

## FLOW PATTERNS AND VOID FRACTION IN DOWNWARD TWO PHASE FLOW

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### ABSTRACT

The two phase flow is of significant interest in chemical and petroleum industries. The two phase flow in horizontal and vertical upward pipe orientation has been studied extensively in the literature. However, limited information is available in the literature on the downward two phase flow. To have a better understanding of this phenomenon, experimental investigation of downward two phase flow was carried out in a 0.0127m diameter tube. The objective of this experimental study was flow visualization and accurate measurements of the void fraction data. It is shown in the present study that the void fraction correlations available for upward flow are also applicable for downward orientation with a modification in the drift velocity. The experimental work is accompanied by thorough literature search and concludes with the identification of best performing void fraction correlations from a pool of available correlations.

### INTRODUCTION

Multiphase flow is a simultaneous flow of several phases which may be a combination of gas, liquid and solid phase. Two phase flow being the simplest case of multiphase flow with distinguished interfaces which separates one phase from another. The complexity in the two phase flow is primarily due to turbulent mixing of the two phases and the compressible nature of the gas phase. In the present study we are concerned and deal with two component two phase air-water flow in vertical downward orientation. The two phase flow has been of prime importance in petroleum, chemical and power industries. The two phase flow has been proven economical in the laying of the oil and natural gas pipelines while it acts to enhance appreciably the heat and mass transfer in heat exchangers and bubble columns. The research done so far in the field of two phase flow can be classified in terms of flow visualization, void fraction, pressure drop and rare investigations of two phase convective heat transfer. Most of this research is dedicated to horizontal and vertical upward two phase flow whereas limited information is available in the literature over the analysis of the vertical downward two phase flow.

The literature reports very few void fraction correlations dedicated to the downward two phase flow. The investigators have proposed correlations for downward two phase flow based on the drift flux model but none of them verified the applicability of the void fraction correlation developed for the upward flow to the downward flow. In the present study the concept of the drift flux was reviewed and the correlations developed for the vertical upward flow were applied to the downward flow data by merely using a negative value of the drift velocity assuming the phase velocities in the flow direction are positive.

The principal interest of this study was to identify the major flow patterns occurring in downward two phase flow followed by the analysis of the available void fraction correlations and finally the recommendation of the best flow pattern independent correlations to predict the void fraction in downward two phase flow.

### NOMENCLATURE

$m_c$	corrected mass (gm)
$m_{tot}$	total mass of the two phase mixture (gm)
$m_{liq}$	mass of liquid accumulated in tank (gm)
$U_{GM}$	drift velocity (m/s)
$U_{sg}$	superficial gas velocity (m/s)
$U_{sl}$	superficial liquid velocity (m/s)
$\alpha$	void fraction

### EXPERIMENTAL SETUP

The experimental setup used in present study is located at the Two Phase Lab, Oklahoma State University and was designed and verified by Cook (2008). This setup is capable of doing flow visualization through transparent test section and measurement of the void fraction, pressure drop and heat transfer data at all orientations from +90° to -90°. The schematic of overall experimental setup and the dimensional details of the flow visualization and void fraction test section are shown in Figs.1 and 2, respectively. In the present study two phase flow was analyzed for air water fluid combination and flowing through 0.0127 m ID polycarbonate pipe. Koflo

model 3-840-C-4-3V2 static mixer was used at inlet to ensure proper mixing of the two phases and good certainty in the flow visualization. A Bell & Gosset series 1535 centrifugal pump was used to pump water through a Whirlpool Aqua Pure AP12T water filter and then passed through ITT standard model BCF 4063 heat exchanger to remove the pumping heat and maintain the constant inlet temperature. It is then passed through Emerson Micro Motion Coriolis flow meter (CMF100) and then to the mixer. Air is supplied by Ingersoll Rand T30 industrial air compressor, passes through a filter and then from a heat exchanger to maintain the air temperature at room temperature. The air is then passed through an Emerson Micro Motion Coriolis flow meter (CMF 025 and LMF3M) for high and low air flow rates respectively. Omega Model TMQSS-06U-6 was installed at inlet and outlet of the test section to measure the two phase mixture temperature. The void fraction measurement was one of the major objectives of the present study. The void fraction section essentially consisted of three solenoid valves to trap two phase flow mixture and collect the residual liquid in the test section. The two solenoid valves were installed at inlet to switch on and off whereas the third solenoid valve severed as a bypass valve for two phase mixture during void fraction measurement. The quick closing solenoid valves were W.E. Anderson Model ABV1DA101 pneumatic ball valves with a closing time of 0.03 seconds. Compressed air was used to drain the water

from test section and was collected in an 8 liter high density polyethylene material tank. Before measuring the void fraction data it was necessary to calibrate the test section since some of the water may get trapped in the fittings and solenoid valves. The mass of water trapped inside the test section was calculated to be 12.5 gm. This mass was used as a correction factor and added to the total mass of water drained from the test section. Thus for any given mass flow rate of air and water the void fraction was calculated using the equation,

$$\alpha = 1 - \frac{m_{liq} + m_c}{m_{tot}}$$

The uncertainty associated with the void fraction measurement was calculated using a method proposed by Kline and McClintock (1953). The maximum uncertainty associated with the measured void fraction data in the present study was 0.0114 or  $\pm 5.40\%$  and the minimum was 0.018 or  $\pm 1.18\%$ . More details about this method are documented in Cook (2008). Flow patterns were observed for varying air and water mass flow rates. In addition to the visual observation the existence of a particular flow pattern was confirmed with the photographic evidences.

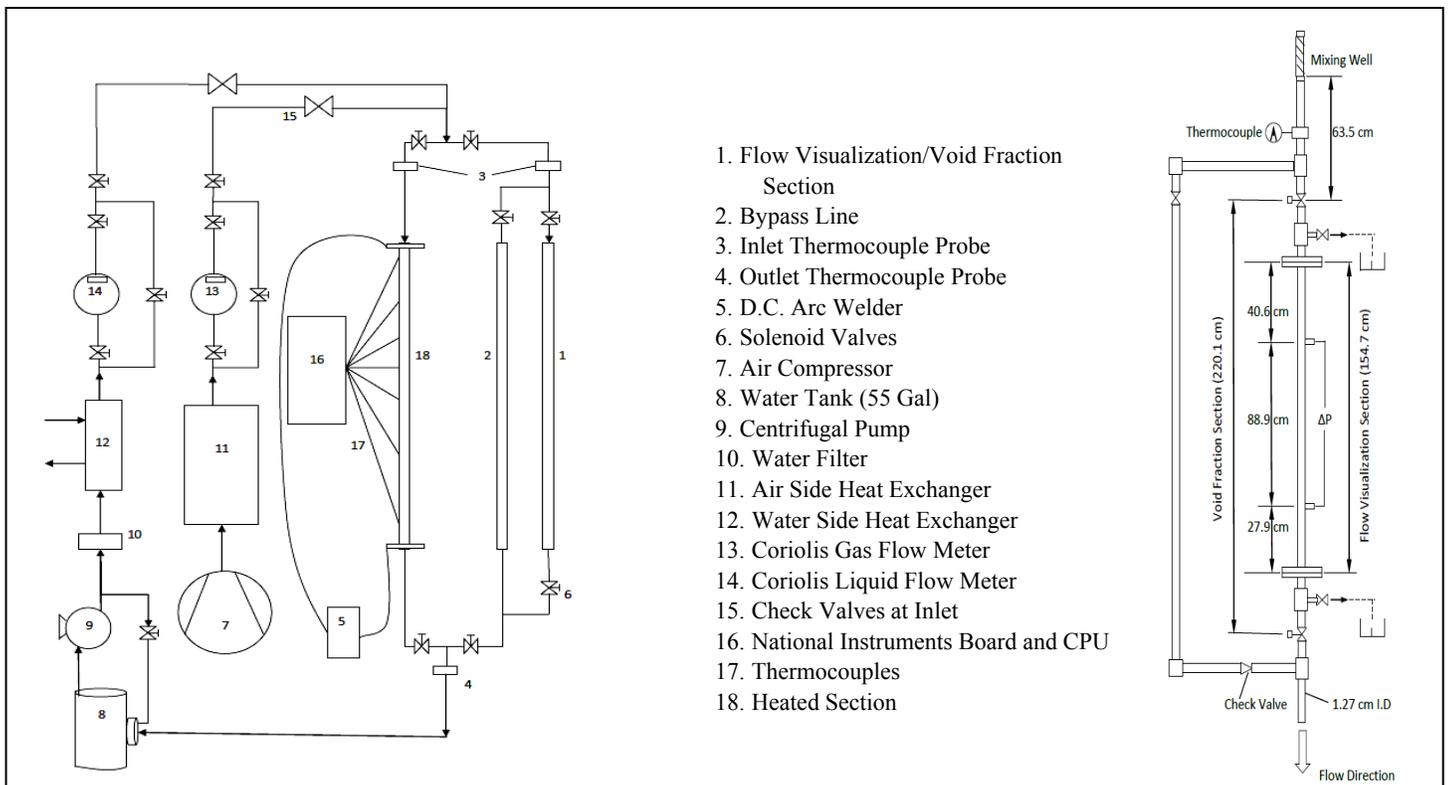
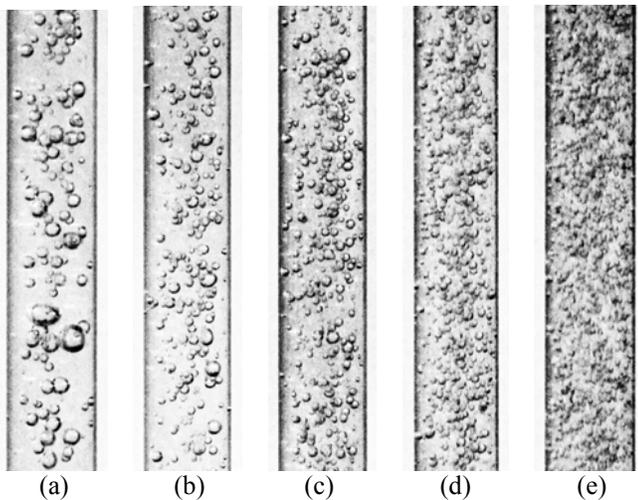


Figure 1 Schematic of experimental setup and the dimensional details of the test section

## EXPERIMENTAL RESULTS

**Flow Patterns:** Visual observation of the flow patterns was necessary in the present study since the two phase flow parameters like void fraction and pressure drop are directly influenced by the flow patterns. Different flow patterns exist in downward two phase flow depending upon the distribution of the individual phases across the pipe cross section. The present experimental study was carried out for constant liquid flow rates starting from low gas flow rates and then moving towards the higher gas flow rates. Five distinct flow patterns were observed namely, Bubbly, Slug, Froth, Falling Film and Annular flow.

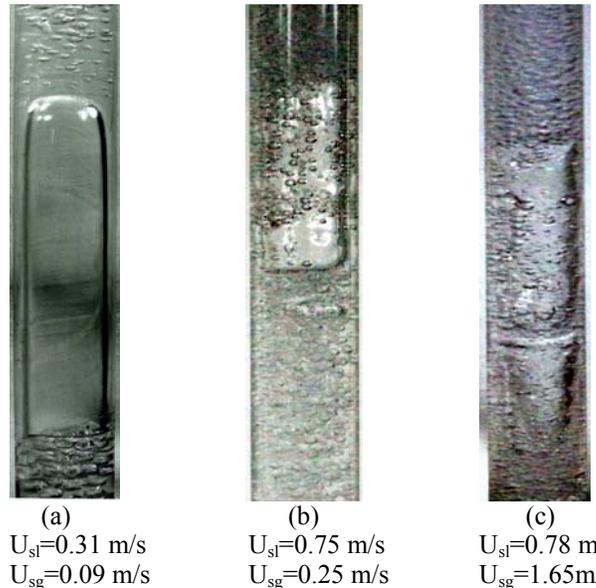
**Bubbly Flow:** The bubbly flow observed in the present study could be subcategorized as bubbly and dispersed bubbly flow. The bubbly flow was observed at moderate liquid flow rates and low gas flow rates. The dispersed bubbly flow was characterized by tiny shaped spherical bubbles dispersed in the continuous liquid phase. It was observed that at the constant gas flow rate the increase in the liquid flow rate caused the large gas bubbles to shear down to tiny spherical bubbles forming dispersed bubbly flow. The physical mechanism governing this transition can be explained from Fig. 3 for constant  $U_{sg} = 0.04$  m/s and  $U_{sl}$  increasing from 0.36 m/s to 1.08 m/s. It is observed that when the two phase mixture flows through a pipe at low gas and liquid flow rates the buoyant forces overcome the inertia forces and the gas bubble has a tendency to rise in the upward direction which is opposed by the inertia of the liquid phase moving in the opposite direction and exerts a shear force on the bubble surface resulting into disintegration of the bubbles into tiny, spherical, finely dispersed and evenly distributed bubbles into continuous liquid phase.



**Figure 2 Effect of increasing liquid superficial velocity on the bubble size, shape and distribution adopted from Ghajar and Tang (2010)**

**Slug Flow:** The slug flow is one of the most common occurring flow patterns in the two phase flow with gas and liquid phase flowing alternately through the pipe cross section. At the onset of the slug flow long gas pockets moving at an irregular interval characterized the slug flow. The gas slug length was observed to reduce with increase in the gas mass

flow rate. The motion of the gas bubble was observed carefully and still photographs were taken to see the effect of the gas and liquid mass flow rates on the bubble shape and the motion in two phase mixture. At low gas and liquid flow rates the bullet shaped Taylor bubble was observed to rise in upward direction due to dominant buoyancy effects over the inertia of the liquid phase. As the individual phase mass flow rates were increased gradually, both ends of the bubble nose were observed to be flat and there was no sign of air bubble trying to rise in upward direction. Eventually with increasing mass flow rates of air and water the air slug was observed to be pointing in the downward direction and moving in the direction of mean flow. Thus it can be speculated that the drift velocity or the bubble velocity do not remain constant in the slug flow. It can be inferred that the bubble velocity or alternatively the drift velocity ( $U_{GM}$ ) is negative at low air and water flow rates since the slug tries to rise in the upward direction; approaches a zero value when the slug nose becomes flat or alternatively when the buoyant forces are balanced by the inertia forces of the liquid phase and finally become positive when the air bubble points and moves in downward direction with the mean flow. The shape of the air slugs observed in the present study, as shown in Fig. 4 are consistent with the observation of Sekoguchi et al. (1996).



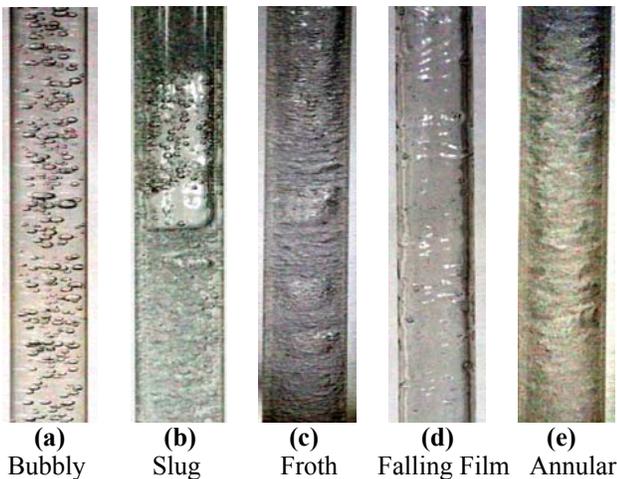
**Figure 3 Influence of the increasing phase mass flow rates over the slug shape and motion**

**Froth Flow:** This flow pattern was observed at high liquid flow rates and moderate gas flow rates in the present study. The onset of froth flow was marked by fast moving distorted slugs. The froth flow could be distinguished from the slug flow only on the basis of the fast moving, distorted and short length slugs with a frothy appearance to the two phase mixture. With the increase in the gas flow rate the gas slug appeared to be more distorted and eventually merged in the liquid phase giving frothy appearance to the flow. The transition to the froth flow can be achieved by increasing the gas flow rate in the slug regime and liquid flow rate in the falling film regime. Oshinowo (1971), Troniewski and Spisak (1987) and Yijun and Rezkallah (1993) have reported the

appearance of the froth flow in the downward two phase flow. The void fraction range associated with the froth flow in the present study is 0.41- 0.73.

**Falling Film Flow:** The falling film flow is a unique type of flow pattern established at very low liquid and moderate gas flow rates. The falling film flow was observed to have a gas phase flowing through the central core surrounded by a liquid film gliding over the pipe wall. Crawford (1983) documented that the falling film flow is essentially a separated flow which takes a form of stratified flow for declined orientations. Occasional dry spots were observed at the pipe surface. Oshinowo (1971) reported similar observations with occasional bubble entrainment in the falling liquid film. Yamazaki and Yamaguchi (1979) observed a similar flow pattern and called it a wetted wall flow. Crawford (1983) observed the liquid film trickling down along the pipe wall under the influence of gravity and called it as low energy or lazy annular flow.

**Annular Flow:** Annular flow appeared at high gas and liquid flow rates. The distribution of the two phases in the pipe cross section is similar to that of the falling film flow except the slow moving liquid film along the pipe wall is replaced by the fast moving agitated liquid film. Though the void fraction range associated with the falling film and annular flow regime are similar, the two phase pressure drop and the heat transfer coefficient associated with these flow patterns can be significantly different. Some investigators subcategorized the annular flow as wispy annular flow featuring liquid drops entrainment in the central gaseous core, however the existence of this flow pattern could not be confirmed in the present study since it was difficult to observe the liquid droplets entrainment in the core. The annular flow regime described above confirms the observations of Golan (1968), Oshinowo (1971), Yamazaki and Yamaguchi (1979), Usui and Sato (1989) and other investigators.



**Figure 4 Flow patterns in downward two phase flow**

**Void Fraction:** It is seen in the literature that very few correlations are available to predict the void fraction in the

downward two phase flow and most of them are based on the concept of the drift flux model. Most of these correlations were developed based on a very limited experimental data set. One of the foremost objectives of this study was to verify the correlations available in the literature and recommend the best performing flow pattern independent void fraction correlations for the downward two phase flow. In order to have a reliable performance of these correlations they were verified against an extensive data set of 909 data points. The experimental database used in the present study is shown in Table1. The analysis of the correlations based on the drift flux model showed that the void fraction correlations used for the vertical upward two phase flow were able to predict the void fraction in the downward flow by using the negative value of the drift velocity ( $U_{GM}$ ) while assuming that the velocities remain positive in the flow direction. The accuracy of the void fraction correlations is confirmed based on the percentage accuracy and the overall root mean square (RMS error). The performance analysis of these correlations was based on their satisfactory performance for the overall data and in the different ranges of the void fraction chosen in the present study. This performance evaluation in the different ranges of void fraction was necessary since the data set available was unevenly scattered in a range of  $0 < \alpha < 1$  and these ranges of the void fraction were chosen to approximate the flow patterns. For example the range  $0 < \alpha \leq 0.25$  approximated the bubbly flow,  $0.25 < \alpha \leq 0.5$  and  $0.5 < \alpha \leq 0.75$  approximated the slug and froth flow regime while the last range of the void fraction accommodated the falling film and annular flow patterns. This assumption though approximate, resembles the experimental data and observation, and is within  $\pm 15\%$  error band of the results of Oshinowo (1971), Usui and Sato (1989) and Yijun and Rezkallah (1993). In the literature there are no universally accepted criteria for the satisfactory performance of the correlations, however in the present study we decided upon the criteria based on the performance of the correlations in the selected ranges of the void fraction mentioned earlier. The criteria set in terms of the acceptable error or desired accuracy for the satisfactory performance is presented in Table 2. It should be also noted that the correlations designated as flow pattern specific are tested against all the flow patterns and for entire range of the void fraction since there is no consensus over the definition of the flow patterns and the range of the void fraction associated with an individual flow pattern.

The experimental investigation showed that the void fraction in a range of 0-0.3 increases rapidly with the increase in the gas flow rate. Due to the small values associated with the lower range of the void fraction large percentage errors are anticipated for  $0 < \alpha \leq 0.25$  and hence a relaxed criterion set for the satisfactory performance of the correlations. However due to large values of void fraction in the higher range the percentage errors between the measured and calculated values was expected to be less and hence a stricter criterion was used for analysis of the correlations.

**Table 1: Experimental data sets used in the present study**

Source	Diameter (m)	Data Points	Fluids	Pressure Range (MPa)	Void Fraction Range
Present Study <sup>1</sup>	0.01267	170	Air-water	0.11 – 0.23	0.025 – 0.93
Hernandez (2002) <sup>1</sup>	0.0545	39	Air-water	NA	0.02 – 0.246
Yijun and Rezkallah (1993) <sup>2</sup>	0.0095	81	Air-water	NA	0.02 – 0.99
Usui and Sato (1989) <sup>3</sup>	0.016	25	Air-water	0.1	0.07 – 0.89
Paras (1982) <sup>5</sup>	0.01905	35	Air-water	NA	0.11 – 0.90
Mukherjee (1979) <sup>6</sup>	0.0381	53	Air-kerosene	0.25 – 0.56	0.32 – 0.99
	0.0381	48	Air-oil	0.28 – 0.45	0.22 – 0.99
Lorenzi and Stogia (1976) <sup>4</sup>	0.044	71	Air-water	NA	0.011 – 0.209
	0.090	44	Air-water	NA	0.015 – 0.169
	0.032 <sup>#</sup>	26	Air-water	NA	0.029 – 0.29
Nguyen (1975) <sup>1</sup>	0.0455	79	Air-water	0.09 – 0.103	0.11 – 0.988
Beggs (1972) <sup>1</sup>	0.0381	13	Air-water	0.54 – 0.66	0.465 – 0.973
	0.0254	12	Air-water	0.43 – 0.63	0.094 – 0.983
Oshinowo (1971) <sup>1</sup>	0.0254	112	Air-water	0.13 – 0.205	0.057 – 0.961
	0.0254	78	Air-glycerin	0.14 – 0.203	0.047 – 0.964

<sup>#</sup>Data from De Rauz (1976) adopted from Lorenzi and Stogia (1976)

<sup>1</sup>Quick closing valve, <sup>2</sup>Gamma ray densitometer, <sup>3</sup>Conductance probe, <sup>4</sup>Manometric method, <sup>5</sup>Manually closing gate type valve,

<sup>6</sup>Capacitance method

**Table 2 Criteria for the performance evaluation of the void fraction correlations**

Void Fraction Range	Criteria
0.0 – 1.0	At least more than 85% points are within $\pm 15\%$ and more than 90% points within $\pm 20\%$ and RMS error $< 15\%$
0 – 0.25	At least more than 85 % points are within $\pm 20\%$ and RMS error $< 15\%$
0.25 – 0.50	At least more than 85 % points are within $\pm 15\%$ and RMS error $< 15\%$
0.50 – 0.75	At least more than 85 % points are within $\pm 15\%$ and RMS error $< 15\%$
0.75 – 1.0	At least more than 90% points within $\pm 15\%$ and RMS error $< 10\%$

The performance of the top performing void fraction correlations for all data set used in the present study is shown in Table 3. The first five top performing correlations identified in each range of the void fraction are presented in this study. The list of void fraction correlations considered in the present study is available in Ghajar and Tang (2010).

**Performance of the correlations for  $0 < \alpha \leq 0.25$ :** The first range of the void fraction 0-0.25 approximating the bubbly flow pattern contained 237 data points. For the relaxed criteria mentioned above eight correlations were found to predict the void fraction satisfactorily with the Gomez et al. (2000) successfully predicting 62.9% of data points in  $\pm 10\%$ , 79.3% in  $\pm 15\%$  and 93.7% in  $\pm 20\%$  error bands, respectively. Cai et al. (1997), Clark and Flemmer (1985), Bonnecaze et al. (1971) and Nicklin et al. (1962) were among the top five performers. It should be noted that all the top performing void fraction correlations in this void fraction range are based on

the concept of the drift flux model (DFM). None of the non DFM correlations performed satisfactorily for  $0 < \alpha \leq 0.25$ . It can be concluded that the drift flux models are most suitable to predict void fraction in the lower range. This capability of drift flux model can be attributed to its feature of accommodating the interaction between the two phases. The Gomez et al. (2000) correlation is recommended for void fraction in a range of  $0 < \alpha \leq 0.25$ .

**Performance of the correlations for  $0.25 < \alpha \leq 0.5$ :** This range of the void fraction approximated the slug flow regime and contained 107 data points. The top performing correlations were shortlisted if they could satisfactorily predict more than 85% of data points within  $\pm 15\%$  error band. In the category of non DFM correlations Yamazaki and Yamaguchi (1979) was the only candidate to perform satisfactorily. It predicted 92.4% of data points satisfactorily within  $\pm 15\%$  error band. Nicklin et al. (1962), Rouhani and Axelsson (1970) and Bonnecaze et al. (1971) were the other top

performers with 91.4% accuracy each within the error tolerance of  $\pm 15\%$ . Though the Gomez et al. (2000) did not qualify among the top five performing correlations, it performed modestly predicting 89.5% of data points successfully within error tolerance of  $\pm 15\%$ .

**Performance of the correlations for  $0.5 < \alpha \leq 0.75$ :**

This range of the void fraction consisted of 127 data points and approximated slug and froth flow regimes. The satisfactory performance criteria set for this range is similar to that of the previous range. Again among the non DFM correlations only Yamazaki and Yamaguchi (1979) emerged as a successful correlation and was able to predict 87.3% of data points well within  $\pm 15\%$  error band. In the category of the DFM correlations Woldesemayat and Ghajar (2007) was the most successful correlation with accuracy of 92.7% followed by Gomez et al. (2000) with accuracy of 86.4% in the error tolerance of  $\pm 15\%$ , respectively.

**Performance of the correlations for  $0.75 < \alpha \leq 1$ :**

Most of the data sets available in the literature had a void fraction in a range of 0.75-1. Among the data set used in the present study 441 data points were scattered in this range. This higher range of the void fraction,  $0.75 < \alpha \leq 1$  approximated the falling film and annular flow regimes. The most restricted criterion was set in this range for the satisfactory performance of the correlations. In the category of non DFM correlations Chen (1986) though not developed for downward flow was the most prominent correlation with accuracy of 90.2% in the error band of  $\pm 10\%$  and RMS error of 8.3%. Among the drift flux based models most of the correlations worked well within the set criteria. Woldesemayat and Ghajar (2007) performed outstandingly followed by Rouhani and Axelsson (1970) and Gomez et al. (2000).

The performance ranking of these correlations on a quantitative basis for the entire data set and for the different void fraction ranges is presented in Tables 3 and 4, respectively. It is evident from these tables that the Gomez et al. (2000) correlation can be designated as the best flow pattern independent correlation to predict the void fraction in the downward two phase flow. However it should be noted that the Gomez et al. (2000) correlation was originally developed for the bubbly flow regime in the upward two phase flow. This outcome validates the flexibility of the drift flux models and supports our claim of employing the DFM based upward two phase void fraction correlations to the downward two phase flow by simply manipulating the sign of the drift velocity term in the equation.

**Table 3 Results of the top five performing correlations for the entire data set used in the present study**

Correlation	Percentage of data points predicted within			RMS error
	$\pm 10\%$	$\pm 15\%$	$\pm 20\%$	
Gomez et al. (2000)	79.1	90.3	96.2	9.2
Cai et al. (1997)	79.2	90.1	95.8	13.6
Rouhani and Axelsson (1970)	76.3	86.7	90.9	11.4
Bonnecaze et al. (1971)	67.9	87.5	94.4	15.3
Nicklin et al. (1962)	67.9	87.5	94.4	15.3

**Table 4 Results of the top five performing correlations for the four specific ranges of the void fraction**

Correlation	Percentage of data points predicted within			RMS error
	$\pm 10\%$	$\pm 15\%$	$\pm 20\%$	
<b><math>0 &lt; \alpha \leq 0.25</math></b>				
Gomez et al. (2000)	62.9	79.3	93.7	11.4
Cai et al. (1997)	62	80.2	91.6	11.5
Clark and Flemmer (1985)	57	74.7	90.7	12.4
Bonnecaze et al. (1971)	48.5	73.5	88.2	13.5
Nicklin et al. (1962)	48.5	73.5	88.2	13.5
<b><math>0.25 &lt; \alpha \leq 0.50</math></b>				
Yamazaki and Yamaguchi (1979)	75.2	92.4	94.3	10.4
Bonnecaze et al. (1971)	75.2	91.4	96.2	10.2
Nicklin et al. (1962)	75.2	91.4	96.2	10.2
Rouhani and Axelsson (1970)	77.1	91.4	94.3	9.9
Kokal and Stainslav (1989)	75.2	90.5	96.2	10.2
<b><math>0.50 &lt; \alpha \leq 0.75</math></b>				
Yamazaki and Yamaguchi (1979)	73.8	87.3	96.8	9.7
Woldesemayat and Ghajar (2007)	80.2	92.1	97.6	8.2
Rouhani and Axelsson (1970)	75.4	86.5	95.2	9.7
Gomez et al. (2000)	75.4	86.5	92.9	10.3
<b><math>0.75 &lt; \alpha \leq 1</math></b>				
Chen (1986)	90.2	93.9	95.9	8.3
Yamazaki and Yamaguchi (1979)	88.2	94.6	96.6	8.2
Woldesemayat and Ghajar (2007)	91.4	95.2	97.3	7.4
Rouhani and Axelsson (1970)	91.2	97.5	98.2	6.8
Gomez et al. (2000)	89.3	97.5	98.9	7.2

**CONCLUSIONS:** The experimental results of the present study showed that the major flow patterns observed in the downward two phase flow and the measured void fraction data are in accordance with those available in the literature. This confirmed the ability and accuracy of our experimental facility to do flow visualization and measure the void fraction data. The drift flux model based correlations designed for vertical upward flow were successfully verified against the downward two phase flow data. It was confirmed that the DFM correlations are the most successful correlations and can account for the void fraction in downward flow by merely

using the negative sign convention for the drift velocity assuming the phase velocities are positive in the flow direction. Following the user defined criteria for satisfactory performance, the analysis of the available void fraction correlations for different ranges of the void fraction gave the best performing correlations in each range. The comparison between the different void fraction correlations concluded with the identification of a best flow pattern independent correlation to predict the void fraction in downward two phase flow.

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