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A HEAT TRANSFER CORRELATION FOR VISCOELASTIC TURBULENT PIPE FLOWS

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A modified version of Kale's heat transfer correlation for viscoelastic turbulent pipe flows in terms of drag reduction ratio and Weissenberg number is presented. The proposed correlation is validated with experimental data of Kwack and this study for Separan AP-273 and Polyox WSR-301 with concentrations ranging from 10 to 1000 ppm in thermally fully developed flow in pipes with diameters of 1.11 and 1.88 cm I.D. under constant wall heat flux. The agreement between the experimental data and the predictions is within a maximum deviation of 30%.

KEYWORDS Heat transfer correlation Viscoelastic fluids Drag reduction Turbulent pipe flow Polymer solutions.

INTRODUCTION

Several attempts have been made to develop heat transfer correlations for drag reducing turbulent pipe flows, for example, Pruitt *et al.* (1966), Poreh and Paz (1968), Wells (1968), Smith *et al.* (1969), Corman (1970), and Kale (1977). However, none of the available correlations are capable of predicting the heat transfer coefficients in the thermally fully developed region for different polymer solutions with wide range of concentrations, i.e. for the minimum to maximum heat transfer reduction cases (Kwack, 1983). The inadequacy of the existing heat transfer correlations is mainly due to the fact that these correlations were developed and established using inaccurate experimental data and incorrect assumptions. As recent major review of the relevant works point out (Cho and Hartnett, 1982), most of the previous experimental studies have been carried without taking into account all of the following important factors: 1) thermal entrance length, 2) polymer degradation, and 3) polymer rheology. In addition, most of the available heat transfer correlations are based on Reynolds analogy, equality between eddy diffusivities of heat and momentum, which has been verified to be invalid for viscoelastic turbulent pipe flows (Kwack, 1983; Yoon and Ghajar, 1984).

In this study, new experiments have been conducted to provide a detailed and reliable data base, which are free from the above mentioned experimental

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deficiencies. Based on this data base, Kale's (1977) heat transfer correlation, which is one of the most recent and sophisticated correlations, was modified with the use of pertinent dimensionless parameters for viscoelastic fluids. The two key dimensionless parameters used are friction drag reduction ratio and Weissenberg number, which can be determined from experimental measurements of pressure drop and rheological properties.

EXPERIMENTS

The present experiments were conducted in the fluid dynamics laboratory at Oklahoma State University. A schematic diagram of the flow circulation system is shown in Figure 1. The test sections used consist of two seamless stainless steel pipes (Type 304) with inside diameters of 1.88 cm ($L/D = 617$) and 1.11 cm ($L/D = 1046$). These test sections ensure the thermally fully developed condition for viscoelastic fluids which require 400 to 500 diameters for the minimum heat transfer asymptote (Toh and Ghajar, 1988). The end connections of test sections consist of copper plates which were silver-arc soldered to the ends of the test section to secure a well defined electric circuit through the end plates. The upstream and downstream sections were electrically insulated from one another. The two-section arrangement allows the operator to choose the location where he desires the temperature profile to begin developing.

The test solutions were prepared in the 2.27 m^3 mixing tank and gravity drained to the 2.27 m^3 storage tank. To minimize mechanical degradation of

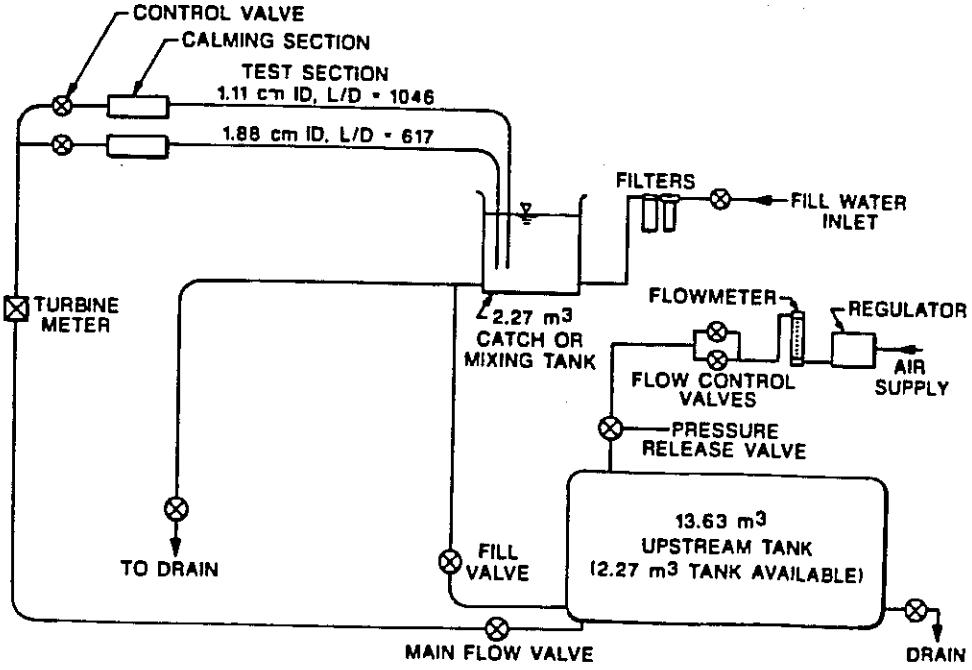

FIGURE 1 Schematic diagram of the flow circulation system.

polymer solutions for these experiments, the 2.27 m^3 upstream stainless steel storage tank was pressurized, forcing the fluid through the test section. While the overall flow system was operated with pressurized air (up to 80 psig) using the once-through mode, the experiments were conducted by maintaining the pressure in the tank from 30 to 40 psig. The 13.63 m^3 tank was used to store tap water which was needed to clean the build-up of polymer on the inner surface of the test section after each experiment. The flow rates which were obtained by this system, even though severely diminished due to considerable friction drag in the test section of small diameter and long length, covered Reynolds numbers based on the apparent viscosity of up to 1.2×10^5.

The constant wall heat flux boundary condition was maintained by a Lincoln DC-600 welder. It can operate in the constant voltage or constant current mode, and has a 100% duty cycle rating at 600 amps and 44 volts. The test sections were insulated from the environment using fiberglass pipe insulation and vapor-proof pipe tape. Double wrapping of fiberglass insulation was deemed enough to produce the well-insulated condition.

In the present flow system, either the hydrodynamic and thermal entrance regions can develop simultaneously from the beginning of the test section, or the velocity profile can be fully developed before heat transfer starts. The measurements of pressure drop and heat transfer were taken at the same time in the thermally fully developed region with the use of one U-tube mercury manometer and #30 gauge copper-constantan thermocouples connected to a 40 channel Monitor Labs Data Logger Model 9302. To ensure fully developed flow, numerous pressure taps and thermocouples were drilled and attached to the test sections. For accurate pressure drop results, the pressure taps were drilled such that the ratio of wall thickness to tap hole diameter for both test sections was greater than 1.5 and less than 2. Pressure drop in the entrance region was measured with a multi-column water manometer. To eliminate the effect of electrical current flowing through the test section on the thermocouple readings, copper oxide cement was used as the adhesive. To obtain the bulk temperature at the end of the test section, a temperature well which consisted of five baffles was installed just downstream of each test section. Since the inlet temperature of the fluid was uniform across the test section, it was measured by means of a thermocouple probe inserted in the calming section.

The flow rate was measured by a one-inch turbine meter located upstream from the test section. This turbine meter monitored by a Hewlett-Packard frequency counter can produce instant or time-averaged readings so that it enables one not only to obtain the average flow rate but also to check the flow stability.

Apparent viscosities of solutions were measured at wide range of shear rates (0.36 to $2 \times 10^4\,\text{sec}^{-1}$) with the use of two Couette viscometers (Brookfield Synchro-Electric Model LVT with UL adaptor and a Fann Model VG) and a capillary tube viscometer (0.94 mm I.D. and $l/d = 325$). Solution samples were taken from the downstream head tank during each run or immediately after each run.

The reliability of the flow circulation system and the experimental procedures were checked with several calibration runs for measurements of friction factors and heat transfer coefficients for a Newtonian fluid (tap water) by comparing the experimental results with well-established Newtonian correlations (Kays, 1960; McAdams, 1954). The uncertainty analyses of the overall experimental procedures for Newtonian and viscoelastic fluids showed that there is 5–8% uncertainty for friction factors and 8–12% uncertainty for heat transfer coefficients. More detailed description of the experimental apparatus and procedures may be found in Yoon (1986).

The viscoelastic fluids used were the well-mixed homogeneous aqueous solutions of polyacrylamide (Separan AP-273) with concentrations of 10, 50, 100, 300, 500, and 1000 ppm and polyethylene oxide (Polyox WSR-301) with concentrations of 100, 300, 500, and 1000 ppm. The apparent viscosity of each polymer solution at wide range of shear rates was measured and the results may be found in Yoon (1986). The measured viscosity data were used to estimate the fluid time scale by the Powell–Eyring model, which has the following expression

$$\eta_a = \eta_\infty + (\eta_0 - \eta_\infty)\left[\frac{\sinh^{-1}\lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right] \qquad (1)$$

The fluid time scale was determined by a linear regression method with the use of all the viscosity data for each solution.

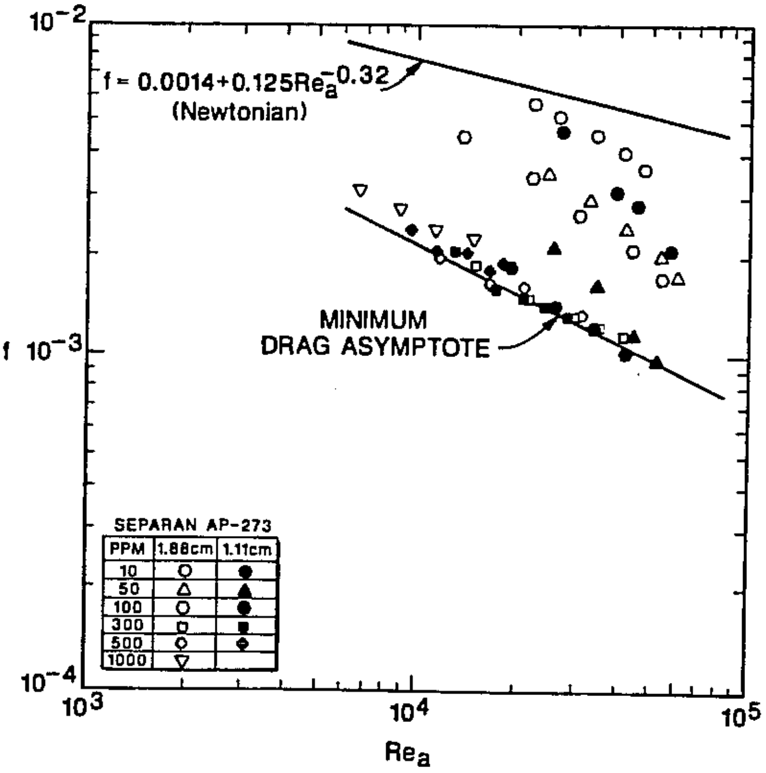

FIGURE 2 Friction factor vs. apparent Reynolds number for Separan AP-273 solutions in the 1.88 and 1.11 cm test sections.

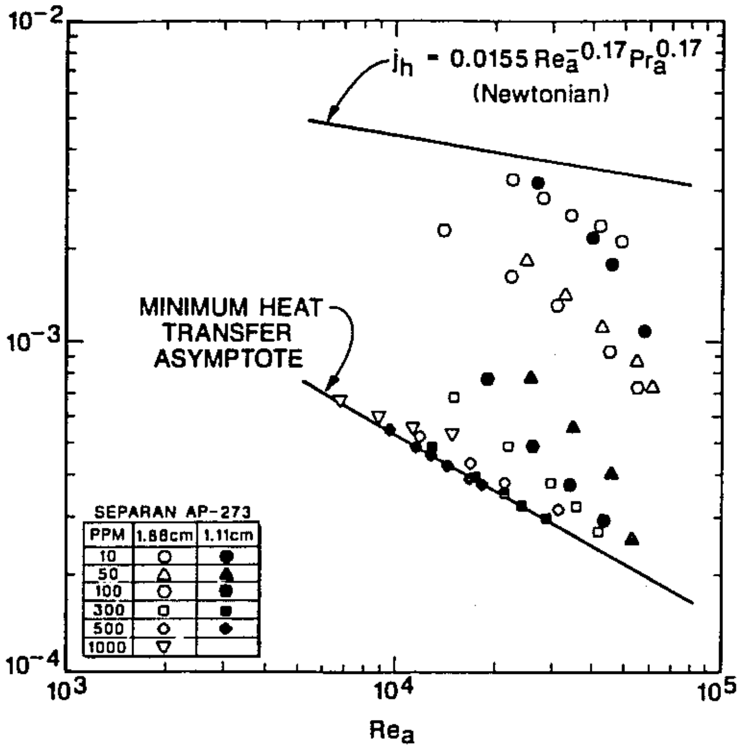

FIGURE 3 Colburn *j*-factor vs. apparent Reynolds number for Separan AP-273 solutions in the 1.88 and 1.11 cm test sections.

The measurements of pressure drop and heat transfer are presented in terms of friction factor and Colburn *j*-factor in Figures 2 and 3 for Separan AP-273 and Figures 4 and 5 for Polyox WSR-301, respectively. These experimental data will be used in the development of the proposed heat transfer correlation.

HEAT TRANSFER CORRELATION

Many rather complicated semi-empirical turbulent heat transfer correlations for non-Newtonian fluids have been developed based on the analogy between heat and momentum transfer (Pruitt *et al.*, 1966; Poreh and Paz, 1968; Wells, 1968; Smith *et al.*, 1969; Corman, 1970; Kale, 1977). These correlations were all designed to yield predictions which are in good agreement with a given set of experimental measurements.

One of the more recent and sophisticated correlations is the one developed by Kale (1977). Kale extended to drag-reducing viscoelastic fluids Reichardt's analysis for heat transfer to Newtonian turbulent flows in smooth pipes. He assumed (incorrectly) that the turbulent Prandtl number ($Pr_t = \varepsilon_m/\varepsilon_h$) is unity for viscoelastic fluids and used a modified Deissler's velocity model to evaluate the eddy diffusivity of momentum, ε_m, and the velocity near the wall including the

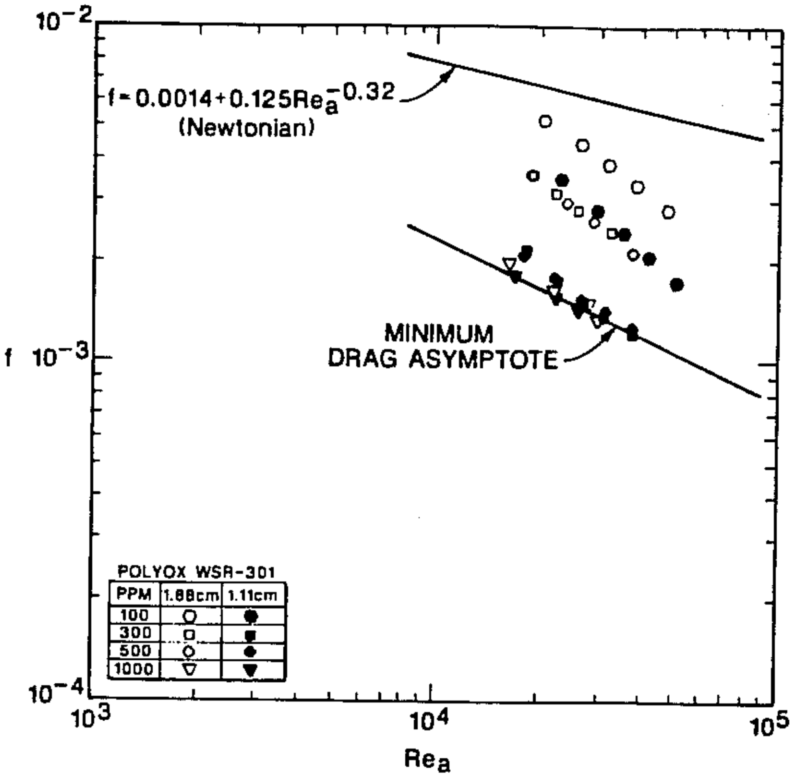

FIGURE 4 Friction factor vs. apparent Reynolds number for Polyox WSR-301 solutions in the 1.88 and 1.11 cm test sections.

viscous sublayer and the transition zone. Kale accounted for the influence of elasticity by incorporating the Deborah number (De) as defined by Seyer and Metzner (1969), $\lambda u_\tau^2/\nu_a$. The final expression for Stanton number was obtained as

$$St = \frac{f/2}{1.2 + (Pr_a - 1)(f/2)^{1/2}\{9.2\,Pr_a^{-0.258} + 1.2\,De\,Pr_a^{-0.236}\}} \tag{2}$$

It is to be noted that, estimation of the fluid time scale (λ) in the Deborah number with the use of Seyer and Metzner's equation, requires normal force measurements, which is not an easily obtainable rheological property, especially for low concentration solutions. In addition, recent studies have shown that the turbulent Prandtl number of concentrated viscoelastic fluids is not unity (Cho and Hartnett, 1982; Kwack, 1983), especially near the wall, where it is important to have accurate values of heat eddy diffusivity for heat transfer calculations.

Kale compared the predictions from Eq. (2) with experimental data from various sources for both Newtonian and viscoelastic fluids. The predicted values were in good agreement with the experimental observations. However, as discussed earlier, most of the experimental data for viscoelastic fluids were obtained under inadequate experimental conditions. As a consequence these measurements do not represent the heat transfer values under fully developed

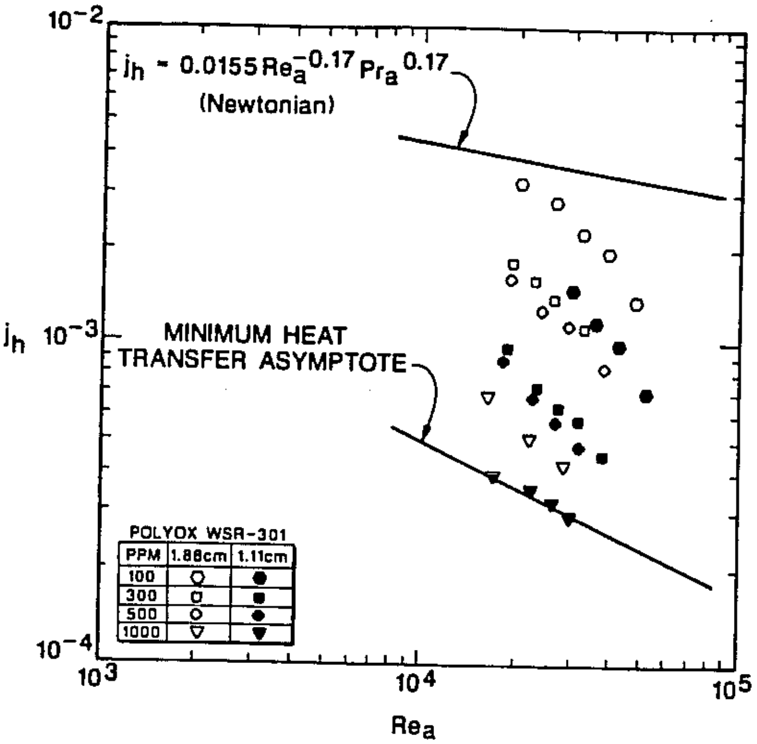

FIGURE 5 Colburn j-factor vs. apparent Reynolds number for Polyox WSR-301 solutions in the 1.88 and 1.11 cm test sections.

conditions. In some cases, the concentration was not high enough to produce the minimum heat transfer asymptote. Degradation and solvent chemistry effects may have been present in many of these investigations.

Using experimental data of Seyer and Metzner (1969), Kale found that the drag reduction approaches the minimum asymptote for a value of the Deborah number approximately equal to 20. He assumed (incorrectly) that the heat transfer coefficient also reaches its minimum asymptote at the same value of the Deborah number. Accordingly, the asymptotic equation for Stanton number was given by Kale as

$$St = \frac{f/2}{1.2 + (Pr_a - 1)(f/2)^{1/2}\{9.2\,Pr_a^{-0.258} + 24\,Pr_a^{-0.236}\}} \tag{3}$$

Comparison of the predictions from Eq. (3) with our experimental data and Kwack's (1983) for the minimum heat transfer asymptote showed very large deviations (as much as 100%). This is mainly due to the incorrect assumptions made by Kale (1977), as discussed earlier.

In order to overcome the shortcomings of Kale's correlation, using the experimental data of this study, the following modifications were made: 1) the numerator $(f/2)$ in Eq. (2) was multiplied by the correction factor, $(1 - FR)^a$, where FR is the friction drag reduction ratio. This is analogous to the modification made by Pruitt *et al.* (1966) for viscoelastic fluids; and 2) the

Deborah number (De) in Eq. (2) was replaced by a function of Weissenberg number (WS^{b}), in which the fluid time scale (λ) is estimated from the viscosity data using the Powell–Eyring fluid model (see Eq. (1)). The adjustable constants **a** and **b** were obtained based on the best fit of the proposed equation to the experimental data of this study. Through a linear regression method, the constants **a** and **b** were evaluated to be equal to 0.6 and 0.565, respectively. The final proposed correlation for Stanton number is

$$St = \frac{f/2(1-FR)^{0.6}}{1.2 + (Pr_a - 1)(f/2)^{1/2}\{9.2\,Pr_a^{-0.258} + 1.2\,Ws^{0.565}\,Pr_a^{-0.236}\}} \qquad (4)$$

It should be noted that, the proposed modifications in Eq. (2) are consistent with our recent published studies (Yoon and Ghajar, 1986 & 1987). In these studies it was established that the turbulent Prandtl number is a function of friction drag reduction ratio (FR) and Weissenberg number (Ws).

The verification of Eq. (4) for viscoelastic fluids was conducted with experimental data of Kwack (1983) for Separan AP-273 and this study for Separan AP-273 and Polyox WSR-301. Both sets of experimental data are free from the experimental deficiencies described earlier. As seen from Figure 6, the predicted values from Eq. (4) are in good agreement with experimental observations for a

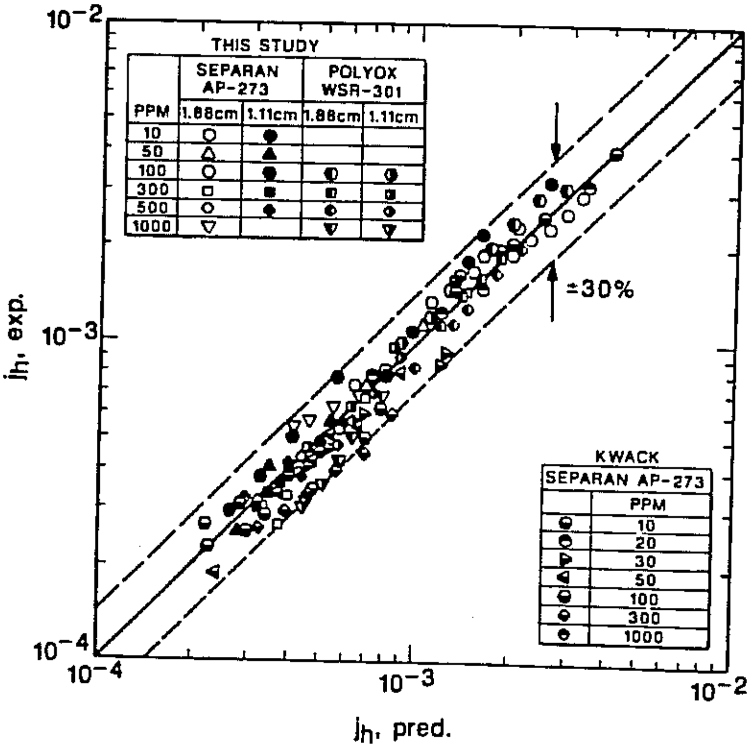

FIGURE 6 Comparison of the predicted Colburn j-factors using the proposed heat transfer correlation with measurements.

wide range of concentrations, including the minimum heat transfer asymptote. At this point it would have been desirable to compare the predictions of Eq. (4) with those of Eq. (2) using the same experimental data. This would have established more credibility to our proposed equation. However, due to the difficulties in obtaining the fluid time scale in the Deborah number, it was not possible to make this comparison. However, using the same experimental data, it was established that the proposed equation is capable of predicting the minimum heat transfer asymptote with good accuracy. As discussed earlier, Kale's asymptotic expression (see Eq. (3)), predicted much larger values for the same experimental data. It should be noted that even though the adjustable constants in the proposed expression were obtained based on the experimental data of this study, the proposed correlation was able to predict comparably the heat transfer coefficients for all polymer concentrations obtained from our study and Kwack's (1983) without any adjustment of the constants.

For Newtonian fluids ($FR = 0$ and $Ws = 0$), Eq. (4) is identical to Kale's expression (see Eq. (2)) with $De = 0$. As reported by Kale, the predictions from his equation compared very favorable with the Newtonian experimental data.

The use of the two key dimensionless parameters in Eq. (4), the Weissenberg number and the friction drag reduction ratio, should not be overlooked. Since the friction and heat transfer behavior of viscoelastic turbulent pipe flows are affected by not only the fluid conditions but also the flow conditions, such as the pipe diameter and the flow rate, the fluid time scale (λ) should be combined with the flow time scale (D/U) to form a complete dimensionless parameter, the Weissenberg number ($\lambda U/D$). The fluid time scale is the most useful and readily measurable material property that can be used to measure the elasticity of a fluid. It can be accurately determined from the rheological properties of the fluid with the use of an appropriate constitutive equation, in our case Powell–Eyring fluid model, even for low concentration solutions (Kwack and Hartnett, 1983). The other important parameter in viscoelastic fluids is the friction drag reduction ratio, which can be determined from the experimental measurements of pressure drop. Since the energy equation is directly related to the equation of motion, in heat transfer calculations involving viscoelastic fluids, friction drag reduction ratio plays a very important role. In general, the use of these two dimensionless parameters plays an important role in correlating friction factor with heat transfer coefficient and can account for several important factors influencing the friction and heat transfer behavior of viscoelastic turbulent pipe flows, such as pipe diameter, solvent chemistry, degradation, as well as the type and the concentration of the polymer.

CONCLUSIONS

Based on the experimental data of this study, Kale's (1977) heat transfer correlation was modified for the better prediction of heat transfer coefficients with the use of pertinent dimensionless parameters for viscoelastic fluids, friction drag reduction ratio (FR) and Weissenberg number (Ws). These parameters can be readily determined from experimental measurements of pressure drop and rheological properties. It was shown that the use of these parameters alleviated the shortcomings of Kale's equation, at least for the minimum heat transfer asymptote, by properly accounting for the influence of elasticity, and the fact that reduction in heat transfer is much more drastic than that in momentum transfer for viscoelastic turbulent pipe flows. Based on these findings it can be concluded that Kale's equation should at least not be used to predict the minimum heat transfer asymptote. In addition, the fluid time scale in the Deborah number is not an easily obtainable rheological property, which in turn makes use of Kale's equation difficult.

The proposed correlation for heat transfer compares very favorably over a wide range of polymer concentrations with experimental data. The comparison data used were obtained for different polymer solutions (Separan AP-273 and Polyox WSR-301), with wide range of concentrations (10–1000 ppm), in the thermally fully developed region of different pipe diameters under constant wall heat flux. The validity of the proposed equation should be further established with more reliable data, as they become available.

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NOMENCLATURE

c	specific heat of fluid
D	inside diameter of test section
De	Deborah number, $De = \lambda u_\tau^2 / \nu_a$
f	friction factor, $f = \tau_w / (\rho U^2/2)$
f_p	friction factor for polymer solution
f_s	friction factor for solvent
FR	friction drag reduction ratio, $FR = (f_s - f_p)/f_s$
h	heat transfer coefficient
j_h	Colburn j-factor, $j_h = St\, Pr_a^{2/3}$
k	thermal conductivity of fluid
L	length of test section
Pr_a	apparent Prandtl number, $Pr_a = \eta_a c / k$
Pr_t	turbulent Prandtl number, $Pr_t = \varepsilon_m / \varepsilon_h$
Re_a	apparent Reynolds number, $Re_a = \rho U D / \eta_a$
St	Stanton number, $St = h / \rho c U$

u_τ	shear velocity, $u_\tau = (\tau_W/\rho)^{1/2}$
U	average velocity
Ws	Weissenberg number, $Ws = \lambda U/D$

Greek Symbols

ε_h	turbulent eddy diffusivity of heat
ε_m	turbulent eddy diffusivity of momentum
η_a	apparent viscosity at the wall
η_0	zero shear rate apparent viscosity
η_∞	infinite shear rate apparent viscosity
$\dot{\gamma}$	shear rate
λ	fluid time scale
ν_a	apparent kinematic viscosity, $\nu_a = \eta_a/\rho$
ρ	fluid density
τ_w	wall shear stress

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