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FLOW PATTERN AND PIPE ORIENTATION INDEPENDENT SEMI-EMPIRICAL VOID FRACTION CORRELATION FOR A GAS-LIQUID TWO PHASE FLOW BASED ON THE CONCEPT OF DRIFT FLUX MODEL

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ABSTRACT

A flow pattern and pipe orientation independent void fraction correlation is proposed in the present study. The correlation is based on the concept of drift flux model and proposes two separate expressions to model distribution parameter and drift velocity. The distribution parameter is expressed as a function of pipe orientation, phase superficial velocities and the void fraction in implicit form, while the drift velocity parameter is modeled as a function of fluid thermo physical properties, pipe orientation and void fraction. The drift velocity equation proposed by Zukoski [1] is extended for downward inclined pipe orientations. The performance of the proposed void fraction correlation is verified against void fraction data set of 5928 data points including the data for fifteen pipe diameters and eight different fluid combinations. The superiority of the proposed correlation is also illustrated by comparing it against the top performing correlations in horizontal, vertical upward and vertical downward pipe orientations and the predictions of the Woldesemayat and Ghajar [2] and Chexal et al. [3] correlations for incline pipe orientations.

INTRODUCTION

The accurate prediction of void fraction for the calculation of pressure drop and heat transfer in the gas liquid two phase flow is of significant interest in energy, nuclear, chemical and petroleum industries. The two major types of void fraction correlation available in the literature are the empirical correlations and those based on the concept of drift flux model but limited to certain flow patterns. It is seen in the literature that there are no universal quantitative measures to predict the existence or the transition of a flow pattern from one to another. Moreover, the flow patterns are observed to be influenced by the parameters like pipe orientation, system pressure and fluid thermo-physical properties. This limits the applicability of the available void fraction correlations to the experimental data they are derived from. The flow patterns and the void fraction appearing in the vertical upward, downward and horizontal two phase flow were studied in detail by [4-6]. The flow patterns and the void fraction for upward inclined and downward inclined pipes were studied by [7-9]. All these studies except that of the [7, 8] confirmed the ability of drift flux model to predict the void fraction for different flow patterns and pipe orientations. The correlations available in the literature such as [10-12] and [2] model the distribution parameter and drift velocity as a function of one or more two phase flow parameters, but are found to be limited in application for a particular flow pattern or void fraction range. Hence, due to these limitations and the importance of the void fraction in the prediction of the two phase pressure drop and heat transfer it is desirable to have a correlation to be able to predict the void fraction irrespective of the flow pattern, pipe orientation and fluid properties. Bhagwat and Ghajar [5] verified the flexibility of the drift flux model to predict the void fraction independent of the flow patterns and the flow direction for vertical two phase flow. Thus it appeared logical to develop a flow pattern and pipe orientation independent void fraction correlation based on the concept of the drift flux model. In the present study two separate equations are proposed to model the distribution parameter and the drift velocity. The distribution parameter is

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expressed as a function of void fraction, phase flow rates, and the pipe orientation. Whereas, the drift velocity is represented as a variable, varying with respect to fluid thermo physical properties, void fraction and the pipe inclination, respectively. The accuracy of the proposed correlation is verified by comparing it against 5928 void fraction data points and top performing correlations for horizontal, vertical upward and downward orientations short listed by [4-6].

NOMENCLATURE

C_o	Distribution parameter
D	Pipe diameter (m)
Eo	Eotvos number, $Eo = (\rho_l - \rho_g)gD^2/\sigma$
g	Accelertion due to gravity (m/s^2)
R	Ratio of liquid to water dynamic viscosity
U	Phase velocity (m/s)
U_b	Bubble rise velocity in stagnant liquid (m/s)
U_{GM}	Drift velocity (m/s)

Greek symbols

α	Void fraction
θ	Pipe orientation from horizontal (degrees)
μ	Phase viscosity (Pa-s)
ρ	Phase density (kg/m^3)
σ	Gas-liquid interface surface tension (N-m)
$\sum$	Non-dimensional surface tension defined by [1] as, $\Sigma = \left(4\sigma/\rho_l g D^2\right)$

Subscripts

g	Gas
l	Liquid
m	Mixture
s	Superficial
w	Water

VOID FRACTION AND DRIFT FLUX MODEL

Drift flux model is one of the two phase flow models intensively incorporated in applications of gas liquid two phase flow for the prediction of the void fraction. It has been observed in the literature that most of the successful flow pattern dependent and independent void fraction correlations are based on the concept of drift flux model. The success of drift flux model in prediction of void fraction is due to flexibility of its modules i.e. distribution parameter and drift velocity to accommodate the change in pipe diameter, fluid properties, pipe orientations and flow patterns. The distribution parameter is a measure of the distribution of the gas phase with respect to the mixture across the pipe cross section while the drift velocity represents the actual gas velocity with reference to the mixture velocity averaged across the pipe cross section. Hence, it was decided in the present study to adopt the concept of drift flux model to develop a semi empirical correlation to predict void fraction independent of flow patterns and pipe orientation. This could be achieved by modeling the distribution parameter and drift velocity as a function of two phase flow variables such as phase velocities, fluid thermo physical properties, pipe orientation and flow patterns in terms of void fraction. The general structure of drift flux model to predict the void fraction is expressed as,

$$\alpha = U_{sg} / \left(C_o U_m + U_{GM}\right) \tag{1}$$

The details of modeling of the distribution parameter (C_o) and the drift velocity (U_{GM}) are presented next.

DISTRIBUTION PARAMETER

The distribution parameter (C_o) is one of the influential drift flux variables representing the distribution of void fraction (α) or distribution of the gas phase across the pipe cross section. Since, the distribution of the gas phase across the pipe cross section changes significantly with respect to the flow patterns or alternatively the void fraction and the pipe orientation; it was decided to express the distribution parameter as a function of phase superficial velocities as a representative of flow patterns, void fraction and the pipe orientation. The documentations of [10,11,13-16] showed that the distribution parameter in the bubbly and slug flow regimes approach a value close to 1.2 for vertical upward flow. For vertical down and up bubbly flow [13] and [15] proposed a distribution parameter of 1.185 and 1.2, respectively. The distribution parameter was defined as a constant assuming value of 1.12 and 1.15 for vertical down and 1.2 for vertical up slug flow, respectively by [14-16]. However, [10-12,17] represented the distribution parameter as a variable and in the form of physical expressions yielding the value of C_o in the vicinity of 1.2 for bubbly flow. The expression of [10] developed for C_o for vertical down two phase flow rendered the distribution parameter to approach a value close to unity as the flow pattern shifted from bubbly to annular flow regime. The distribution parameter was expressed in terms of quality (x) by [11] to reduce the C_o value from 1.2 in bubbly flow to some finite value close to one in annular regime. Usui and Sato [12] related the dependency of the distribution parameter on Eotovos number (Eo). Kawanishi et al. [17] used three different equations for the distribution parameter in terms of the density ratio of two phases. The distribution parameter was presented as a function of the phase superficial velocities and densities by [2]. Though this expression treated the distribution parameter as a variable it assigned the distribution parameter; values considerably less than unity for bubbly and slug flows and values higher than unity for the high void fraction flow regime. In addition to the flow patterns and void fraction, it is estimated that the distribution parameter is also influenced by the pipe inclination. To our knowledge there is no expression in the literature with the exception of Chexal et al. [3] that models the distribution parameter as a function of the pipe orientation. Figure 1 illustrates the geometry of the gas bubble (slug) observed in case of horizontal (A), upward inclined (B) and vertical upward (C) pipes. Similar bubble geometries were observed by [9]. For vertical upward orientation, the gas bubble

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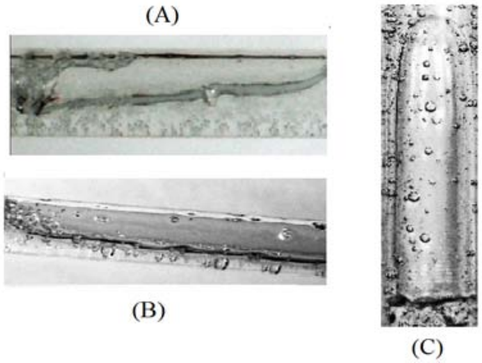
occupies almost the entire cross section of the pipe and is symmetric about the pipe axis. For such a case, [13] proposed the value of distribution parameter to be 1.2. For the case of upward inclined flow, the gas bubble loses symmetry about the pipe axis and flows offset and in the vicinity of the top wall of the pipe. Similar bubble geometry is observed for the horizontal pipe orientation. Due to significant density difference between the two phases, the gas bubble is always oriented close to the top wall. This means that the gas bubble occupies less cross section as compared to that in the vertical flow.



(A) Horizontal (B) Upward inclined (C) Vertical upward

Figure 1 Bubble shape for three different pipe orientations

Thus it is conjectured from Figure 1 that the distribution parameter for upward inclined and horizontal orientations should be less than that in the vertical upward orientation. The analysis of the experimental data in the present study as reported in Table 1 revealed that the distribution parameter less than unity predicted the void fraction satisfactorily for the bubbly and slug flows in the upward inclined and horizontal orientations, respectively; while a distribution parameter slightly greater than unity approximately in a range of 1-1.05 gave satisfactory results in the annular flow regime. Similar concept is applicable for the distribution parameter in the downward inclined orientations. Thus it is obvious that the distribution parameter should vary with respect to both flow pattern or the void fraction and the pipe inclination. For vertical upward and downward flows, the distribution parameter should assume a value close to 1.2 and approach unity as the flow pattern transits to the annular flow regime. However, for the horizontal, upward inclined and downward inclined orientations the distribution parameter should be less than unity and approach unity value as the void fraction increases or the flow pattern transits from bubbly to annular flow. To satisfy the above mentioned requirements, the following expression is proposed in the present study for the variation of distribution parameter (C_o) with respect to the superficial phase flow rates or the void fraction and the pipe orientation (θ) measured in degrees.

$$C_o = \left[\frac{1}{(1+\cos\theta)^{1.25}}\right]^{(1-\alpha)^{0.5}} + 0.18\left(\frac{U_{sl}}{U_m}\right)^{0.1} \tag{2}$$

In Equation (2), the exponents 1.25, 0.5, 0.1 and the multiplying factor 0.18 are empirical constants.

DRIFT VELOCITY

In context to the drift flux model the drift velocity is defined as the actual cross sectional averaged velocity of the gas phase with respect to the averaged mixture velocity crossing a reference plane in the pipe. The influence of viscosity, surface tension and the pipe inclination on the bubble rise velocity in stagnant liquid was demonstrated by [1]. The drift velocity of bubble in stagnant liquid was defined as the rise velocity of a single bubble (U_b) injected in stagnant liquid by [18, 19]. They reported that the velocity of a single bubble injected in a stagnant liquid is influenced by the pipe inclination and liquid-gas interface surface tension. The drift velocity or alternatively the rise velocity of a single bubble in stagnant liquid in any pipe orientation is an outcome of the interaction between the gravity and the buoyancy force exerted on the lighter phase. In case of the vertical upward flow, the buoyancy force is always acting in the direction of the mean flow aided by the liquid inertia, whereas, for downward flow as documented by [5] the bubbles try to rise in the vertical upward direction, due to dominant buoyant force, i.e. against the direction of the mean flow. Thus intuitively, at low void fraction the drift velocity in the vertical down flow is approximately equal and opposite in magnitude as compared to the vertical up flow. This observation is consistent with the reports of [14,15]. Accordingly the drift velocity equations proposed by [20] and [16] for the bubbly and slug flow can be used for both vertical up and down orientations merely by flipping the sign of the drift velocity from positive to negative. It should be noted that this assumption holds good only for the low values of the void fraction or low phase flow rates or alternatively for the bubbly and slug flows. It was observed by [1] and [18] that the rise velocity of the bubbles in stagnant liquid increase with decreasing angle from vertical position up to an angle between 30 and 50 degrees where the bubble rise velocity is maximum. Further this point, the bubble rise velocity was observed to decrease with decreasing pipe inclination and approach some finite value at horizontal orientation. The bubble rise velocity in stagnant liquid at horizontal position was observed to be greater than that at the vertical orientation. This trend of variation of the bubble rise velocity in stagnant liquid in a non-dimensional form or alternatively the non-dimensional drift velocity with varying pipe inclination presented by [18] is expressed as,

$$U_b \Big/ \sqrt{gD} = 0.35\sin\theta + 0.54\cos\theta \tag{3}$$

In the present study we extend this equation for downward inclined orientations. Figure 2 shows the variation of non-dimensional drift velocity with respect to the pipe orientation from +90 to -90 degrees. Figure 3 reveals the effect of non-dimensional surface tension on the non-dimensional drift velocity as reported by [1]. For the proposed correlation in this study, the non-dimensional drift velocity taking into account different fluid combinations and hence the varying liquid

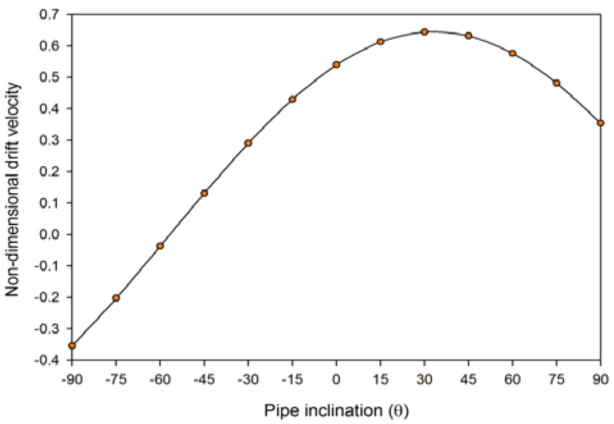


Figure 2 Extension of [18] non-dimensional drift velocity as a function of pipe orientation.

dynamic viscosities as shown in Figure 4 can be represented in form of following equation as,

$$\frac{U_b}{\sqrt{gD}} = \left(\frac{\mu_l}{\mu_w}\right)^{-0.25} \left(0.35\sin\theta + 0.54\cos\theta\right) \qquad (4)$$

It is observed from Figure 2 that the drift velocity remains positive up to a certain downward inclination, becomes zero at a downward inclination near -60° and is a decreasing function of the increasing downward inclination from the horizontal. It is speculated that for downward inclined near horizontal orientations, due to the reduced tangential component of the buoyancy effect, the dominant liquid inertia force carry the gas bubbles with it in the direction of the mean flow. Further in context to Figure 2, it is logical to claim that for a smooth variation in the drift velocity from horizontal to downward inclined and vertical downward orientation, the drift velocity should stay positive and gradually reduce down with increasing downward inclination from the horizontal and attain negative drift velocity at the vertical downward orientation, i.e. where the buoyancy effect is significant. However, the experimental results of [18] showed that for $-30^{o} \leq \theta < 0^{o}$, the bubble rise velocity is equal and opposite in sign to its upward inclined counterpart. It should be noted that here we logically extend the bubble rise velocity equation to the downward inclined orientations and this needs to be verified by the experimental data. On another note the experimental results of [18] for $-30^{o} \leq \theta < 0^{o}$ is open to question since his results have not been compared or verified by other investigators.

It was shown by [1] that in addition to the pipe orientation, the bubble rise velocity is also affected by the surface tension at the gas-liquid interface. He experimentally showed that the drift velocity decreases with increasing surface tension. The effect of non-dimensional surface tension ($\sum$), on the drift velocity, experimentally measured by [1] is shown in Figure 3. In the proposed correlation for drift velocity (U_{GM}) as shown in Equation (5), the influence of the fluid properties on the drift velocity at any given orientation is accounted by the inclusion of the ratio of dynamic viscosity of the liquid phase normalized by the dynamic viscosity of the water at system temperature and pressure. The expected variation in the drift velocity with changing ratio of dynamic viscosity of the given liquid phase to that of water measured at system temperature and pressure is indicated by '*R*' as shown in Figure 4.

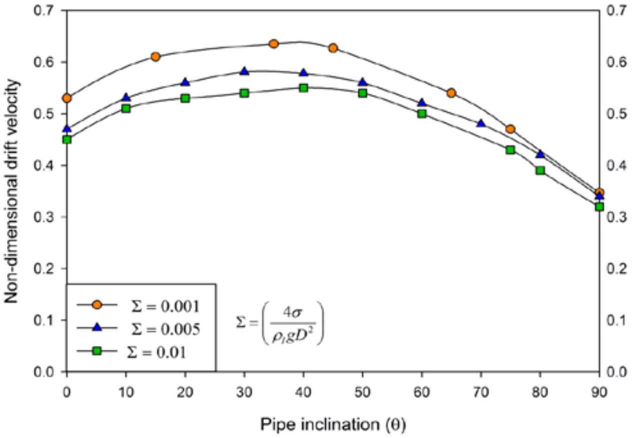


Figure 3 Variation of the non-dimensional drift velocity with pipe inclination and surface tension experimentally measured by [1].

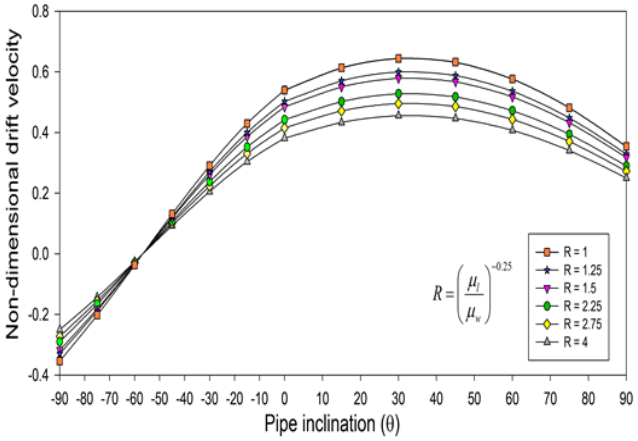


Figure 4 Variation of non-dimensional drift velocity with the non-dimensional liquid phase viscosity and pipe inclination.

The effect of the inclusion of the viscosity ratio term in the correlation on the prediction of void fraction could not be verified due to the lack of data for different fluid combinations in the void fraction range of $0 < \alpha \leq 0.4$. However, comparison of Figures 3 and 4 clearly indicates a similar trend of the variation of the drift velocity with respect to non-dimensional surface tension at the gas-liquid interface and the ratio of the liquid phase dynamic viscosity to the dynamic viscosity of water, respectively. Gomez et al. [21] proposed a drift flux model to predict the void fraction in vertical upward two phase flow. They modified the drift velocity expression proposed by [20] by multiplying with a term of $\left(1-\alpha\right)^{0.5}$ to take care of bubble swarm effect. In the proposed correlation for drift

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velocity we account for the variation of the drift velocity with respect to varying void fraction and pipe orientation. To consider this phenomenon a term containing inclination and void fraction effect $\left(1\Big/(1-\alpha)^{\frac{\sin\theta}{2}}\right)$ is included in the expression of drift velocity. The present correlation considers all these complexities that may occur with varying void fraction, fluid properties and pipe inclination. To take into account the pipe orientation effect the term, $0.35\sin(\theta)+0.54\cos(\theta)$ proposed by [18] is included in the drift velocity expression and finally the expression for the drift velocity is presented by the following equation written as,

$$U_{GM}=\left(\frac{\mu_l}{\mu_w}\right)^{-0.25}(0.35\sin\theta+0.54\cos\theta)\sqrt{\frac{gD\Delta\rho}{\rho_l}}\left(\frac{1}{(1-\alpha)^{\frac{\sin\theta}{2}}}\right) \quad (5)$$

The exponent -0.25 is an empirical constant. Thus the void fraction is predicted by putting Equations (2) and (5) given for C_o and U_{GM} in the drift flux model expressed as Equation (1).

PERFORMANCE VERIFICATION OF THE PROPOSED VOID FRACTION CORRELATION

The performance of the proposed correlation and its competency with respect to other top performing void fraction correlations in vertical upward, downward, horizontal and inclined two phase flow is verified against a comprehensive data set of 5928 data points. The top performing correlations in each orientation used for comparing the proposed correlation come from the recommendations of [5] and [22]. The details of the experimental data set used in this study for the performance verification are documented in Table 1. Due to space limitation it is not possible to report the details of the comparison of the correlation against all data and other correlations. The accuracy of the proposed correlation is verified for four specified ranges of the void fraction i.e. $0 < \alpha \le 0.25$, $0.25 < \alpha \le 0.5$, $0.5 < \alpha \le 0.75$ and $0.75 < \alpha \le 1$. These four specific void fraction ranges were decided based upon approximate range of the void fraction associated with individual flow patterns. More justification on choosing these ranges of the void fraction is given by [5]. The performance of the correlation is defined in terms of the percent RMS error and the percentage of data points that fall within ±10%, ±15% and ±20% error bands. The criteria for satisfactory performance of the void fractions in these three orientations are given by [22]. The % RMS error is given by the following equation as,

$$\%\ \text{RMS error} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left[\frac{(\alpha_{calc})_i-(\alpha_{meas})_i}{(\alpha_{meas})_i}\right]^2}\times 100$$

Where, N is the number of data points and the subscripts *calc* and *meas* refer to the calculated and measured parameters, respectively.

Table 1 Experimental data set used for the verification of proposed void fraction correlation.

Source	No. of data points					Pipe diameter (mm)	Fluid combination
	UP	DN	H	UI	DI		
Two phase flow lab[1*]	153	193	184	-	-	12.7	Air-water
Abdul-Majeed [23][1]	-	-	88	-	-	58	Air-kerosene
Arosio and Stogia [24][4]	-	141	-	-	-	32, 44, 90	Air-water
Badie et al. [25][2]	-	-	30 36		-	78 36	Air-oil Air-water
Beggs [7][1]	27	25	58	219	235	25.4, 38	Air-water
Chen [26][3]	-	-	48	-	-	77.9	Air-kerosene
Chokshi [27][2]	103	-	-	-	-	76	Air-water
Eaton [28][1]	-	-	238	-	-	54, 102	Natural gas-water
Fernandes [29][1]	88	-	-	-	-	57	Air-water
Franca and Lahey [30][1]	-	-	88	-	-	19	Air-water
Hernandez [31][1]	-	39	-	-	-	58	Air-water
Isbin et al. [32][2]	22	-	-	-	-	22.1	Steam-water
Minami and Brill [33][1]	-	-	57 54	-	-	77.9	Air-kerosene Air-water
Mukherjee [8][3]	65	53 48	75	483	319	38	Air-Kerosene Air-Lube Oil
Nguyen [9][1]	224	79	270	910	307	45	Air-water
Oshinowo [34][1]	153 172	112 78	-	-	-	25.4	Air-water Air-glycerin
Ottens (1998)[#]	-	-	42	17	53	51	Air-water
Paras [35][5]	-	35	-	-	-	19.1	Air-water
Schmidt et al. [36][2]	20	-	-	-	-	54	Nitrogen-water
Sujumnong [37][1]	77 104	-	-	-	-	12.7	Air-water Air-glycerin
Usui and Sato [12][6]	-	25	-	-	-	16	Air-water
Yujun and Rzkallah [38][2]	-	81	-	-	-	9.5	Air-water

UP – Vertical upward, DN – Vertical downward, H – Horizontal, UI – Upward inclined, DI – Downward inclined

[1] Quick closing valve, [2] Gamma-ray absorption, [3] Capacitance sensor, [4] Manometer, [5] Neutron scattering, [6] Conductance probe,

*Data from Oklahoma State University contributed by [4-6], [#] Data from Prof. D. Barnea, Tel-Aviv University, Israel

VERTICAL UP AND DOWN FLOW

The prediction performance of the proposed correlation for vertical upward and downward flow was compared against the top three performing correlations proposed by [5] and [22] as shown in Table 2. It is observed that for vertical upward flow the overall performance of the proposed correlation is comparable to that of the other recommended correlations in the restricted error bands of ±10% except for the void fraction range of $0.5 \le \alpha \le 0.75$. For a relaxed criteria of ±20%, the proposed correlation gives highest accuracy especially in the lower region of the void fraction i.e. for a void fraction range of $0 < \alpha \le 0.25$ by predicting more than 70% of the data points within ±20% error bands and RMS error of 52.8% being the lowest in comparison to other three correlations. In case of vertical down flow for $0 < \alpha \le 0.25$, the proposed correlation exhibits outstanding performance by predicting the maximum of 70.8% and 92.8% of the void fraction data in ±10% and ±20%

Table 2 Comparison of the proposed correlation against the top performing correlations for vertical up and down flow.

Correlations (Vertical upward flow)	**Percentage of data points predicted within**			**RMS error (%)**	**Correlations (Vertical downward flow)**	**Percentage of data points predicted within**			**RMS error (%)**
	±10%	±15%	±20%			±10%	±15%	±20%	
$0 < \alpha \le 0.25$					**$0 < \alpha \le 0.25$**				
Greskovich and Cooper [39]	45.2	57.8	67.8	55.1	Gomez et al. [21]	62.9	79.3	93.7	11.4
Nicklin et al. [16]	37.2	54.3	70.9	53.4	Hasan et al. [14][S]	70.1	81	91.6	11.5
Rouhani and Axelsson [11]	40.2	62.8	71.9	56.5	Cai et al. [15][S]	62	80.2	91.6	11.5
Proposed correlation	39.8	55.6	74.5	52.8	Proposed correlation	70.5	86.5	92.8	11.7
$0.25 < \alpha \le 0.5$					**$0.25 < \alpha \le 0.5$**				
Bonnecaze et al. [40]	53.2	80.5	85.3	16.9	Rouhani and Axelsson [11]	77.1	91.4	94.3	9.9
Kokal and Stainslav [41]	53.2	81.6	84.7	16.9	Yamazaki and Yamaguchi [42]	75.2	92.4	94.3	10.4
Nicklin et al. [16]	53.2	80.5	85.3	16.8	Nicklin et al. [16][#]	75.2	91.4	96.2	10.2
Proposed correlation	51.9	80	85.9	16.3	Proposed correlation	74.3	88.6	94.3	11.6
$0.5 < \alpha \le 0.75$					**$0.5 < \alpha \le 0.75$**				
Bonnecaze et al. [40]	80.3	93.4	96.6	9.7	Gomez t al. [21]	75.4	85.5	92.9	10.3
Kokal and stainsalv [41]	80.1	93.2	96.6	9.7	Rouhani and Axelsson [11]	75.4	85.5	95.2	9.7
Nicklin et al. [16]	80.3	93.4	96.6	9.7	Woldesemayat and Ghajar [2]	80.2	92.1	97.6	8.2
Proposed correlation	67.3	91.2	96.7	10.3	Proposed correlation	75.4	83.3	92.1	10.7
$0.75 < \alpha < 1$					**$0.75 < \alpha < 1$**				
Armand [43]	94.7	99.4	100	5.3	Gomez et al. [21]	89.3	97.5	98.9	7.2
Dix [44]	93.8	98.7	99.8	5.2	Rouhani and Axelsson [11]	91.2	97.5	98.2	6.8
Rouhani and Axelsson [11]	94	99.4	99.6	5.6	Woldesemayat and Ghajar [2]	91.4	95.2	97.3	7.4
Proposed correlation	90.8	98.5	99.6	6.6	Proposed correlation	93.7	97.1	98.6	6.7

[#] Similar performance given by [40]. [S] Slug, indicates two or more flow pattern specific correlations given by the same author.

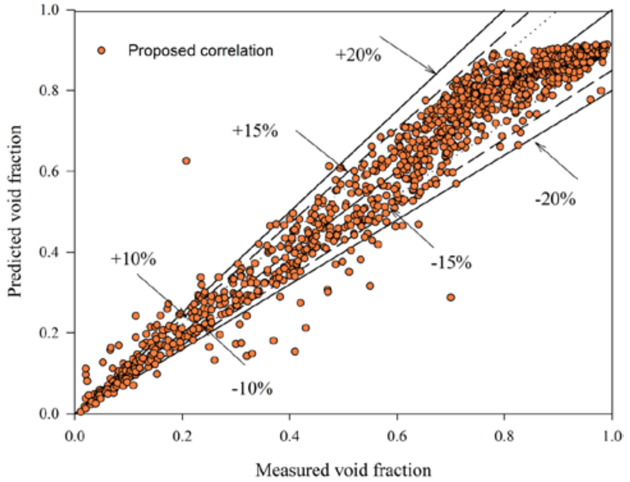

Figure 5 Performance of the proposed correlation for vertical upward flow (1208 data points).

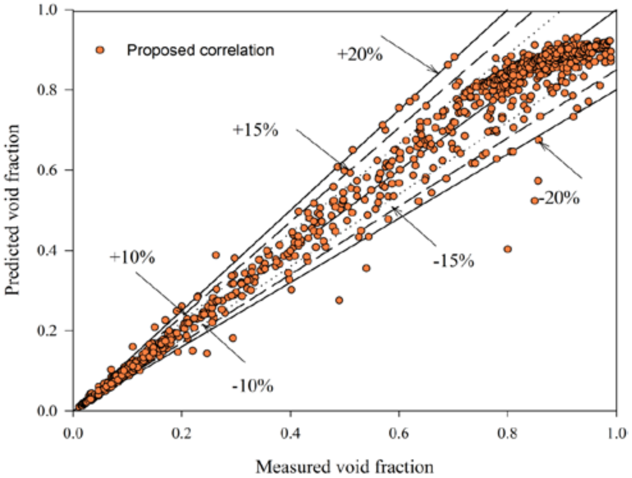

Figure 6 Performance of the proposed correlation for vertical downward flow (909 data points).

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error bands, respectively. For the void fraction range of $0.5 < \alpha \leq 0.75$, though the proposed correlation underperforms for the ±10% error tolerance, its accuracy in ±20% error bands is highest compared to other three correlations. Finally for the last void fraction range of $0.75 < \alpha < 1$, the proposed correlation displays competence with other top performing correlations by predicting highest percentage (93.7%) of the data points within stringent ±10% tolerance bands and the RMS error of 6.6%. The performance of the proposed correlation for vertical up and down flow is illustrated graphically in Figures 5 and 6, respectively.

HORIZONTAL FLOW

The performance of the proposed correlation for 1268 data points in the horizontal pipe orientation is compared against the top performing correlations short listed by [22] as shown in Table 3. The superiority of the proposed correlation over others is of interest for this orientation as most of the available correlations fail to predict void fraction accurately typically in a range of $0 < \alpha \leq 0.25$. For the lowest void fraction range of $0 < \alpha \leq 0.25$, the proposed correlation gives the maximum accuracy of 47.5%, 60% and 77.5% in error bands of ±10%, ±15% and ±20%, respectively. Albeit, the accuracy of the proposed correlation drops down a little in the intermediate void fraction ranges of $0.25 < \alpha \leq 0.5$ and $0.5 < \alpha \leq 0.75$. However, its performance in the less restrictive error bands of ±20% can be considered comparable to top three performers.

Table 3 Comparison of the proposed correlation against the top performing correlations in horizontal flow.

Correlations	**Percentage of data points predicted within**			**RMS error (%)**
	±10%	±15%	±20%	
$0 < \alpha \leq 0.25$				
Huq and Loath [45]	27.5	47.5	65	33.9
Permoli et al. [46]	35	50	62.5	37.5
Rouhani and Axelsson [11]	32.5	50	62.5	34.2
Present correlation	47.5	60	77.5	42.1
$0.25 < \alpha \leq 0.5$				
Minami and Brill [33]	43.4	61.4	75.9	17.6
Mukherjee [8]	54.4	65.5	82.1	14.6
Woldesemayat and Ghajar [2]	47.6	67.6	80.7	16.2
Present correlation	45.5	62.2	72.4	18.2
$0.5 < \alpha \leq 0.75$				
Rouhani and Axelsson [11]	70.6	84.8	91.3	11.7
Sun et al. [47]	69.2	87.2	90.3	12.3
Woldesemayat and Ghajar [2]	73.7	86.2	92.7	12
Present correlation	47.4	76.8	88.6	14.9
$0.75 < \alpha < 1$				
Armand [43]	77	99.9	100	4.9
Lockhart and Martinelli [48]	95	98.6	99.6	4.6
Permoli et al. [46]	97.2	98.9	99.1	4.1
Present correlation	86.0	95.7	97.8	9.5

For the last range of the void fraction the proposed correlation excels by predicting more than 85% and 95% of the data within ±10% and ±15%, respectively. Overall, for the error bands of ±20%, the proposed correlation displays competitive performance in all four specified ranges of the void fraction. The performance of the proposed correlation for horizontal flow data is presented graphically in Figure 7. The decreasing percentage accuracy of the correlations with the decreasing void fraction values is essentially because of the low values of the void fraction. However, with decreasing void fraction the absolute deviation of the predicted values from the measured void fraction data is acceptable.

UPWARD AND DOWNWARD INCLINED FLOW

It was observed in the literature review that very few investigations are dedicated to the understanding of upward and downward inclined two phase flow. The correlations to predict void fraction in upward and downward inclined pipes were developed by [2, 3, 7, 8, 14, 21, 39]. The correlations of [7, 8] were the outcome of a regression program and were limited to their own experimental data. The correlations of [2, 3, 14, 21, 39] are based on the concept of drift flux model and incorporate the pipe orientation term (θ) in the drift velocity expression to account for the effect of pipe orientation. The correlations of [39, 49] are limited in their application for $0^o \leq \theta \leq 90^o$, and cannot be applied for downward inclined pipes.

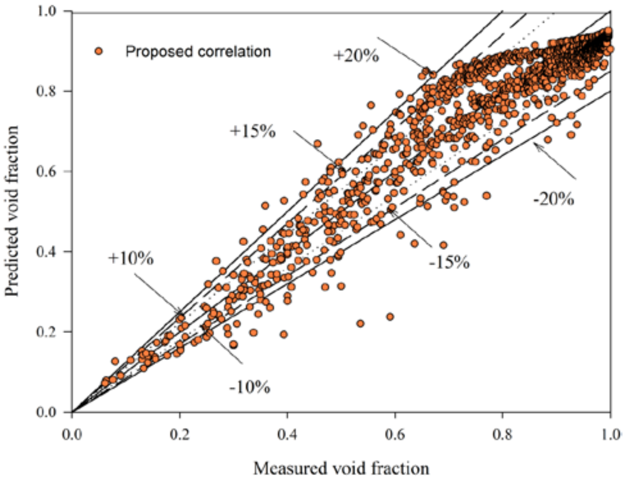

Figure 7 Performance of the proposed correlation for horizontal flow (1268 data points).

The performance analysis of the void fraction correlations showed the correlation of [2] to predict the void fraction satisfactorily for upward inclined and downward inclined pipes but at an expense of a significant loss in accuracy for vertical upward and downward orientations particularly in the void fraction range of $0 < \alpha \leq 0.5$. Unlike the vertical and horizontal orientations, due to scarcity of the void fraction data it was not possible to analyze the performance of the proposed correlation for the four ranges of the void fraction. Due to page limitation, we have only presented the comparison of the proposed correlation against that of the [2, 3] for upward and downward inclined orientations in Tables 4 and 5, respectively.

The performance of the proposed correlation for upward and downward orientations is shown graphically in Figures 8 and 9, respectively. It should be noted that the proposed correlation predicts the void fraction satisfactorily in downward inclined orientations for all flow patterns except for the stratified flow.

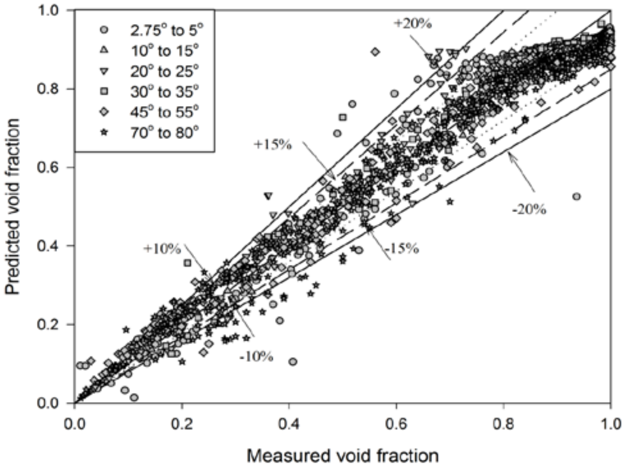

Figure 8 Performance of the proposed correlation for upward inclined orientation (1629 data points).

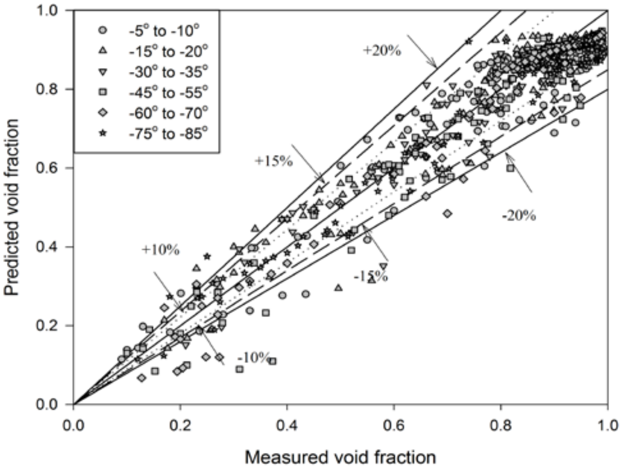

Figure 9 Performance of the proposed correlation for downward inclined orientation (914 data points).

Table 4 Performance of the proposed correlation against the other correlations in upward inclined flow.

Correlation	Pipe inclination	2.75° to 5°	10° to 15°	20° to 25°	30° to 35°	45° to 55°	70° to 80°
	Data points	338	66	308	110	307	500
Proposed correlation	±10%	77.5	89.1	76.5	84.3	69.2	70.3
	±15%	86.5	98.4	88.2	90.7	77.0	81.7
	±20%	91.3	100	93.5	93.5	83.0	87.6
	RMS	13.0	7.0	10.6	11.0	30.9	1.0
Chexal et al. [3]	±10%	71.5	81.3	72.5	75.9	70.2	67.5
	±15%	78.1	92.2	82.4	84.3	79.3	78.3
	±20%	80.8	93.8	85.9	90.7	83.9	82.7
	RMS	53.4	7.5	15.0	12.3	40.1	42.7
Woldesemayat and Ghajar [2]	±10%	80.5	84.4	66.3	73.1	60.3	65.7
	±15%	88	89.1	74.2	86.1	66.9	72.6
	±20%	91.6	90.6	76.1	90.7	70.2	78.7
	RMS	15.6	13.4	30.8	17.1	61.6	39.7

Table 5 Performance of the proposed correlation against other correlations in downward inclined flow.

Correlation	Pipe inclination	-5° to -10°	-15° to -20°	-30° to -35°	-45° to -55°	-60° to -70°	-75° to -85°
	Data points	224	214	86	163	136	91
Proposed correlation	±10%	80.6	59.4	70.2	83.9	81.3	78.7
	±15%	89.6	79.7	82.1	87.0	82.8	88.8
	±20%	92.8	85.8	92.9	90.7	87.3	94.4
	RMS	12.8	19.9	11.1	15.5	18.8	11.8
Chexal et al. [3]	±10%	80.2	50.9	60.7	72.0	72.4	58.4
	±15%	87.4	69.8	79.8	80.1	74.6	75.3
	±20%	89.2	77.8	88.1	85.1	78.4	79.8
	RMS	16.8	20.0	14.2	17.6	22.9	14.7
Woldesemayat and Ghajar [2]	±10%	82.9	52.8	63.1	59.0	76.9	50.6
	±15%	87.4	71.7	82.2	68.3	82.8	75.3
	±20%	89.2	76.9	91.7	75.8	84.3	84.3
	RMS	24.4	22.1	12.9	55.0	17.6	17.3

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It is seen that for each range of inclination in upward and downward orientations, the proposed correlation gives better accuracy than the correlations of Woldesemayat and Ghajar [2] and Chexal et al. [3] for each error band. It should be noted that the correlation presented in this study gives good accuracy for all the flow patterns in downward orientations except the stratified flow pattern.

CONCLUSIONS

A void fraction correlation independent of the flow pattern and pipe orientation is proposed in this study. The correlation is based on the concept of drift flux model and expresses the distribution parameter and the drift velocity as the functions of the void fraction, phase superficial velocities and the pipe orientation. Two separate equations are proposed for the distribution parameter and the drift velocity. The non-dimensional drift velocity equation for upward inclined flow proposed by [18] is extended for downward inclined orientations and finally presented in a dimensional form giving the drift velocity (U_{GM}) in m/s for different pipe orientations. A ratio of dynamic viscosity of given liquid to the dynamic viscosity of water at system temperature and pressure is incorporated in the drift velocity equation to account for the variation in the fluid properties. The trend of the drift velocity variation curve obtained theoretically by implementing this ratio is qualitatively similar to the experimental results of [1]. The proposed void fraction correlation is compared with the existing top performing correlations for vertical upward, downward and horizontal orientations and is found to compete with them satisfactorily. The performance of the proposed correlation is also compared with Woldesemayat and Ghajar [2] and Chexal et al. [3] correlations and is found to give better results than these correlations for all upward and downward inclined pipe orientations. It should be noted that the proposed correlation works well for the range of pipe diameters analyzed in the present study, i.e. for $9.5 \text{ mm} \leq D \leq 102 \text{ mm}$, however, since the drift velocity expression contains the diameter term in it, the upper and lower limit of pipe diameter for which this correlation holds good should also be tested. Though the proposed void fraction correlation is tested against a comprehensive data set for data consisting of different flow patterns and fluid properties, more verification of this correlation against the low range of void fraction ($0 < \alpha < 0.3$) in upward and downward inclined pipe orientations and variable fluid properties is highly recommended.

REFERENCES

[1] Zukoski, E. E., 1966, "Influence of viscosity, surface tension and inclination angle on motion of long bubbles in closed tubes," Journal of Fluid Mechanics, **25**, pp. 821-837.

[2] Woldesemayat, M. A., and Ghajar, A. J., 2007, "Comparision of void fraction correlations for different flow patterns in horizontal and upward inclined pipes," International Journal of Multiphase Flow, **33**, pp. 347-370.

[3] Chexal, B., Lellouche, G. S., Horowitz, J., and Healzer, J., 1992, "A Void Fraction Correlation for Generalized Applications," Progress in Nuclear Energy, **27**, pp. 255-295.

[4] Godbole, P. V., Tang, C. C., and Ghajar, A. J., 2011, "Comparison of void fraction correlations for different flow patterns in upward vertical two phase flow," Heat Transfer Engineering, **32**(10), pp. 843-860.

[5] Bhagwat, S.M. and Ghajar, A.J., 2012, "Similarities and Differences in the Flow Patterns and Void Fraction in Vertical Upward and Downward Two Phase Flow". Experimental Thermal and Fluid Science: doi:10.1016/j.expthermflusci.2012.01.026.

[6] Mathure, N. N., 2010, "Flow patterns and void fraction in horizontal two phase flow," MS thesis, Oklahoma State University, Stillwater

[7] Beggs, H. D., 1972, "An experimental study of two phase flow in inclined pipes," Ph.D. thesis, University of Tulsa, Tulsa.

[8] Mukherjee, H., 1979, "An Experimental Study of Inclined Two Phase Flow," Ph.D. thesis, The University of Tulsa, Tulsa.

[9] Nguyen, V. T., 1975, "Two Phase Gas-Liquid Cocurrent Flow : An Investigation of Hold Up, Pressure Drop and Flow Pattern in a pipe at Various Inclinations," Ph.D. thesis, The University of Auckland.

[10] Goda, H., Hibiki, T., Kim, S., Ishii, M., and Uhle, J., 2003, "Drift Flux Model for Downward Two Phase Flow," International Journal of Heat and Mass Transfer, **46**, pp. 4835-4844.

[11] Rouhani, S. Z., and Axelsson, E., 1970, "Calculation of void volume fraction in the subcooled and quality boiling regions," International Journal of Heat and Mass Transfer, **13**, pp. 383-393.

[12] Usui, K., and Sato, K., 1989, "Vertically Downward Two Phase Flow (I) Void Distribution and Average Void Fraction," Journal of Nuclear Science and Technology, **26**(7), pp. 670-680.

[13] Zuber, N., and Findlay, J. A., 1964, General Electric, GEAP-4592.

[14] Hasan, A. R., 1995, "Void Fraction in Bubbly and Slug Flow in Downward Vertical and Inclined systems," Society Of Petrolium Engineers Production and Facilities, **10**(3), pp. 172-176.

[15] Cai, J., Chen, T., and Ye, Q., 1997, "Void fraction in bubbly and slug flow in downward air-oil two phase flow in vertical tubes," Proceedings of International Symposium on Multiphase Flow, pp. 68-73.

[16] Nicklin, D. J., Wilkes, J. O., and Davidson, J. F., 1962, "Two Phase Flow in Vertical Tubes," Institute of Chemical Engineers, **40**, pp. 61-68.

[17] Kawanishi, K., Hirao, Y., and Tsuge, A., 1990, "An experimental study on drift flux parameters for two phase flow in vertical round tubes," Nuclear Engineering and Design, **120**, pp. 447-458.

[18] Bendiksen, K. H., 1985, "An experimental investigation of the motion of long bubbles in inclined tubes," International Journal of Multiphase Flow(10), pp. 467-483.

[19] Alves, I. N., Shoham, O., and Taitel, Y., 1993, "Drift Velocity of Elongated Bubbles in Inclined Pipes," Chemical Engineering Science, **48**(17), pp. 3063-3070.

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[20] Haramathy, T. Z., 1960, "Velocity of Large Drops and Bubbles in Media of Infinite or Restricted Extent," AIChE, **6**, pp. 281-288.

[21] Gomez, L. E., Shoham, O., Schmidt, Z., Choshki, R. N., and Northug, T., 2000, "Unified Mechanistic Model for Steady State Two Phase Flow: Horizontal to Upward Vertical Flow," Society of Petrolium Enginners Journal, **5**, pp. 339-350.

[22] Ghajar, A. J., and Tang, C. C., 2012, "Void Fraction and Flow Patterns of Two-Phase Flow in Upward and Downward Vertical and Horizontal Pipes," Advances in Multiphase Flow and Heat Transfer **4**, pp. 175-201.

[23] Abdul-Majeed, G. H., 1996, "Liquid holdup in horizontal two phase gas-liquid flow," Journal of Petroleum Science and Engineering, **15**, pp. 271-280.

[24] Arosio, L., and Stogia, G., 1976, "A Downward Two Phase Flow- Experimental Investigation," Energia Nucleare, **23**(7), pp. 396-401.

[25] Badie, S., Hale, C. P., and Hewitt, G. F., 2000, "Pressure gradient and holdup in horizontal two phase gas-liquid flows with low liquid loading " International Journal of Multiphase Flow, **26**(1525-1543).

[26] Chen, J. J., 1986, "A Further Examination of Void Fraction in Annular Two-phase Flow," International Journal of Heat and Mass Transfer, **29**, pp. 4269-4272.

[27] Chokshi, R., 1994, "Prediction of pressure drop and liquid holdup in vertical two phase flow through large diameter tubing," Ph.D. thesis, University of Tulsa, Tulsa.

[28] Eaton, B. A., 1966, "The prediction of flow patterns, liquid holdup and pressure losses occuring during continuous two phase flow in horizontal pipelines," Ph.D. thesis, University of Texas, Austin.

[29] Fernandes, R. C., 1981, "Experimental and therotical studies of isothermal upward gas-liquid flows in vertical tubes," Ph.D. thesis, University of Houston.

[30] Franca, H., and Lahey, R. T., 1992, "The use of drift flux techniques for the analysis of horizontal two phase flows," International Journal of Multiphase Flow, **18**, pp. 787-801.

[31] Hernandez, A., 2002, "Experimental Research on Downward Two Phase Flow," Proceedings of SPE Annual Technical Conference and Exhibition

[32] Isbin, H. S., Shear, N. C., and Eddy, K. C., 1957, "Void fractions in steam water two phase flow," AIChE, **3**, pp. 136-142.

[33] Minami, K., and Brill, J. P., 1987, "Liquid holdup in wet gas pipelines," Society of Petrolium Enginners **2**, pp. 36-44.

[34] Oshinowo, O., 1971, "Two Phase Flow in a Vertical Tube Coil," Ph.D. thesis, University of Toronto, Toronto.

[35] Paras, G., 1982, "Characterization of Downward Two Phase Flow By Neutron Noise Analysis," MS thesis, University of Washington.

[36] Schmidt, J., Giesbrecht, H., and Van der Geld, C. W., 2008, "Phase and velocity distributions in vertically upward high viscosity two phase flows," International Journal of Multiphase Flow, **34**, pp. 363-374.

[37] Sujumnong, M., 1998, "Heat tranfer, pressure drop and void fraction in two phase two component flow in a vertical tube," PhD thesis, University of Manitoba.

[38] Yijun, J., and Rezkallah, K., 1993, "A Study on void fraction in vertical co-current upward and downward two-phase gas-liquid flow - I: Experimental Results," Chemical Engineering Communication, **126**, pp. 221-243.

[39] Greskovich, E. J., and Cooper, W. T., 1975, "Correlation and prediction of gas-liquid holdups in inclined upflows," AIChE, **21**, pp. 1189-1192.

[40] Bonnecaze, R. H., Erskine, and Greskovich, E. J., 1971, "Hold up and pressure drop for two phase slug flow in inclined pipelines," AIChE, **17**, pp. 1109-1113.

[41] Kokal, S. L., and Stainslav, J. F., 1989, "An Experimental Study of Two Phase Flow in Slightly Inclined Pipes II: Liquid Holdup and Pressure Drop," Chemical Engineering Science, **44**, pp. 681-693.

[42] Yamazaki, Y., and Yamaguchi, K., 1979, "Characteristics of Two Phase Downflow in Tubes: Flow Pattern ,Void Fraction and Pressure Drop," Journal of Nuclear Science and Technology, **16**(4), pp. 245-255.

[43] Armand, A. A., 1946, "Resistance to two phase flow in horizontal tubes," IZV,VTI (in Russian), **15**, pp. 16-23.

[44] Dix, G. E., 1971, "Vapor void fractions for forced convection with subcooled boiling at low flow rates," Ph.D. thesis, University of California, Berkeley.

[45] Huq, R. H., and Loath, J. L., 1992, "Analytical Two-Phase Void Fraction Prediction Method," Journal of Thermophysics, **6**, pp. 139-144.

[46] Permoli, A., Francesco, D., and Prima, A., 1970, "An emperical correlation for evaluating two phase mixture density under adiabatic conditions," European Two Phase Flow group Meeting Milan, Italy.

[47] Sun, K. H., Duffey, R. B., and Peng, C. M., 1981, "The prediction of two phase mixture level and hydrodynamically controlled dryout under low flow conditions," International Journal of Multiphase Flow, **7**, pp. 521-543.

[48] Lockhart, R. W., and Martinelli, R. C., 1949, "Proposed correlation of data for Isothermal Two Phase, Two Component flow in Pipes," Chemical Engineering Progress, **45**, pp. 39-48.

[49] Hasan, A. R., and Kabir, C. S., 1988, "A Study of Multiphase Flow Behavior in Vertical Wells," SPE Production Engineering, **3**, pp. 263-272.

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