable improvement from its performance for the other inclined data sets where it had excessive over predictions in the lower void fraction range. [Filimonov et al. \(1957\)](#) also has good uniformity in its prediction with slight under predictions for void fractions less than 0.7. The [Hughmark \(1962\)](#) correlation over predicted the data while [Dix \(Coddington and Macian, 2002\)](#) gave a slight random scatter for the lower void fraction range. The excellent performance by [Dix \(Coddington and Macian, 2002\)](#) for the very high void fraction range should be well

Table 5

Percentage of vertical data points correctly predicted within  $\pm 15\%$  by the best performing correlations for the different data sources

Correlation/data source	<a href="#">Beggs (1972)</a>	<a href="#">Spedding and Nguyen (1976)</a>	<a href="#">Mukherjee (1979)</a>	<a href="#">Sujumnong (1997)</a>
Fluids considered	AW	AW	AK	AW
Total data points	26	224	52	101
<a href="#">Filimonov et al. (1957)</a>	92.3	80.4	86.5	88.1
<a href="#">Hughmark (1962)</a>	96.2	75.4	88.5	88.1
<a href="#">Nicklin et al. (1962)</a>	88.5	80.8	76.9	84.2
<a href="#">Premoli et al. (1970)</a>	92.3	79.0	86.5	82.2
<a href="#">Rouhani I<sup>a</sup></a>	96.2	84.4	82.7	91.1
<a href="#">Bonnecaze et al. (1971)</a>	88.5	80.8	76.9	84.2
<a href="#">Dix (1971)<sup>b</sup></a>	96.2	83.9	82.7	72.3
<a href="#">Kokal and Stanislav (1989)</a>	88.5	79.9	76.9	84.2
<a href="#">Toshiba (1989)<sup>b</sup></a>	92.3	89.7	86.5	95.0

<sup>a</sup> [Rouhani and Axelsson \(1970\)](#).

<sup>b</sup> [Coddington and Macian \(2002\)](#).

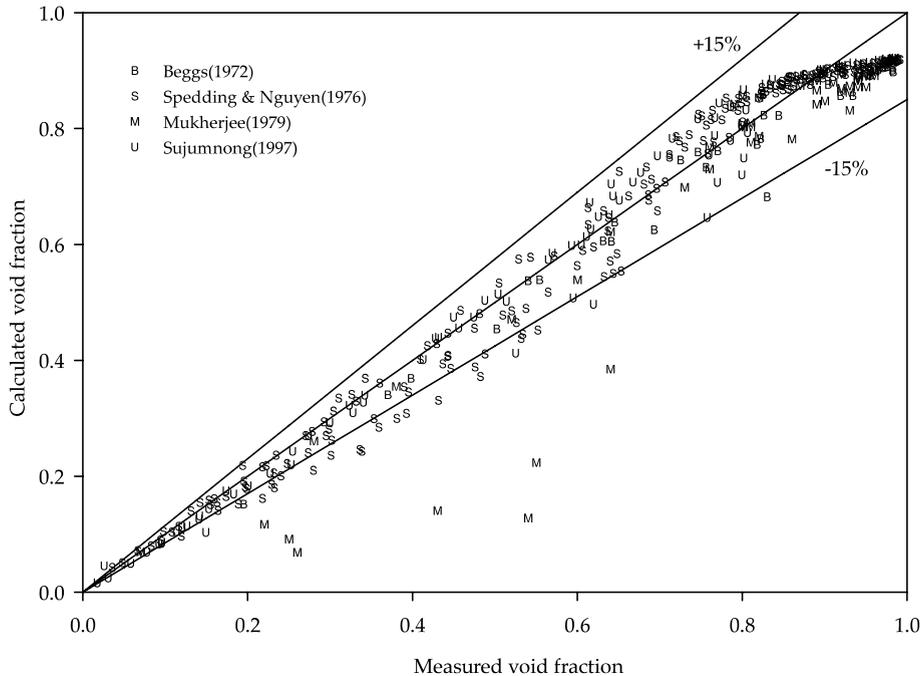


Fig. 3. Comparison of Toshiba (Coddington and Macian, 2002) correlation with measured combined vertical experimental data.

noted where measurement of experimental data is usually very difficult. Premoli et al. (1970) repeated its tendency to over predict the data as observed for the other inclination angles though there has been a significant improvement for the vertical case. It also gave an excellent result for the higher void fraction range comparable to Dix (Coddington and Macian, 2002).

The combined air–water data of Beggs (1972), Spedding and Nguyen (1976) and Sujumnong (1997) is well captured by Toshiba (Coddington and Macian, 2002) that predicted 81.5% of the data within the 15% error index. The correlations of Rouhani I (1970) and Filimonov et al. (1957) gave comparably close predictions to that of Toshiba (Coddington and Macian, 2002). The next best predictions are from Dix (Coddington and Macian, 2002), Premoli et al. (1970) and Hughmark (1962) which gave results of 81.5%, 80.9% and 80.6% respectively for the same error index.

Owing to the small number of data in the air–kerosene vertical data set of Mukherjee (1979), fifteen correlations were able to predict 85% and above for the 15% error index. Armand–Massina (Leung, 2005) gave 90.4% while Chisholm (1973), Armand (1946) predicted 88.5% each, respectively. The correlations by Filimonov et al. (1957), Premoli et al. (1970), Sun et al. (1980), Chisholm (1983), Armand (1946), Chen (1986) and Toshiba (Coddington and Macian, 2002) all predicted 86.5% of the data sets within the wider error band of 15%.

## 7. Summarized comparison and discussion for all data sets

Based on the analysis for the three scenarios considered, six correlations are recommended for acceptably predicting void fraction for horizontal and upward inclined pipes regardless of flow regimes. The percentage of data points correctly predicted for the whole database within the three error indices for each correlation is given in Table 6. Basing our judgment of the capability of the correlations only on the total number of data points correctly predicted, we can have two groups of correlations where the three correlations within each group have very close performance to one another.

The first group of correlations consists of Toshiba (Coddington and Macian, 2002), Rouhani I (1970) and Dix (Coddington and Macian, 2002) predicting 85.3%, 84.2% and 83.1% of the total data points within the

Table 6

Total number and percentage of data points correctly predicted by the top six and modified correlation for the whole database (all sources)

Correlation	Total data points correctly predicted for the whole database			Percentage of data points correctly predicted for the whole database		
	Total points 2845			Total points 2845		
Percentage within	±5%	±10%	±15%	±5%	±10%	±15%
Toshiba (1989) <sup>a</sup>	1065	2137	2427	37.4	75.1	85.3
Rouhani I <sup>b</sup>	1082	2059	2395	38.0	72.4	84.2
Dix (1971) <sup>a</sup>	1597	2139	2363	56.1	75.2	83.1
Hughmark (1962)	1244	2003	2322	43.7	70.4	81.6
Premoli et al. (1970)	1643	2084	2304	57.8	73.3	81.0
Filimonov et al. (1957)	1369	1953	2294	48.1	68.6	80.6
Present study	1718	2234	2436	60.4	78.5	85.6

<sup>a</sup> Coddington and Macian (2002).<sup>b</sup> Rouhani and Axelsson (1970).

15% error index, respectively. The second group is comprised of Hughmark (1962), Premoli et al. (1970) and Filimonov et al. (1957) capturing 81.6%, 81% and 80.6% of the total data points correctly within the 15% error index, respectively.

Hence, the correlation by Toshiba (Coddington and Macian, 2002) has shown best prediction capability in the inclined and vertical flow cases with a very good agreement with the horizontal experimental data set making it the top correlation to be worth a general recommendation. The next on the list would be that of Rouhani I (1970) with a very close performance to that of Toshiba (Coddington and Macian, 2002). Moreover, the Rouhani I (1970) correlation with its relatively good consistency across the whole void fraction range for the total data set has been given weight over the excellent consistency shown by Dix (Coddington and Macian, 2002) for the combined inclined experimental data set with a wide scatter for the horizontal data which is placed third. However, it should be pointed out that the Dix (Coddington and Macian, 2002) correlation has the best prediction with the tighter error indices of 5% and 10% above all of the correlations considered in this analysis.

On the second group of correlations, though ranked lower than Hughmark (1962) in Table 6 based on the 15% error index for the whole data set, Premoli et al. (1970) correlation has the best capability in the tighter indices which is very comparable to that of Dix (Coddington and Macian, 2002). Moreover, the Premoli et al. (1970) correlation has predicted all the databases with very good accuracy and is the second only correlation to do so behind Rouhani I (1970). Hence it would deserve a general recommendation above that of Hughmark (1962) and Filimonov et al. (1957) within this group.

The dilemma to chose between the correlations of Rouhani I (1970) and Premoli et al. (1970) over Dix (Coddington and Macian, 2002) and Filimonov et al. (1957) respectively could be resolved if one is interested to predict void fraction in inclined flow only. For this case the superior performances of the latter two correlations would out weigh the general applicability of the former ones.

In addition to the correlations discussed above, the correlations of Chisholm (1973, 1983), Armand (1946) and Armand–Massina (Leung, 2005) have also given reasonable prediction for the whole database with quite a very good performance for horizontal data sets. Their performances are very much close to one another which are to be expected as they are combinations and/or derivations of each other. The scatter plots for the correlations of Toshiba (Coddington and Macian, 2002), Rouhani I (1970) and Dix (Coddington and Macian, 2002) with the whole data set are given in Figs. 4–6.

We have mentioned the general failure of all the correlations in predicting any of the experimental data within the 5% error index. The accuracy of the data set within this range is also one issue which we have raised as open to question and may not at all be the problem of the correlations. To make a conclusive statement on this and also in trying to predict the compiled experimental database with greater accuracy than what we have achieved so far, an effort is made to modify one of the best performing correlations so as to remove some of its weaknesses.

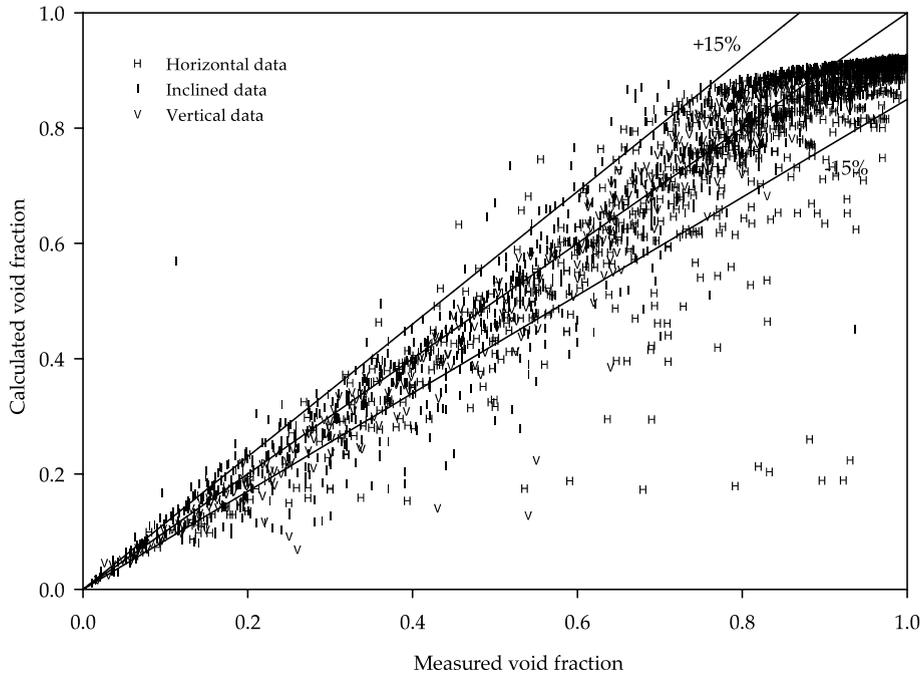


Fig. 4. Comparison of Toshiba (Coddington and Macian, 2002) correlation with measured combined total experimental data.

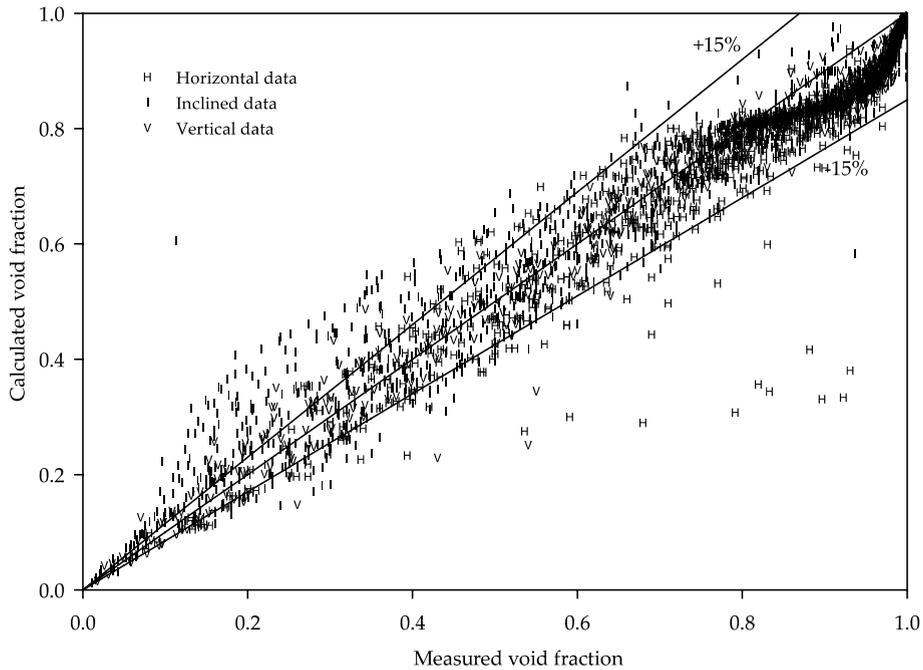


Fig. 5. Comparison of Rouhani I (1970) correlation with measured combined total experimental data.

### 8. Modified correlation

Considering the fact that the Dix (Coddington and Macian, 2002) correlation has an outstanding consistency in all its predictions over the entire void fraction range and the fact that it has the highest prediction of data points within the tighter 5% error index makes it a prime candidate for further improvement.

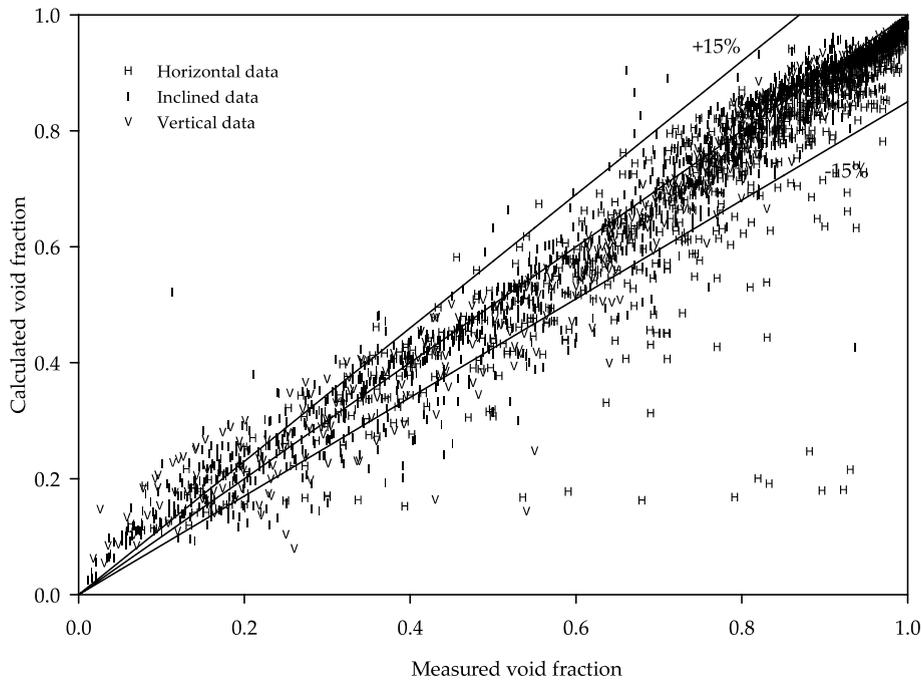


Fig. 6. Comparison of Dix (Coddington and Macian, 2002) correlation with measured combined total experimental data.

The correlation of Dix (Coddington and Macian, 2002) and all the other correlations have significantly underperformed for the data of Spedding and Nguyen (1976). A close look at the data reveals that the system pressure is quite different from that of all the other data sets considered here. The prediction of the correlations further deteriorates with inclination angle for this data set. The other variable with this data set is the slightly “off standard” pipe diameter used in their experiment. Using these inputs, a damping correction factor is suggested which takes into account some of the observations made in previous void fraction developments.

The inclination angle and the system pressure effect on the measured void fraction data were closely analyzed. Bankoff (1960) has shown that the non uniformity in phase distribution is a function of pressure while Zuber and Findlay (1965) have stated that the drift velocity is a function of the concentration profiles and also depends on the momentum transfer between the phases. The concentration profiles across the pipe for a given inclination are generally assumed constant for flows without mass transfer along the pipe length. Integrating these observations and noting that this effect is normal to the pipe inclination, a correction factor of the form  $[1 + \cos \theta]^{0.25}$  is introduced.

The functional dependency of drift velocity on the momentum transfer between the phases assuming one dimensional force interaction would be a maximum for pipes with very high inclination angles operating near atmospheric pressure. This effect could conveniently be captured by the factor,  $[1 + \sin \theta]^{P_{atm}/P_{system}}$ , where  $P_{atm}$  and  $P_{system}$  are the atmospheric and system pressures, respectively.

These two correction factors have been introduced into the drift velocity expression of Dix (Coddington and Macian, 2002) correlation. Hence the modified correlation becomes,

$$\varepsilon = \frac{U_{SG}}{U_{SG} \left( 1 + \left( \frac{U_{SL}}{U_{SG}} \right) \left( \frac{\rho_G}{\rho_L} \right)^{0.1} \right) + 2.9 \left[ \frac{gD\sigma(1 + \cos \theta)(\rho_L - \rho_G)}{\rho_L^2} \right]^{0.25} (1.22 + 1.22 \sin \theta)^{P_{atm}/P_{system}}}$$

The performance of the top ranked Toshiba (Coddington and Macian, 2002) correlation could also be improved following a similar approach.

The performance of the present study correlation on the whole database in comparison to the best performing correlations is also included in Table 6 while the scatter plot for the whole data set is presented in Fig. 7. It

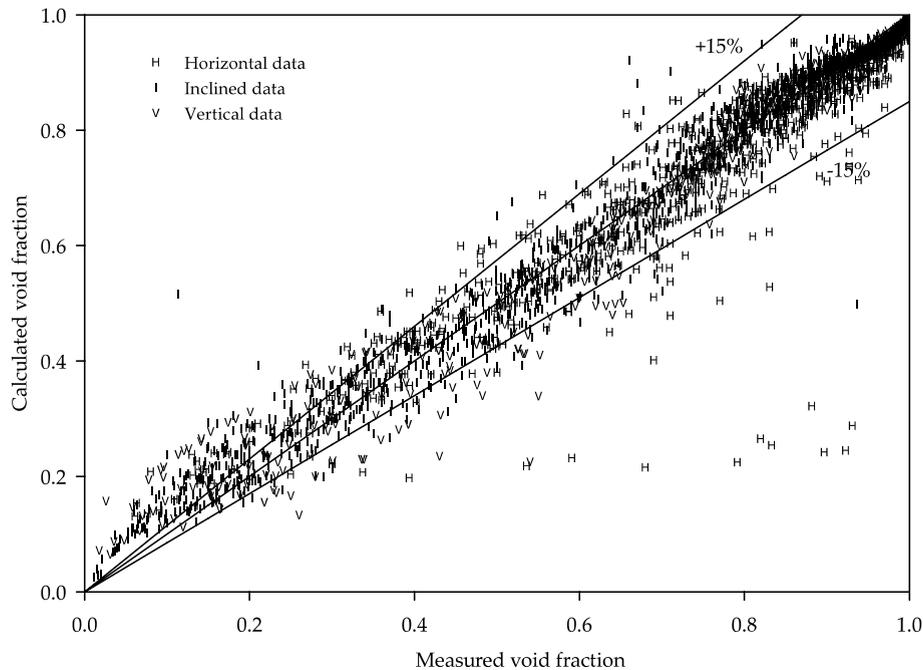


Fig. 7. Comparison of present study correlation with measured combined total experimental data.

can be seen that the under prediction by the original Dix (Coddington and Macian, 2002) correlation on a substantial number of horizontal, inclined, and vertical data points has been improved.

## 9. Summary and conclusions

A very extensive comparison of most of the void fraction correlations available in the open literature was made against 2845 experimental data points of which 900 were horizontal, 1542 inclined, and 403 for vertical pipe setups.

The best performing correlations considering total number of data points predicted as well as their relative consistency in performance was highlighted. Appropriate recommendations were also given. This work is by far the most comprehensive and unbiased comparison done than any of the previous similar evaluations.

It was confirmed that the drift flux analysis method is a powerful tool in developing void fraction correlations as well as analyzing experimental data. Out of the six best performing correlations for the whole void fraction database, all except one were developed based on the drift flux model.

An improvement to the correlation of Dix (Coddington and Macian, 2002) was made by systematically introducing appropriate physical parameters. It was shown that a total of 121 data points were additionally captured within the restrictive 5% error index than that was predicted by the original correlation. This improvement made the correlation to be the top correlation in predicting the whole database within each of the three error indices.

A refined void fraction data set is compiled and made available which could be used for further validation of void fraction correlations as well as any other analysis. The complete data set is available from the second author (Professor Ghajar).

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