

## PREDICTION OF THERMAL CONDUCTIVITY AND VISCOSITY FOR SOME FLUIDS IN THE NEAR-CRITICAL REGION

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*(Received January 28, 1984; in final form July 31, 1985)*

The objective of this work was to develop general, simple and accurate expressions to predict the peculiar behavior of thermal conductivity and viscosity in the near-critical region. Expressions were determined for (a) variation of thermal conductivity as a function of temperature and density or pressure for carbon dioxide, water, and ammonia, and for (b) variation of viscosity as a function of temperature and density or pressure for carbon dioxide, water, and nitrogen. The predicted results based on the developed expressions showed a very good agreement with the available thermal conductivity experimental data and an excellent agreement with the available viscosity experimental data.

KEYWORDS Thermal Conductivity Viscosity Near-critical region

### 1. INTRODUCTION

Accurate knowledge of thermal conductivity and viscosity in the near-critical region should help in the better utilization of heat transfer correlations developed for the near-critical region. Accurate knowledge of thermal conductivity and viscosity should also assist in the critical evaluation of the existing heat transfer correlations which require thermal conductivity and viscosity as a function of temperature and density or pressure.

Heat transfer to fluids by free or forced convection may typically be predicted by evaluating the physical properties in the following common expressions at a reference temperature.

For free convection [1]

$$(Nu)_{Tr} = a(Gr)_{Tr}^b (Pr)_{Tr}^c \left( \frac{T_\infty}{T_w - T_\infty} \right)^d \quad (1)$$

For forced convection [2]

$$(Nu)_{Tr} = a(Re)_{Tr}^b (Pr)_{Tr}^c \left( \frac{T_w}{T_b} \right)^d \quad (2)$$

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The effect of inaccuracies in transport properties  $\mu$  and  $k$  can be illustrated easily by combining the definitions of  $Nu$ ,  $Gr$ ,  $Re$  and  $Pr$  with Eqs. (1) and (2) which yields:

For free convection

$$h = aL^{(3b-1)}g^b[\rho^{2b}\mu^{(-2b+c)}k^{(1-c)}C_p^c]_{Tr}\left(\frac{\rho_\infty - \rho_w}{\rho_w}\right)^b\left(\frac{T_\infty}{T_w - T_\infty}\right)^d \quad (3)$$

For forced convection

$$h = aD^{(b-1)}V^b[\rho^b\mu^{(c-b)}k^{(1-c)}C_p^c]_{Tr}\left(\frac{T_w}{T_b}\right)^d \quad (4)$$

If in Eqs. (3) and (4)  $\rho$  and  $C_p$  are held constant we could evaluate the effect of inaccuracies in  $\mu$  and  $k$  on  $h$ . For example, from Eq. (3) we could easily see that any error in  $\mu$  and  $k$  will translate directly into an error in  $h$ . For illustrative purposes if  $b = 0.8$  and  $c = 0.4$ , a 15% error in  $k$  will result in a 8.7% error in  $h$ . Likewise, a 15% error in  $\mu$  will result in a 15.4% error in  $h$ . The compounded error is about 25%. Thus, if accurate values for  $h$  are desired,  $\mu$  and  $k$  must be known accurately. Also, any error in  $\rho$  and  $C_p$  will have about the same effect on  $h$ . Development of accurate expressions for  $\rho$  and  $C_p$  is the subject of another publication [3]. Therefore in order to solve convective heat transfer problems under variable fluid property conditions, such as the ones proposed by Ghajar and Parker [1, 4] and Ghorbani and Ghajar [61], accurate knowledge of the transport properties is essential.

## 2. RELATION-TO-THE-STATE-OF-THE-ART

It has been confirmed by several investigators that the transport properties,  $k$  and  $\mu$ , exhibit a singular behavior in the vicinity of the critical point. This behavior is not fully understood [5], so all the representative equations are more or less based on assumptions and mathematical models used to describe the anomalies. Most of these expressions are rather complicated and require laborious computations of the derivatives of the equation of state [5-7], so in order to predict the necessary values one has to obtain the derivatives which are not very accurate in the vicinity of the critical point [8, 9].

In spite of these most recent and complicated representative equations, large errors still exist between the predicted values and the experimental data. For example, deviations in the order of 5 to 10% are common [5, 7, 10-12] for thermal conductivity.

Tables I and II summarize the available analytical studies for thermal conductivity and viscosity, respectively. These tables list for each substance under study the source reference for the analytical expressions of  $k$  and  $\mu$ , the year they were published, the experimental data set used to construct the equations, the estimated accuracy of the equations, and finally some remarks on the method or form of the expressions.

TABLE I

Thermal conductivity—analytical studies

Fluid	Year	Ref. no.	Exp. data used [ref. no.]	Estimated accuracy %	Remarks
CO <sub>2</sub>	1968	13	14	×	Based on diffusion and dissociation of clusters
	1970	15	14	5-15	Based on scaling laws (simple version)
	1971	12	14	×	Based on scaling laws (simple version)
	1972	16	14	×	Based on scaling laws
	1980	17	13, 14	2.5	Exponential form of density, good for the supercritical region
	1981	5	14	<10	Based on scaling laws, used with scaled equation of state
H <sub>2</sub> O	1973	6	6	×	Based on scaling laws
	1977	18	29, 30	2-8	Based on scaling laws
	1978, 79	19, 20	×	×	Based on scaling laws
	1981	11	29, 30	5-16	Based on scaling laws
	1981	5	29, 30	5-8	Based on scaling laws, used with scaled equation of state
N <sub>2</sub>	1974	7, 21	33, 34	13	Based on scaling laws
	1975	22	×	×	Based on scaling laws (simple version)
CH <sub>4</sub>	1971	12	35	large	Based on scaling laws (simple version)
	1975, 77	23, 24	×	×	Based on scaling laws
H <sub>2</sub>	1970	25	25	×	Function of $C_p$
O <sub>2</sub>	1971, 75	12, 22	×	×	Based on scaling laws (simple version)
	1974	7, 26	×	15	Based on scaling laws
C <sub>2</sub> H <sub>6</sub>	1979	27	×	×	Based on scaling laws
NH <sub>3</sub>	1977	28	×	×	Based on scaling laws

× Not available.

TABLE II

Viscosity—analytical studies

Fluid	Year	Ref. no.	Exp. data used [Ref. no.]	Estimated accuracy %	Remarks
CO <sub>2</sub>	1972	36	41	2.4	Eight parameter equation. Polynomial of reduced $T$ and $\rho$
	1975	22	42, 43	×	Polynomial of $T$ and $\rho$
	1980	17	41, 44	1.4	Exponential term of density, good for the supercritical region
H <sub>2</sub> O	1981	37	37	×	Scaling laws
	1979	38	×	2-10	Polynomial of density and temperature
	1980	10, 39	45 <sup>a</sup>	1	Based on scaling laws
	1981	11	45	1-4	Based on scaling laws
N <sub>2</sub>	1979	5,40	46	1-3	Based on scaling laws

<sup>a</sup> Used re-evaluated data from [45].

× Not available.

### 3. METHOD OF APPROACH

Development of expressions for the transport properties were based on the scaling-laws to predict the near-critical behavior and a polynomial of density to describe the behavior further away from the critical-point. Both expressions can become infinite at the critical point.

#### 3.1. Thermal Conductivity

Sengers and Keyes [15], devised a scaling procedure for determining the near-critical thermal conductivity of carbon dioxide. It was shown that the experimental data for the thermal conductivity of carbon dioxide satisfy scaling-laws relations very similar to those previously established for equilibrium properties near the critical point. They demonstrated that the reduced thermal conductivity data can be represented by a single-valued function of the scaling parameter, for relatively large ranges of temperatures and densities. The essence of their work can be represented by the following equation [12], which approximated their data very well:

$$y = \left[ 1 + \frac{0.9}{(x^\beta)^{2.86}} \right]^{(-0.6)} \quad (5)$$

where

$$y = \frac{(k_c - k)\rho_r^{-0.5}}{e_1 T^{*-0.6}} \quad (5a)$$

$$x^\beta = \left( \frac{T^*}{\rho^{*2.86}} \right)^{0.35} \quad (5b)$$

in which

$$T^* = \left| \frac{T - T_c}{T_c} \right| \quad (5c)$$

$$\rho^* = \left| \frac{\rho - \rho_c}{\rho_c} \right| \quad (5d)$$

The dimensionless parametric groupings  $y$  and  $x^\beta$  in Eq. (5), represent the reduced thermal conductivity and the scaling parameter, respectively. In Eq. (5a),  $e_1$  is the proportionality constant, which has the same dimensions as the thermal conductivity. The exponent  $\beta$  in Eq. (5b) represents the exponent of the coexistence curve. Further details about the parametric groupings, Eqs. (5a) and (5b), and the development of Eq. (5) may be found elsewhere [15, 12].

Simplifying Eq. (5) and rearranging the terms results in

$$k_c - k = e_1 \left[ T^* \left( 1 + \frac{0.9\rho^{*2.86}}{T^*} \right) \right]^{-0.6} \rho_r^{0.5} \quad (6)$$

The "normal" thermal conductivity (without critical enhancement) has often been described as a polynomial of density, so if we write it as a third order polynomial

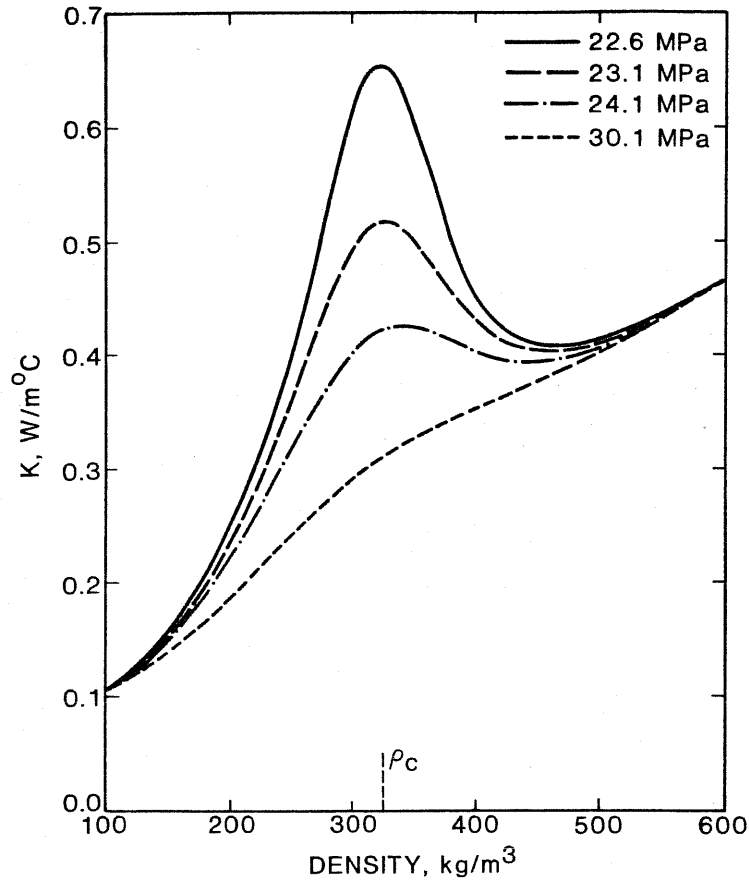


FIGURE 1 Variation of thermal conductivity vs. density for water at different pressures, data from [29–32].

of density, we have

$$k = a_1 + b_1\rho + c_1\rho^2 + d_1\rho^3 \quad (7)$$

combining Eqs. (6) and (7) we obtain:

$$k_c = a_1 + b_1\rho + c_1\rho^2 + d_1\rho^3 + e_1 \left[ T^* \left( 1 + \frac{0.9\rho^{*2.86}}{T^*} \right) \right]^{-0.6} \rho_r^{0.5} \quad (8)$$

Equation (8) describes the thermal conductivity in the critical and the near-critical region (with critical enhancement) and overlaps very well with the other accurate available expressions outside the near-critical region. Typical variation of thermal conductivity of water in the near-critical region based on data of [29–32] is shown in Figure 1.

Note that Eq. (8) has only five constants to be curve-fitted and in general is a fairly simple expression. Also it should be noted that the expression is highly density dependent which requires accurate density values from an equation of

state. The analytical values of density used in this expression were provided by a Benedict–Webb–Rubin type equation of state. Further details may be found in [3].

### 3.2. Viscosity

It has been shown [5, 11] that the critical enhancement of viscosity is much smaller than that of thermal conductivity for the same thermodynamic region, but shows the same trend as the thermal conductivity.

Ghajar and Parker [17] reported that the same expression could be applied to the viscosity and thermal conductivity in the near-supercritical region due to the similar behavior of these transport properties. In other literature different forms of representative equations were reported. However, in this work the same form of equation will be applied.

An expression similar to Eq. (6) developed for thermal conductivity can be developed for viscosity, the resulting equation is:

$$\mu_c - \mu = e_2 \left[ T^* \left( 1 + \frac{0.9\rho^{*2.86}}{T^*} \right) \right]^{-0.6} \rho_r^{0.5} \quad (9)$$

The “normal” viscosity (without critical enhancement) can be described as a polynomial of density, similar to the thermal conductivity. Following a procedure similar to the one for thermal conductivity, we can apply a third order polynomial of density and express the “normal” viscosity as:

$$\mu = a_2 + b_2\rho + c_2\rho^2 + d_2\rho^3 \quad (10)$$

combining Eqs. (9) and (10) yields:

$$\mu_c = a_2 + b_2\rho + c_2\rho^2 + d_2\rho^3 + e_2 \left[ T^* \left( 1 + \frac{0.9\rho^{*2.86}}{T^*} \right) \right]^{-0.6} \rho_r^{0.5} \quad (11)$$

Equation (11) describes the viscosity (with critical enhancement) in the critical and the near-critical region and overlaps very well with other accurate available expressions outside the near-critical region. Typical variation of viscosity of water in the near-critical region based on data of [45] is shown in Figure 2.

Note that Eq. (11) has the same characteristics as Eq. (8), that is; has only five curve-fitted constants and is relatively simple. Also, it is highly dependent on the density.

### 3.3. Equation of State

Equations (8) and (11) are highly dependent on density, so it is of great importance that accurate analytical density values are used in these expressions. In relation to this work a simple Benedict–Webb–Rubin type equation of state has been curve-fitted by Najjar and Ghajar [3]. This equation has only eight curve-fitted constants and has proved to be accurate in the near-critical region.

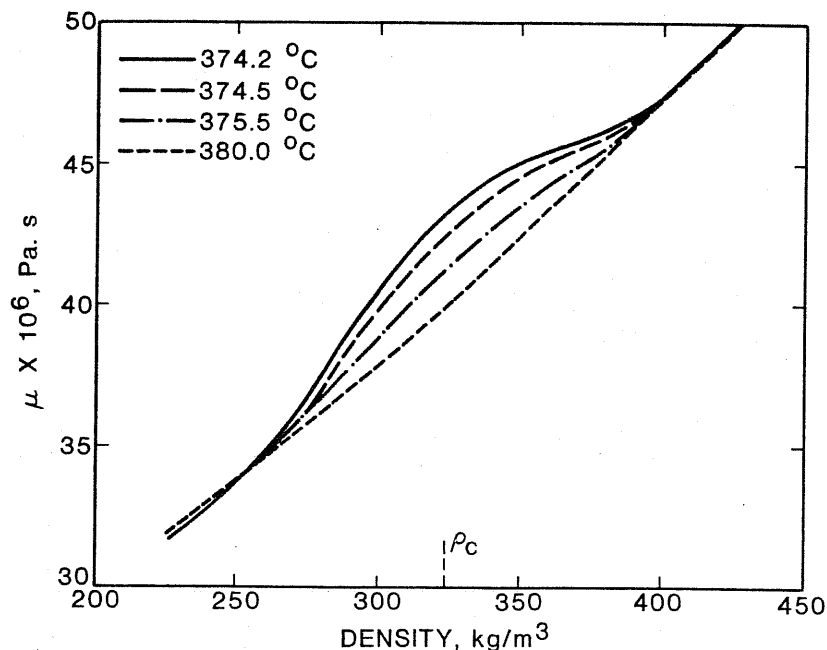


FIGURE 2 Variation of viscosity vs. density for water at different pressures, data from [45].

The form of the equation is:

$$\begin{aligned}
 P = & RT/v + (B_0RT - A_0 - C_0/T^2)/v^2 \\
 & + (bRT - a)/v^3 + a\alpha/v^6 \\
 & + [c(1 + \gamma/v^2)\exp(-\gamma/v^2)]/(v^3T^2)
 \end{aligned}
 \tag{12}$$

The eight parameters  $B_0$ ,  $A_0$ ,  $C_0$ ,  $b$ ,  $a$ ,  $c$ ,  $\alpha$  and  $\gamma$  are numerical constants which are different for different gases. These constants were obtained by curve fitting the equation to experimental  $P$ - $v$ - $T$  data. The units and constants to be used in the equation of state, and its range of application are tabulated in Table III for water, carbon dioxide, ammonia, and nitrogen.

Due to the fact that the equation of state is explicit only in pressure, it was necessary to use numerical techniques (half interval search) to obtain values for density. Further details may be found in [3].

#### 4. EXPERIMENTAL DATA USED

An intensive literature survey was carried out to obtain the available experimental data for the substances under study. The results are shown in Tables IV and V for thermal conductivity and viscosity, respectively. These tables list for each substance under study, the source references, the year the experiments were

TABLE III

Units and constants for equation of state

Parameter	Description	Units	Water	Carbon dioxide	Ammonia	Nitrogen
$P$	Pressure	Atm.	—	—	—	—
$T$	Temperature	°K	—	—	—	—
$v$	Specific volume	cm <sup>3</sup> /gmol	—	—	—	—
$R$	Gas constant	cm <sup>3</sup> -Atm/gmol-K	82.05	82.05	82.05	82.05
$B_0$	Constant	—	$4.366083 \times 10^1$	$6.348170 \times 10^1$	$3.377578 \times 10^1$	$4.515999 \times 10^1$
$A_0$	Constant	—	$3.455640 \times 10^6$	$2.978731 \times 10^6$	$3.128520 \times 10^6$	$1.189430 \times 10^6$
$C_0$	Constant	—	$1.403741 \times 10^{12}$	$1.411484 \times 10^{11}$	$1.878376 \times 10^{11}$	$5.594108 \times 10^9$
$b$	Constant	—	$-5.285869 \times 10^2$	$2.675216 \times 10^3$	$1.609715 \times 10^3$	$2.913385 \times 10^3$
$a$	Constant	—	$6.437270 \times 10^6$	$1.050408 \times 10^8$	$4.715941 \times 10^7$	$2.777842 \times 10^7$
$\alpha$	Constant	—	$2.537106 \times 10^5$	$8.947460 \times 10^4$	$3.751366 \times 10^4$	$1.692999 \times 10^4$
$c$	Constant	—	$6.846662 \times 10^{13}$	$1.446193 \times 10^{13}$	$1.505006 \times 10^{13}$	$5.711593 \times 10^{11}$
$\gamma$	Constant	—	$1.660712 \times 10^3$	$4.586610 \times 10^3$	$2.376220 \times 10^3$	$2.409941 \times 10^3$
Range of application			Water	Carbon dioxide	Ammonia	Nitrogen
$T_r = T/T_c$			0.9779–1.1944	0.9801–1.1450	0.8968–1.1100	0.9511–1.9809
$P_r = P/P_c$			0.8107–1.3122	0.7018–1.7752	0.7129–1.8742	0.4569–4.5135

TABLE IV

Thermal conductivity—experimental studies

Fluid	Year	Ref. no	No. of points	Range			Apparatus	Accuracy %
				$T_r$	$\rho_r$	$P_r$		
CO <sub>2</sub>	1962	13	39	0.91 to 1.15	0.86 to 1.19		coaxial cylinder	3
	1962	14 <sup>c</sup>	119	0.98 to 1.03	0.7 to 1.52		parallel plate	×
	1970	47	53	1.0 to 1.018	0.96 to 1.06		parallel plate	3
	1973	6	47	1.00 to 1.16	0.2 to 1.5		coaxial cylinder	×
	1977	48 <sup>b</sup>	157	×	×		parallel plate	×
H <sub>2</sub> O	1973	6	129	1.0 to 1.21	0.3 to 2.0		coaxial cylinder	2
	1976–1981	29, 30, 31 32 <sup>c</sup> , 49	140	0.999 to 1.084	0.43 to 2.04		parallel plate	×
N <sub>2</sub>	1952	33 <sup>a</sup>	3	0.71 to 1.51		0.02 to 2.0	coaxial cylinder	5
	1958	34 <sup>a</sup>	10	0.67 to 1.6		0.3 to 4.0	coaxial cylinder	×
NH <sub>3</sub>	1964	50	35	0.87 to 1.64		0.01 to 3.4	coaxial cylinder	1
	1965	51 <sup>c</sup>	115	0.72 to 1.11		0.1 to 4.3	coaxial cylinder	×
CH <sub>4</sub>	1967	35	440	0.57 to 1.25		0.02 to 10.9	coaxial cylinder	1
	1963	52 <sup>a</sup>	×	×		×	×	×
	1973	53 <sup>a</sup>	×	0.63 to 2.1		0.54 to 15.2	hot wire	1
H <sub>2</sub>	1970	25 <sup>a</sup>	231	0.52 to 6.09		0.04 to 11.65	parallel plate	2
O <sub>2</sub>	1955	54 <sup>a</sup>	66	0.52 to 1.3		0.02 to 2.6	coaxial cylinder	×
He-3	1969	55 <sup>a</sup>	×	0.45 to 1.19		0.0 to 29.6	parallel plate	3

<sup>a</sup> Near-critical region not covered.<sup>b</sup> Thermal conductivity derived from thermal diffusivity data.<sup>c</sup> Experimental data used in this study.

× Not available.



TABLE V

Viscosity—experimental studies

Fluid	Year	Ref. no.	No. of points	Range			Apparatus	Accuracy %
				$T_r$	$\rho_r$	$P_r$		
CO <sub>2</sub>	1957	44	194	0.98 to 1.14	0.1 to 2.5		capillary	×
	1964	44	107	0.98 to 1.06	0.18 to 1.24		oscillating disk	×
	1981	37 <sup>b</sup>	383	0.98 to 1.062		0.1 to 1.96	oscillating disk	×
H <sub>2</sub> O	1975	45 <sup>c</sup>	78	1.0 to 1.25		1.0 to 1.25	capillary	1
	1977	56 <sup>a</sup>	93	0.95 to 1.07		0.0045 to 1.08	oscillating disk	×
CH <sub>4</sub>	1970	57 <sup>a</sup>	102	0.50 to 0.98		0.2 to 2.2	oscillating disk	×
	1980	58 <sup>a</sup>	12	0.52 to 1.57		0.2 to 6.5	oscillating quartz crystal	2
N <sub>2</sub>	1974	46 <sup>b</sup>	110	1.0 to 1.07		0.01 to 1.45	oscillating disk	×

<sup>a</sup> Near-critical region not covered.<sup>b</sup> Experimental data used in this study.<sup>c</sup> Experimental data of [45] as re-evaluated by [10] was used in this study.

× Not available.

reported, the number of reported experimental data points, the range of temperature and pressure or density, the type of experimental apparatus used and finally, the investigator's estimated accuracy of their experimental data.

## 5. DETERMINATION OF THE NUMERICAL CONSTANTS

The constants in the thermal conductivity expression were obtained by curve fitting the expression to the experimental thermal conductivity data of water [32], carbon dioxide [14], and ammonia [51]. The constants in the viscosity expression were obtained by curve fitting the expression to the experimental viscosity data of water [10], carbon dioxide [37], and nitrogen [46]. A computer program called MARQ [59] which performs a nonlinear least squares fit was used for curve fitting purposes.

Two sets of curve-fitted constants were obtained, one set for the analytical values of density and another set for the experimental values of density. The reason for this is that the proposed analytical expression is highly dependent on the density, and when comparisons were made, better results were obtained by using the experimental density values as opposed to the analytical values of density for the region very close to the critical point. So any small deviation from the experimental density generated high deviation between the predicted transport property and the experimental value.

The two sets of constants are reported to give an alternative in case a very accurate equation of state in the region very close to the critical point is available. Such equations have been developed and are based on the scaling laws [5, 9].

They are only valid in a very narrow range of the critical point. The future user could use either the equation of state used in this work along with the scaled equation of state with the analytical set of constants or alternatively use the set of constants based on the experimental values of density.

Investigators [5, 10] have stated that in general it would be necessary for each experiment dealing with the properties in the near-critical region to define its own set of critical constants due to the uniqueness of each experiment. Also, the near-critical properties are highly sensitive to the values of the critical constants when they are predicted by expressions based on the scaling-laws. Therefore, sometimes it is necessary to adjust the critical constants in order to obtain an accurate expression which is a strong function of these constants. This is especially true for the expressions based on scaling-laws; expressions similar to Eqs. (8) and (11). In this work it was necessary to adjust the critical density of ammonia in order to improve the prediction of the analytical expression. This was carried out by substituting the critical density in Eq. (8) by a curve-fitted constant. This curve-fitted constant was then used as the "critical density".

The results presented here are for the substances with reliable reported experimental data for thermal conductivity (water, carbon dioxide and ammonia) and viscosity (water, carbon dioxide and nitrogen). For the experimental data of Roder and Diller [25] for hydrogen the obtained curve-fitted expression was not acceptable due to the lack of reliable experimental data. The thermal conductivity data of Sokolova and Golubev [35] for methane were also used for curve-fitting purposes. The results were far from being acceptable due to the fact that the experimental data did not show the same behavior in the near-critical region as observed for the other substances under study. Further details may be found in [60].

The units and constants to be used in the expressions for thermal conductivity and viscosity are given in Tables VI and VII, respectively. In these tables two sets of constants along with the critical constants used in the development of the transport property expressions are reported. One set is based on the analytical values of density from [3], and the other is based on the experimental density data.

Tables VIII and IX give the ranges of reduced densities and temperatures for which the developed relationships of this study are applicable, see Eqs. (8) and (11). Based on these tables, the ranges of densities and temperatures for the thermal conductivity and viscosity expressions for the three fluids are:

#### Thermal Conductivity

- (1) Carbon Dioxide,  $\rho = 176.32$  to  $928.00$  kg/m<sup>3</sup> and  $T = 298.13$  to  $334.63$  °K
- (2) Water,  $\rho = 86.94$  to  $640.78$  kg/m<sup>3</sup>, and  $T = 634.11$  to  $711.76$  °K
- (3) Ammonia,  $\rho = 81.16$  to  $446.38$  kg/m<sup>3</sup> and  $T = 398.66$  to  $441.78$  °K

#### Viscosity

- (1) Carbon Dioxide,  $\rho = 176.32$  to  $742.40$  kg/m<sup>3</sup> and  $T = 304.21$  to  $323.07$  °K
- (2) Water,  $\rho = 128.80$  to  $579.60$  kg/m<sup>3</sup> and  $T = 669.83$  to  $704.03$  °K
- (3) Nitrogen,  $\rho = 109.91$  to  $596.66$  kg/m<sup>3</sup> and  $T = 126.20$  to  $135.03$  °K

TABLE VI

Units and constants for thermal conductivity

Parameter	Description	Units	CO <sub>2</sub>		H <sub>2</sub> O		NH <sub>3</sub>	
			Ana. <sup>a</sup>	Exp. <sup>b</sup>	Ana. <sup>a</sup>	Exp. <sup>b</sup>	Ana. <sup>a</sup>	Exp. <sup>b</sup>
<i>k</i>	Thermal conductivity	W/m <sup>2</sup> K	—	—	—	—	—	—
<i>M</i>	Molecular weight	kg/kg-mole	4.4016 × 10 <sup>1</sup>	—	1.8016 × 10 <sup>1</sup>	—	1.703 × 10 <sup>1</sup>	—
<i>T<sub>c</sub></i>	Critical temperature	°K	3.0421 × 10 <sup>2</sup>	—	6.4705 × 10 <sup>2</sup>	—	4.068 × 10 <sup>2</sup>	—
<i>P<sub>c</sub></i>	Critical pressure	KPa	7.383 × 10 <sup>3</sup>	—	2.209 × 10 <sup>4</sup>	—	1.1627 × 10 <sup>4</sup>	—
<i>ρ<sub>c</sub></i>	Critical density	kg/m <sup>3</sup>	4.64 × 10 <sup>2</sup>	—	3.22 × 10 <sup>2</sup>	—	2.029 × 10 <sup>2c</sup>	—
<i>a<sub>1</sub></i>	Constant	—	-4.406180 × 10 <sup>-2</sup>	-4.423736 × 10 <sup>-2</sup>	1.822742 × 10 <sup>-2</sup>	1.153930 × 10 <sup>-2</sup>	1.265895 × 10 <sup>-1</sup>	1.342923 × 10 <sup>-1</sup>
<i>b<sub>1</sub></i>	Constant	—	5.981447 × 10 <sup>-4</sup>	5.937360 × 10 <sup>-4</sup>	8.095208 × 10 <sup>-4</sup>	8.678986 × 10 <sup>-4</sup>	-1.279279 × 10 <sup>-3</sup>	-1.425101 × 10 <sup>-3</sup>
<i>c<sub>1</sub></i>	Constant	—	-1.331925 × 10 <sup>-6</sup>	-1.313111 × 10 <sup>-6</sup>	-1.340863 × 10 <sup>-6</sup>	-1.420788 × 10 <sup>-6</sup>	6.900752 × 10 <sup>-6</sup>	7.661185 × 10 <sup>-6</sup>
<i>d<sub>1</sub></i>	Constant	—	1.022006 × 10 <sup>-9</sup>	1.100290 × 10 <sup>-9</sup>	1.995175 × 10 <sup>-9</sup>	1.954992 × 10 <sup>-9</sup>	-7.376971 × 10 <sup>-9</sup>	-8.489347 × 10 <sup>-9</sup>
<i>e<sub>1</sub></i>	Constant	—	2.739334 × 10 <sup>-3</sup>	2.694979 × 10 <sup>-3</sup>	1.581311 × 10 <sup>-2</sup>	1.566376 × 10 <sup>-2</sup>	1.077567 × 10 <sup>-2</sup>	1.076967 × 10 <sup>-2</sup>

<sup>a</sup> Ana. = Curve-fitted constants by using analytical values of density.<sup>b</sup> Exp. = Curve-fitted constants by using experimental values of density.<sup>c</sup> Adjusted critical parameter.

TABLE VII

Units and constants for viscosity

Parameter	Description	Units	CO <sub>2</sub>		H <sub>2</sub> O		N <sub>2</sub>	
			Ana. <sup>a</sup>	Exp. <sup>b</sup>	Ana. <sup>a</sup>	Exp. <sup>c</sup>	Ana. <sup>a</sup>	Exp. <sup>b</sup>
$\mu$	Viscosity	N-s/m <sup>2</sup>	—	—	—	—	—	—
$M$	Molecular weight	kg/kg-mole	$4.4016 \times 10^1$	—	$1.8016 \times 10^1$	—	$2.801 \times 10^1$	—
$T_c$	Critical temperature	°K	$3.0421 \times 10^2$	—	$6.4705 \times 10^2$	—	$1.262 \times 10^2$	—
$P_c$	Critical pressure	kPa	$7.383 \times 10^3$	—	$2.209 \times 10^4$	—	$3.4 \times 10^3$	—
$\rho_c$	Critical density	kg/m <sup>3</sup>	$4.64 \times 10^2$	—	$3.22 \times 10^2$	—	$3.1403 \times 10^2$	—
$a_2$	Constant	—	$1.232416 \times 10^{-5}$	$1.419889 \times 10^{-5}$	$1.705897 \times 10^{-5}$	$1.828227 \times 10^{-5}$	$1.080524 \times 10^{-6}$	$4.195232 \times 10^{-6}$
$b_2$	Constant	—	$3.097280 \times 10^{-8}$	$1.303647 \times 10^{-8}$	$5.650212 \times 10^{-8}$	$4.207545 \times 10^{-8}$	$-1.981027 \times 10^{-8}$	$4.884292 \times 10^{-8}$
$c_2$	Constant	—	$-3.265285 \times 10^{-12}$	$4.847735 \times 10^{-11}$	$3.508322 \times 10^{-11}$	$8.989557 \times 10^{-11}$	$1.622664 \times 10^{-10}$	$-6.384812 \times 10^{-11}$
$d_2$	Constant	—	$6.713438 \times 10^{-14}$	$2.421705 \times 10^{-14}$	$3.228291 \times 10^{-14}$	$-3.602178 \times 10^{-14}$	$-7.359191 \times 10^{-14}$	$1.686762 \times 10^{-13}$
$e_2$	Constant	—	$1.449306 \times 10^{-8}$	$1.131738 \times 10^{-8}$	$2.577068 \times 10^{-8}$	$2.648099 \times 10^{-8}$	$1.1942711 \times 10^{-8}$	$1.135415 \times 10^{-8}$

<sup>a</sup> Ana. = Curve-fitted constants by using analytical values of density.<sup>b</sup> Exp. = Curve-fitted constants by using experimental values of density.<sup>c</sup> Exp. = Curve-fitted constants by using re-evaluated values of density [10].

TABLE VIII  
Summary of thermal conductivity equation

Substance	Set of constants	Eqn. no.	Source ref. no.	Number of points	Average percent absolute deviation	Maximum percent absolute deviation	Curve-fitted range $\rho_r$	Curve-fitted range $T_r$	Range of application $\rho_r$	Range of application $T_r$
Carbon dioxide	Analytical density	8	14	83	3.48	16.04	0.38-1.52	0.98-1.03	0.38-2.0	0.98-1.1
	Experimental density	8	14	83	3.09	11.03	0.38-1.52	0.98-1.03	0.38-2.0	0.98-1.1
Water	Analytical density	8	32	63	4.83	18.2	0.27-1.99	0.97-1.083	0.27-1.99	0.98-1.1
	Experimental density	8	32	63	3.97	12.01	0.27-1.99	0.97-1.083	0.27-1.99	0.98-1.1
Ammonia	Analytical density	8	51	35	3.45	11.1	0.43-2.08	0.98-1.06	0.4-2.2	0.98-1.086
	Experimental density	8	51	35	3.12	11.1	0.98-2.08	0.98-1.06	0.4-2.2	0.98-1.086

TABLE IX  
Summary of viscosity equation

Substance	Set of constants	Eqn. no.	Source ref. no.	Number of points	Average percent absolute deviation	Maximum percent absolute deviation	Curve-fitted range $\rho_r$ , $T_r$	Range of application $\rho_r$ , $T_r$
Carbon dioxide	Analytical density	11	37	193	1.29	4.7	0.38-1.6	0.38-1.6
	Experimental density	11	37	193	0.78	4.7	0.38-1.6	0.38-1.6
Water	Analytical density	11	10	64	1.16	3.49	0.67-1.66	0.4-1.8
	Experimental density	11	10	64	0.73	3.49	0.67-1.66	0.4-1.8
Nitrogen	Analytical density	11	46	72	1.5	4.86	0.60-1.66	0.35-1.9
	Experimental density	11	46	72	1.0	4.05	0.60-1.66	0.35-1.9

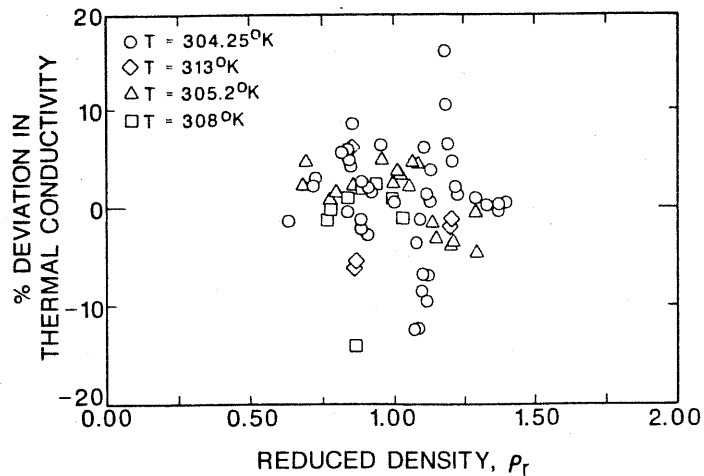


FIGURE 3 Percent deviation between predicted thermal conductivity and experimental data of [14] for carbon dioxide using analytical density values.

## 6. RESULTS

The fit of the proposed expression for thermal conductivity to the available experimental data was very good for all the three substances. The results of comparison with the experimental data are summarized in Table VIII. Some of these results are also shown in Figures 3 and 4 for carbon dioxide.

The proposed viscosity expression predicted the available experimental data excellently and the results of the comparison are presented in Table IX. Some of these results are also shown in Figures 5 and 6 for carbon dioxide.

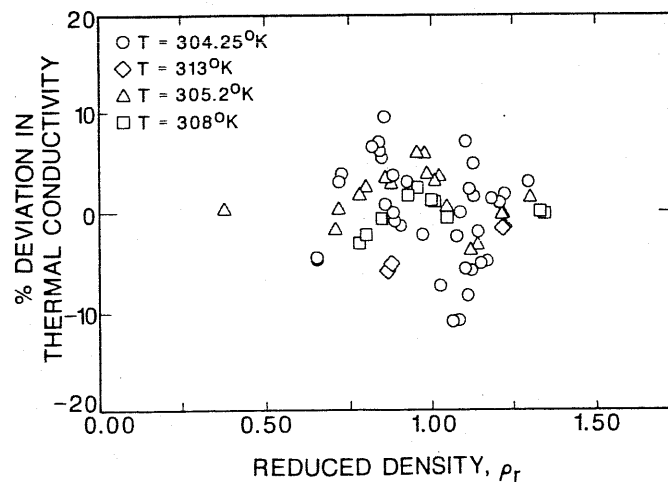


FIGURE 4 Percent deviation between predicted thermal conductivity and experimental data of [14] for carbon dioxide using experimental density data.

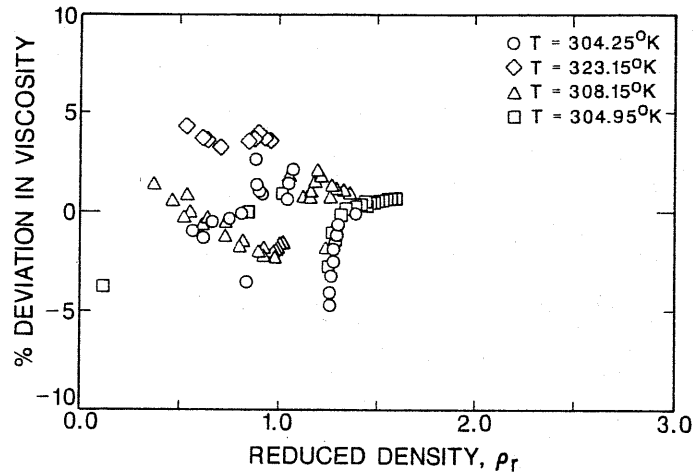


FIGURE 5 Percent deviation between predicted viscosity and experimental data of [37] for carbon dioxide using analytical density values.

A detailed comparison between the developed expressions and the other analytical equations was not possible. This was mainly due to the fact that the investigators did not report a point by point comparison between their analytical predictions and the experimental data. However, whenever possible, an attempt was made to estimate the accuracy of some of the analytical expressions for the transport properties as shown in Tables I and II. Based on these tables the present expressions show better or comparable accuracy.

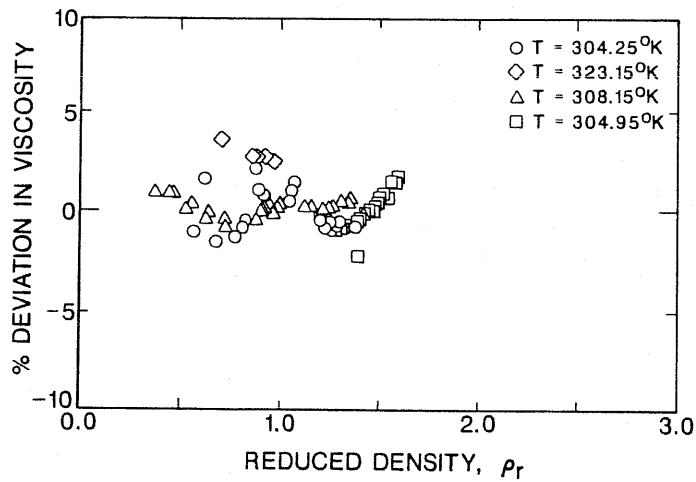


FIGURE 6 Percent deviation between predicted viscosity and experimental data of [37] for carbon dioxide using experimental density data.



## 7. CONCLUSIONS

The comparison of the predicted thermal conductivity values with the available experimental data showed that the simple analytical expression developed in this work predicts the experimental thermal conductivity data very well. This expression also shows better or comparable accuracy when compared to the long and complicated expressions available in the literature (see Tables I and VIII).

The comparison of the predicted viscosity values with the available experimental data showed that the simple analytical expression developed in this work excellently predicts the experimental viscosity data. This expression also shows a comparable accuracy when compared to the long and complicated expressions available in the literature (see Tables II and IX).

## NOMENCLATURE

$a, b, c, d$	Constants used in Eqs. (1) and (2)
$a_1, b_1, c_1, d_1, e_1$	Constants used in Eqs. (6-8)
$a_2, b_2, c_2, d_2, e_2$	Constants used in Eqs. (9-11)
$B_0, A_0, C_0, b, a, c, \alpha, \gamma$	Constants used in Eq. (12)
$C_p$	Specific heat at constant pressure
$D$	Diameter
$Gr$	Grashof number, $Gr = \frac{gL^3\rho^2}{\mu^2} \left( \frac{\rho_\infty - \rho_w}{\rho_w} \right)$
$h$	Heat transfer coefficient
$k$	Thermal conductivity without critical enhancement ("normal" conductivity)
$k_c$	Thermal conductivity with critical enhancement
$L$	Length
$Nu$	Nusselt number, $Nu = hL/k$ or $hD/k$
$P$	Pressure
$Pr$	Prandtl number, $Pr = \mu C_p/k$
$R$	Gas constant
$Re$	Reynolds number, $Re = \rho VD/\mu$
$T$	Temperature
$T^*$	Dimensionless temperature defined by Eq. (5c)
$Tr$	Reference temperature
$v$	Specific volume
$V$	Velocity

$x^\beta$	Parameter defined by Eq. (5b)
$y$	Parameter defined by Eq. (5a)
$\mu$	Absolute viscosity without critical enhancement ("normal" viscosity)
$\mu_c$	Absolute viscosity with critical enhancement
$\rho$	Density
$\rho^*$	Dimensionless density defined by Eq. (5d)

### Subscripts

$b$	Evaluated at bulk temperature
$c$	Refers to critical property
$r$	Refers to reduced property, as $T_r = T/T_c$
$w$	Evaluated at the wall temperature
$\infty$	Evaluated at the free stream temperature

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