

EMPIRICAL CORRELATIONS FOR FRICTION FACTOR IN
DRAG-REDUCING TURBULENT PIPE FLOWS

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ABSTRACT

Measurements of friction factor for solutions of Separan AP-273 and AP-30 at various concentrations are presented. From these data the critical Weissenberg number for friction for Separan AP-273 and AP-30 is found to be of the order of 7-13 ($20,000 \leq Re_a \leq 40,000$) and 5-10 ($10,000 \leq Re_a \leq 20,000$), respectively. For each polymer solution an empirical correlation in terms of the Reynolds number, Weissenberg number, and critical Weissenberg number is developed. The proposed correlations predict the experimental data with good accuracy.

Introduction

It is well-known that the addition of small amounts of certain high molecular weight polymers to a solvent results in a viscoelastic fluid. The friction drag of such a viscoelastic fluid under turbulent flow conditions is lower than the values associated with the pure solvent. Another interesting aspect of these fluids is that their turbulent friction behavior is affected by various factors such as polymer concentration, the level of mechanical degradation, solvent chemistry, pipe diameter, and flow rate. In general, the drag reduction increases with increasing flow rate, polymer molecular weight and polymer concentration, while it decreases with increasing diameter of pipe [1]. Mechanical degradation of the polymer is associated with an increase in friction factor [2]. The effect of the solvent chemistry on the friction is more complicated [3].

The polymer concentration, the level of mechanical degradation, and the effect of solvent chemistry, influence the fluid time scale (i.e. elasticity), while the flow rate and pipe diameter determine the flow time scale [4]. The ratio of the fluid time scale (λ) to the flow time scale (D/U) forms a dimensionless parameter called Weissenberg number (Ws). This parameter has been found to adequately account for the influence of polymer concentration, mechanical degradation, pipe diameter, and solvent chemistry on the friction

behavior of viscoelastic fluids [1,4-10].

Kwack et al. [5] found that for aqueous polyacrylamide solutions (Separan AP-273) flowing turbulently in circular pipes, the fully established friction factor is a function of both Reynolds and Weissenberg numbers. For a fixed Reynolds number, the friction factor decreases with an increase in the Weissenberg number up to certain critical Weissenberg number for friction (Ws_{cf}). If $Ws \geq Ws_{cf}$ friction factor reaches its minimum asymptotic value and can be expressed as a function of Reynolds number only. Stated in functional terms,

$$f = f(Re_a, Ws) \quad \text{when } Ws < Ws_{cf} \quad (1)$$

$$f = f(Re_a) \quad \text{when } Ws \geq Ws_{cf} \quad (2)$$

where the critical Weissenberg number (based on the Powell-Eyring fluid model) for friction for aqueous polyacrylamide solutions is of the order of 5-10 for the Reynolds number range of 20,000 to 30,000 [5].

Based on the above observations, Kwack and Hartnett [8] correlated their experimental data for 1000 ppm solutions of Separan AP-273, for two fixed Reynolds numbers, using a nonlinear curve fitting technique. They proposed the following empirical correlations for friction factor for Reynolds numbers of 20,000 and 30,000, respectively.

$$f = 0.00172 + 0.00472 (1.0 + 8.65Ws + 0.0527Ws^5)^{-0.485} \quad (3)$$

$$f = 0.00142 + 0.00458 (1.0 + 11.0Ws + 0.0260Ws^5)^{-0.469} \quad (4)$$

It is to be noted that the above correlations are of very limited use since they are restricted to one type of polymer, one concentration, and a fixed Reynolds number.

In a recent study, Kwack and Hartnett [9] extended their previous empirical correlations [8], to cover a wider range of Reynolds numbers. Based on their experimental results obtained with 300 and 500 ppm solutions of Separan AP-273 for the Reynolds number range from 10,000 to 80,000 the following correlation was proposed.

$$f = f_v + (f_n - f_v) [1.0 + C_1 Ws + C_2 Ws^5]^{-0.50} \quad (5)$$

where

$$f_n = 0.079 Re_a^{-0.25} ; f_v = 0.20 Re_a^{-0.48}$$

$$C_1 = 7.1 \times 10^9 Re_a^{-2.6} ; C_2 = 0.15 Re_a^{-0.37}$$

According to the authors [9], Eq. (5) predicts their experimental data with a maximum deviation of 10% with the majority of predictions being within 5%. It is to be noted that although Eq. (5) is in a general form, it does not include the influence of the critical Weissenberg number on the friction factor as expressed by Eqs. (1) and (2). Furthermore, the adjustable constants in the relations for C_1 and C_2 are restricted to one type of polymer and two concentrations. In addition, the general applicability of the correlation to other polymer solutions should be examined. One additional note about Eq.

(5), the minus sign in the exponent of Eq. (5) was omitted in the published work [9].

From the foregoing discussion, it can be concluded that the Weissenberg number, the critical Weissenberg number, and the Reynolds number are the major dimensionless parameters that control the friction factor behavior in viscoelastic turbulent pipe flows. Therefore, if an empirical relationship in terms of these parameters can be obtained, the friction factor of an aqueous polymer solution flowing in a circular pipe could be predicted. The aim of this study is to develop such a correlation.

To address the above objective, measurements of friction factor for two polyacrylamide solutions (Separan AP-273 and AP-30) with different concentrations and Reynolds numbers were carried out. Based on this data and the rheological properties of the fluid, the Weissenberg number and the critical Weissenberg number for friction for each polymer type was established. This information was then used to develop the desired correlation for friction factor in terms of the pertinent dimensionless parameters (Ws , Ws_{CF} , Re_a).

Experiments

The flow circulation experimental setup used in this study is the same setup used in our previous studies with viscoelastic fluids for measurements of pressure drop and heat transfer in the entrance and fully developed regions [1,10,11]. Therefore, only the features pertinent to this study will be briefly described.

The test section used has an inside diameter of 1.88 cm ($L/D = 617$) and is made of stainless steel (Type 304). To minimize mechanical degradation of polymer solutions, the overall flow system was operated with pressurized air. The pressure drop measurements were taken in the fully developed region with a U-tube mercury manometer. To ensure fully developed flow, numerous pressure taps were drilled into the test section. Pressure drop in the entrance region was measured with a multi-column water manometer. The flow rate was measured by a one-inch turbine meter located upstream from the test section. The fluids used were the well-mixed homogeneous aqueous solutions of Separan AP-273 with concentrations of 10, 50, 100, 300, 500, and 1000 ppm in the once-through flow mode and Separan AP-30 with a starting concentration of 1500 ppm in the recirculation-dilution flow mode. For dilution, the proper amount of tap water was added to the existing solution after each

recirculation and data collection to obtain the desired concentration. Apparent viscosities of the solutions were measured at a wide range of shear rates (0.36 to $2 \times 10^4 \text{sec}^{-1}$) with two Couette viscometers (Brookfield Synco-Electric Model LVT with UL adaptor and a Fann Model VG) and a capillary tube viscometer (0.94 mm I.D. and $l/d = 325$). The measured viscosities for Separan AP-273 and AP-30 are presented in Figs. 1 and 2, respectively.

In order to evaluate the Weissenberg number, a fluid time scale must be calculated. The measured viscosity data were used to estimate the fluid time scale (λ) by the Powell-Eyring fluid model, which has the following expression

$$\eta_a = \eta_\infty + (\eta_0 - \eta_\infty) \left[\frac{\sinh^{-1} \lambda \dot{\gamma}}{\lambda \dot{\gamma}} \right] \quad (6)$$

The fluid time scale was determined by a linear regression method with the use of all the viscosity data for each solution. Further details on the determination of the fluid time scale and the model used may be found elsewhere [10,12,13].

The measurements of the pressure drop are presented in terms of friction factor as a function of apparent Reynolds number for Separan AP-273 and AP-30 in Figs. 3 and 4, respectively. An increase in the polymer concentration generally results in a decrease in the friction factor up to a certain asymptotic limit. For the studies shown in Figs. 3 and 4, this asymptotic limit was reached at a concentration of 300 ppm for Separan AP-273 and 800 ppm for Separan AP-30. This indicates that Separan AP-273 solution is a much more effective drag-reducer than the Separan AP-30 solution. This difference is mainly due to the higher average molecular weight of Separan AP-273 (6×10^6) compared to Separan AP-30 (4×10^6) which makes Separan AP-273 more elastic.

The reliability of the flow circulation system and the experimental procedures were checked with several calibration runs for measurements of friction factor for a Newtonian fluid (tap water) by comparing the experimental results with well-established Newtonian correlations [14,15]. The uncertainty analyses of the overall experimental procedures for Newtonian and viscoelastic fluids showed that there is 5-10% uncertainty in friction factor measurements. More detailed description of the experimental apparatus and procedures are presented elsewhere [1,10-12].

Critical Weissenberg Number Correlations

The critical Weissenberg number is one of the key dimensionless

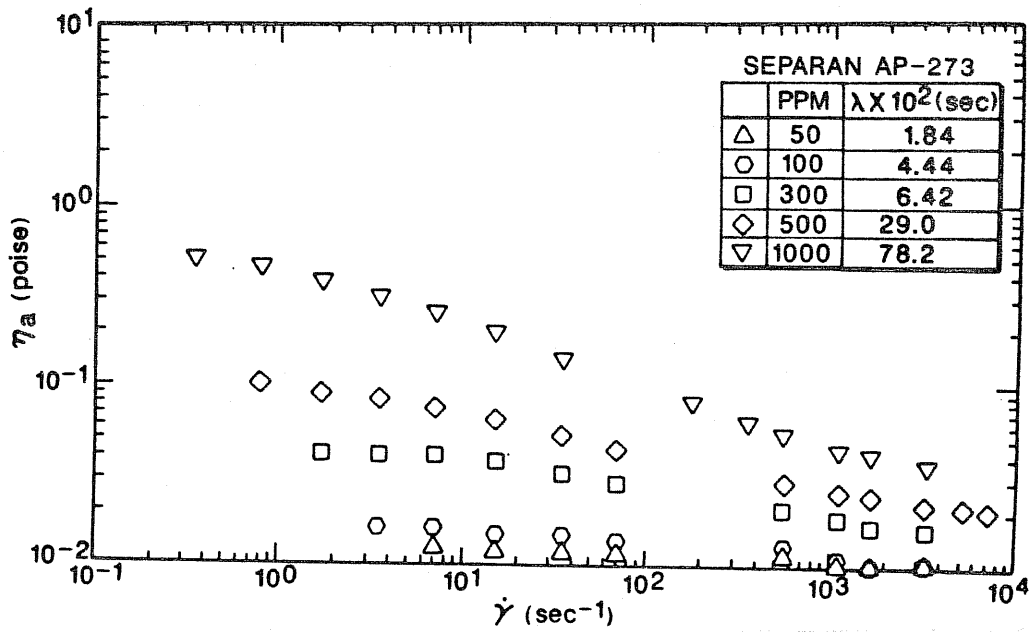


FIG. 1
Apparent viscosity vs. shear rate for various Separan AP-273 solutions.

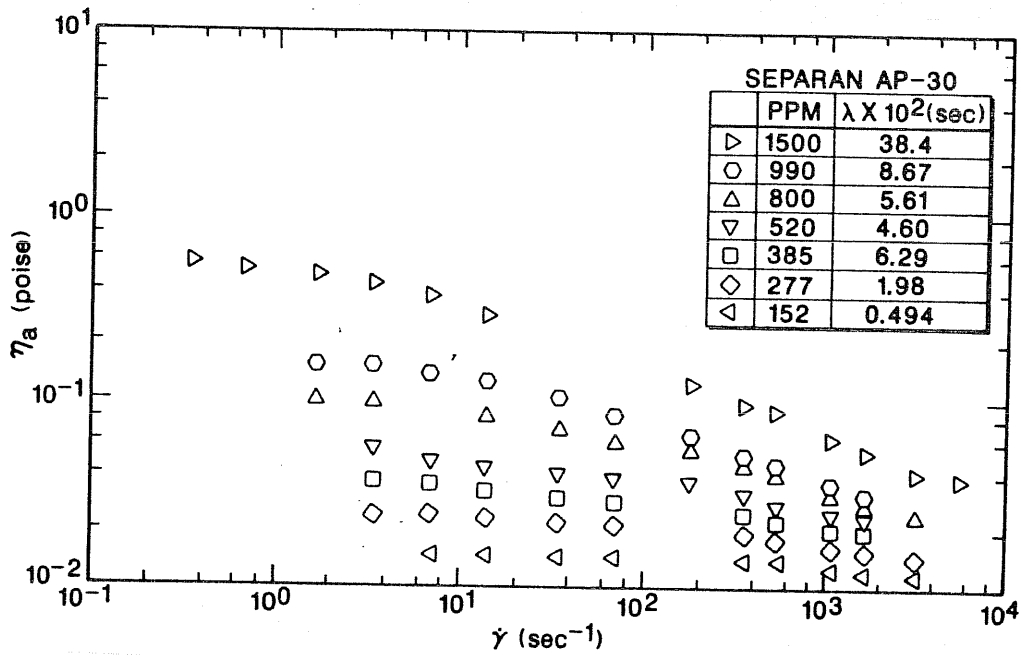


FIG. 2
Apparent viscosity vs. shear rate for various Separan AP-30 solutions.

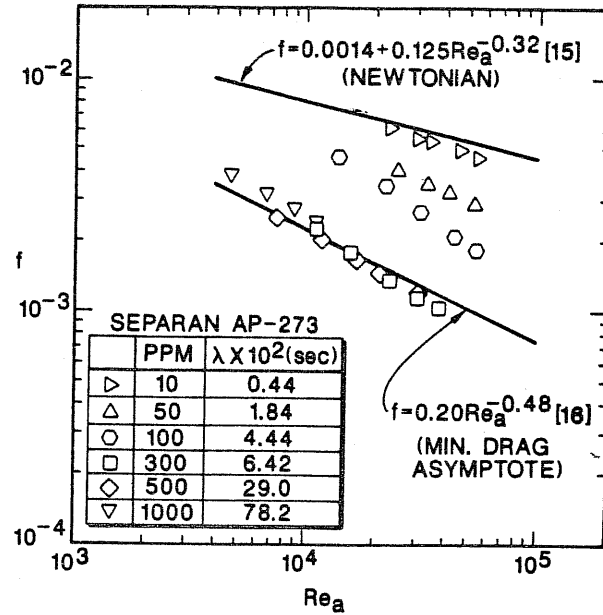


FIG. 3
Friction factor vs. apparent Reynolds number for various Separan AP-273 solutions.

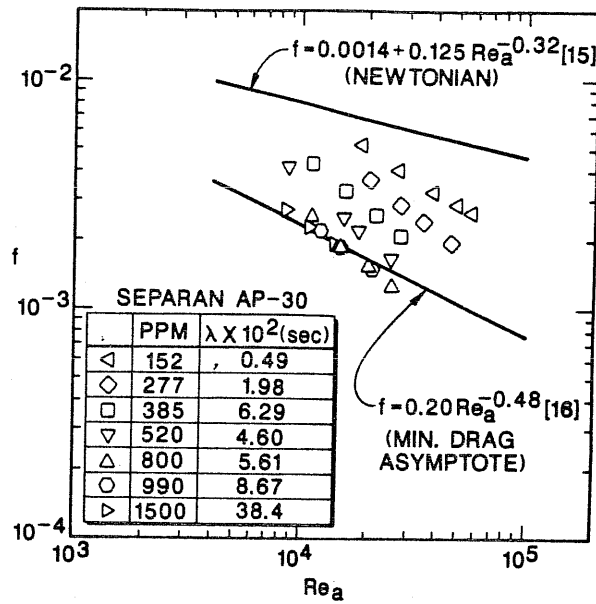


FIG. 4
Friction factor vs. apparent Reynolds number for various Separan AP-30 solutions.

parameters that controls the friction behavior in viscoelastic turbulent pipe flows. In addition, the critical Weissenberg number can suggest the optimum concentration compromising the performance and the economics of polymer addition. Therefore, accurate determination of its value is very important.

The results presented in Figs. 3 and 4 are replotted in terms of friction factor as a function of Weissenberg number for three different Reynolds numbers ranging from 10,000 to 40,000 as shown in Figs. 5 and 6 for Separan AP-273 and AP-30, respectively. These figures show that the turbulent friction factor remains constant (lies on the minimum friction asymptotic line) in the high Weissenberg number region. However, as the Weissenberg number is decreased for a fixed Reynolds number a critical Weissenberg number for friction is reached; below this critical value, the friction factor increases with decreasing Weissenberg number, approaching the Newtonian value as the Weissenberg number approaches zero. From Fig. 5 the critical Weissenberg number for the Separan AP-273 solutions is of the order of 7 to 13 for the Reynolds numbers of 20,000, 30,000 and 40,000 (see dark symbols in Fig. 5). This is in good agreement with the reported results of Kwack et al. [5] and Kwack and Hartnett [9] for Separan AP-273 which were of the order of 5 to 10 for Reynolds numbers of 20,000 and 30,000 [5] and 4 to 18 for the Reynolds number range from 10,000 to 70,000 [9]. Similarly, from Fig. 6 the critical Weissenberg number for the Separan AP-30 solutions is of the order of 5 to 10 for the Reynolds numbers of 10,000, 15,000 and 20,000 (see dark symbols in Fig. 6).

The results presented in Figs. 5 and 6 indicate that the critical Weissenberg number for friction is a function of both the Reynolds number and the polymer type. The critical Weissenberg number shows an increase with increasing Reynolds number for both polymer solutions. The critical Weissenberg number at the same Reynolds number ($Re_a = 20,000$) is lower for Separan AP-273 than AP-30 (see Figs. 5 and 6). This difference is rendered to the effectiveness of the polymers considered. As previously discussed in connection with Figs. 3 and 4, Separan AP-273 is more effective in drag reduction than Separan AP-30, since a smaller concentration of the Separan AP-273 was necessary to reach the minimum drag asymptote. A smaller concentration translates into a smaller "effective concentration", which is the minimum concentration that forces the friction coefficient to reach the asymptotic condition, and therefore a smaller critical Weissenberg number.

The dependency of the critical Weissenberg number on the Reynolds number

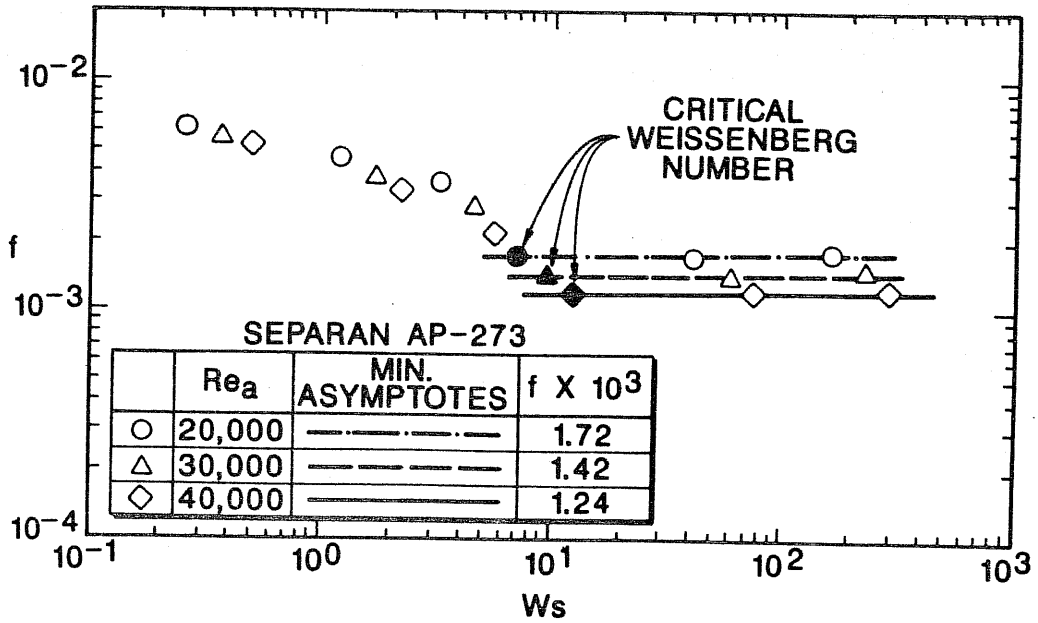


FIG. 5
Friction factor of Separan AP-273 vs. Weissenberg number for various Reynolds numbers.

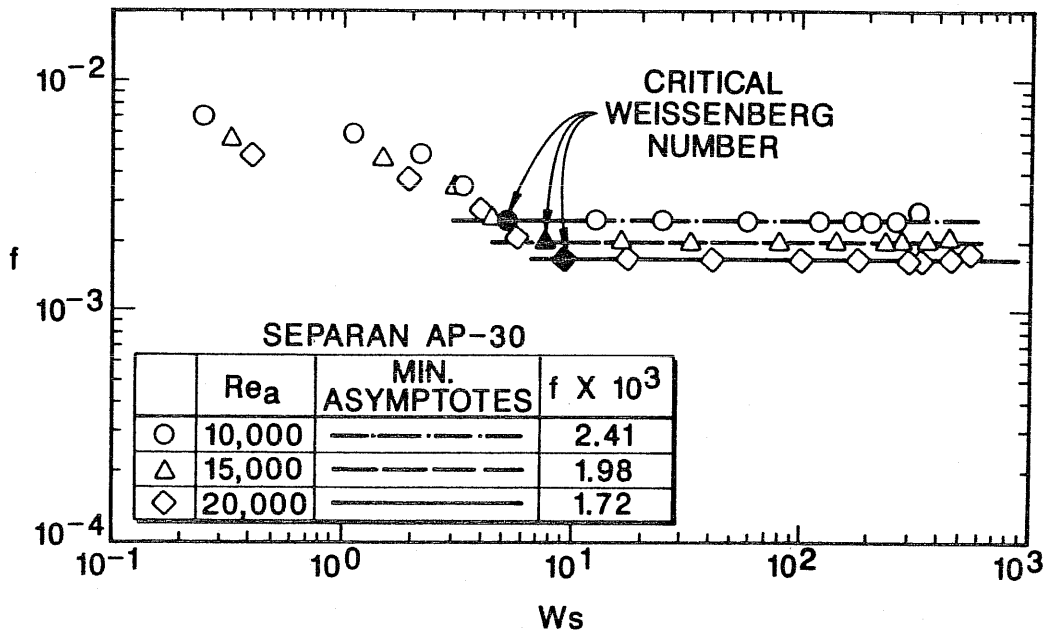


FIG. 6
Friction factor of Separan AP-30 vs. Weissenberg number for various Reynolds numbers.

can be expressed as

$$Ws_{cf} = c_1 Re_a^{c_2} \quad (7)$$

where c_1 and c_2 are curve-fitted constants. For the polymer solutions used, the final results are

$$Ws_{cf} = 1.5 \times 10^{-3} Re_a^{0.85} \quad \text{for Separan AP-273} \quad (8)$$

$$Ws_{cf} = 2.1 \times 10^{-3} Re_a^{0.85} \quad \text{for Separan AP-30} \quad (9)$$

These correlations predict the critical Weissenberg numbers determined from Figs. 3 and 4 within a maximum error of $\pm 1.5\%$.

Equations (8) and (9) suggest that the variation of the critical Weissenberg number with the Reynolds number, for both polymers, follows the same trend; in fact both have the same value of the exponent c_2 with the difference occurring only in the leading constant c_1 . As discussed earlier, this difference is due to the effectiveness of the polymers used.

Friction Factor Correlations

The friction factor as a function of apparent Reynolds number for different concentrations of Separan AP-273 and AP-30 were shown in Figs. 3 and 4, respectively. From these figures it can be observed that beyond the 300 ppm concentration for Separan AP-273 and 800 ppm concentration for Separan AP-30, no further reduction in friction factor is obtained. For the moderately higher concentrations the friction factors fall on a single line, an indication that the minimum drag asymptote is reached. Cho and Hartnett [16] showed that the minimum drag asymptote for Separan AP-273 and Polyox WSR-301 solutions can be described by the following equation:

$$f = 0.20 Re_a^{-0.48} \quad (10)$$

Yoon and Ghajar[1] showed that their minimum drag asymptote in two different pipe diameters (1.11 and 1.88 cm) is within $\pm 10\%$ of Eq. (10) for Separan AP-273 and Polyox WSR-301 solutions. This study's experimental data for Separan AP-273 and AP-30 solutions is also within $\pm 10\%$ of Eq. (10) as shown in Figs. 3 and 4. Since these deviations are within the uncertainty range of the experimental data, therefore Eq. (10) is considered reliable for representing the minimum drag asymptote.

Figures 5 and 6 show that for Weissenberg numbers equal or greater than the critical Weissenberg number, the friction factors fall on a line that coincides with the minimum drag asymptote given by Eq. (10). This suggests that for $Ws \geq Ws_{cf}$ the friction factor is a function of Reynolds number only. However, for $Ws < Ws_{cf}$ the friction factor is a function of both

Reynolds and Weissenberg numbers. Actually the friction factor increases exponentially as the Weissenberg number decreases.

These facts suggest that if a correlation, that would estimate the friction factor of polymer solutions at any concentration, would exist, it should comprise of two parts:

- (a) A monomial function in terms of Re_a only, that would describe the asymptotic line of any curve of Fig. 5 or 6 when $Ws \geq Ws_{cf}$.
- (b) An exponential function in terms of Ws and Ws_{cf} that would describe the increasing trend of any curve of Fig. 5 or 6, when $Ws < Ws_{cf}$, and that would die out at the transition point where $Ws = Ws_{cf}$. In functional terms, requirements (a) and (b) are stated as

$$f = g(Re_a) \cdot h(Ws, Ws_{cf}) \quad (11)$$

But Eq. (10) could fit requirement (a) and requirement (b) is conceived as

$$h(Ws, Ws_{cf}) = \text{Exp}[K(1-Ws/Ws_{cf})^c] \quad (12)$$

where Ws can be determined from the measured apparent viscosity data and the Powell-Eyring fluid model (see Eq. (6)) and Ws_{cf} from Eq. (8) or (9). Therefore Eq. (11) becomes

$$f = 0.20 Re_a^{-0.48} \text{Exp}[K(1-Ws/Ws_{cf})^c] \quad (13)$$

Equation (13) does not apply to the case when $Ws > Ws_{cf}$. For this case, the term (Ws/Ws_{cf}) should be set to unity. In addition, due to the nature of Eq. (13), this correlation is not applicable to Newtonian fluids when Weissenberg number is zero. The constants c and K in Eq. (13) are curve-fitted constants and for the polymer solutions used, the final results are

$$f = 0.20 Re_a^{-0.48} \text{Exp}[1.4(1-Ws/Ws_{cf})^{1.1}] \quad \text{for Separan AP-273} \quad (14)$$

$$f = 0.20 Re_a^{-0.48} \text{Exp}[1.0(1-Ws/Ws_{cf})^{1.1}] \quad \text{for Separan AP-30} \quad (15)$$

The predicted friction factors with the use of both correlations for various concentrations of Separan AP-273 and AP-30 are compared with the experimental data in Fig. 7. This figure suggests that the parameters Re_a , Ws and Ws_{cf} used in the proposed correlations fully represent the characteristics of the friction factor for the polymer solutions used. The correlations predict all the experimental data for both polymer solutions with a maximum deviation of $\pm 15\%$, with the majority of the data points within $\pm 10\%$.

It should be noted that the predictive capability of the recent correlation of Kwack and Hartnett [9], see Eq. (5), was also tested with our friction factor experimental data. Their correlation predicted some of our experimental data for Separan AP-273 with good accuracy. The data for 10, 300, 500, and 1000 ppm solutions were predicted with an average absolute error

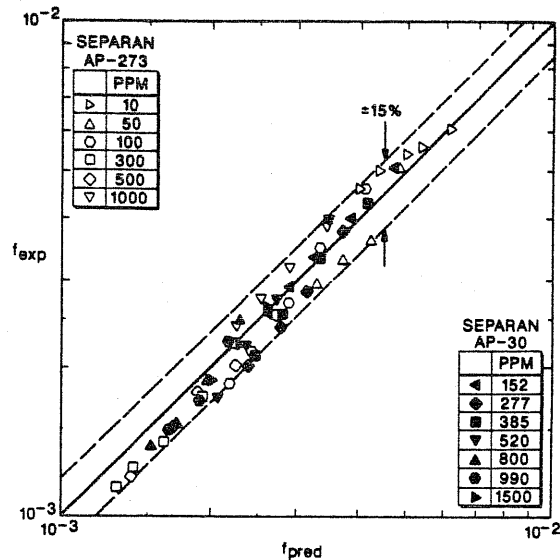


FIG. 7

Comparison of the predicted friction factors using the proposed correlations with measurements.

of 10%. However, the predicted results for 50 and 100 ppm solutions showed large deviations (as much as 60%) compared to the experimental data. The good agreement for these particular concentrations of Separan AP-273 is to be expected since (1) the constants in Eq. (5) were based on the two specific concentrations of 300 and 500 ppm of Separan AP-273 and (2) the predicted friction factors for concentrations of 10 and 1000 ppm correspond very closely to the limiting values of Eq. (5) which are the well-established and general Newtonian friction factor correlation and the minimum drag asymptote. Comparison of the predictions from Eq. (5) with our experimental data for Separan AP-30 for concentrations below 800 ppm showed large deviations (as much as 80%). However, as expected, their correlation predicted the experimental data at or near the minimum drag asymptote with good accuracy. These data were predicted with an average absolute error of 11%.

Comparison of Eqs. (14) and (15) reveals that the value of the curve-fitted constant c is the same for both correlations. Suggesting that with the decrease in the Weissenberg number, the exponential increase in the friction factor has the same trend for both polymer solutions, as evidenced from Figs. 5 and 6. The only difference between the two correlations is in the value of the constant K . This difference is due to the effectiveness of the polymers

considered. This fact was observed earlier when only 300 ppm of Separan AP-273 as opposed to 800 ppm of Separan AP-30 was necessary to reach the minimum drag asymptote (see Figs. 3 and 4).

The effectiveness of a polymer solution is directly proportional to its elasticity. Or the elasticity is closely associated with the fluid time scale (λ). For a given concentration, a much larger λ corresponds to a more effective polymer solution. Or a larger λ gives a larger W_s but a smaller $W_{s_{cf}}$, as the previous section results showed (see Figs. 5 and 6). That makes the term $(1 - W_s/W_{s_{cf}})$ of the correlations smaller in the case of Separan AP-273 than it is for Separan AP-30. Thus a higher value of the constant K for Separan AP-273 is indicative of more effectiveness of this polymer solution as compared with Separan AP-30.

It is speculated that the constant K is proportional to the molecular weight ratio of the two polymers. In fact the ratio of the constant K for Separan AP-273 to that of Separan AP-30 is 1.4, while the ratio of the molecular weights of Separan AP-273 (6×10^6) and Separan AP-30 (4×10^6) is 1.5. This hypothesis also extends to Eqs. (8) and (9) for the critical Weissenberg number. However, in this case since the critical Weissenberg number decreases with an increase in the polymer molecular weight (at a fixed Reynolds number), the ratio of the leading constant for Separan AP-273 to that of Separan AP-30 is inversely proportional to the ratio of the molecular weights. Considering the uncertainty reported for the experimental friction factor data, the agreement between the two ratios is considered to be very close. Therefore, based on the proposed hypothesis, it might be possible to use a single correlation to predict the friction factor behavior of different polymer solutions by properly accounting for the influence of the molecular weight on the friction factor.

Conclusions

The current experimental results provided additional confirmation of the existence of critical Weissenberg number for friction, above which the fully established friction factor reaches its minimum asymptotic value. In addition, the generality of the minimum drag asymptote was confirmed. The results indicate that the critical Weissenberg number is a function of both Reynolds number and the polymer type. The critical value increases with increasing Reynolds number and decreases with an increase in the polymer molecular weight (at a fixed Reynolds number). The critical Weissenberg

number for aqueous polyacrylamide solutions of Separan AP-273 and AP-30 is found to be of the order of 7 to 13 ($20,000 \leq Re_a \leq 40,000$) and 5 to 10 ($10,000 \leq Re_a \leq 20,000$), respectively. For each polymer solution an empirical correlation for critical Weissenberg number as a function of Reynolds number was developed. The fully established friction factors were successfully correlated as a function of Re_a , Ws , and Ws_{cf} numbers for each polymer solution. Using the proposed correlations, the experimental data were predicted with a maximum deviation of $\pm 15\%$. It was hypothesized that it might be possible to use a single empirical correlation to predict the drag reduction behavior of different viscoelastic fluids by properly accounting for the influence of the molecular weight on the friction factor.

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Nomenclature

c	constant in Eqs. (12) and (13)
c_1, c_2	constants in Eq. (7)
C_1, C_2	constants in Eq. (5)
D	inside diameter of test section
f	friction factor, $f = \tau_w / (\frac{1}{2} \rho U^2)$
f_n	friction factor of Newtonian fluid, see Eq. (5)
f_v	minimum drag asymptote, see Eq. (5)
K	constant in Eqs. (12) and (13)
L	length of test section
Re_a	apparent Reynolds number, $Re_a = \rho U D / \eta_a$
U	average velocity
Ws	Weissenberg number, $Ws = \lambda U / D$
Ws_{cf}	critical Weissenberg number for friction
η_a	apparent viscosity at the wall
η_0	zero shear rate apparent viscosity
η_∞	infinite shear rate apparent viscosity
$\dot{\gamma}$	shear rate
λ	fluid time scale
ρ	density of fluid
τ_w	wall shear stress

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