

Improved Forced Convective Heat-Transfer Correlations for Liquids in the Near-Critical Region

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The objective of this study was to compare the existing empirical approaches for forced convective heat transfer in the near-critical region and improve their predictive capabilities. Heat-transfer correlations based on accurate values for physical property inputs and accurate experimental heat-transfer data were developed for carbon dioxide and water in turbulent pipe flows. The important result of this study was that turbulent forced convective heat transfer to fluids in the near-critical region could be predicted by Dittus-Boelter-type heat-transfer correlations while using the property ratio method to account for large variations of physical properties in the near-critical region. The predicted results based on the developed correlations showed good agreement with the available experimental data. The proposed correlations also show better accuracy when compared to other correlations available in the literature.

Nomenclature

a, \dots, d	= curve-fitted constants, Eq. (1) and Table 1
a_1, \dots, e_1	= constants, Eq. (5)
a_2, \dots, e_2	= constants, Eq. (7)
$A_0, B_0, C_0,$ $a, b, c,$	= constants, Eqs. (2-4)
A_1, \dots, A_8	= constants, Eq. (6)
$A_{10}, A_{11}, A_{20}, A_{21}$	= constants, Eq. (8)
$A_{30}, A_{31}, A_{40}, A_{41}$	= constants, Eq. (8)
C_p	= specific heat at constant pressure
\bar{C}_p	= integrated mean specific heat, $= (i_w - i_b) / (T_w - T_b)$
D	= diameter
G	= mass velocity, $= \rho V$
h	= heat-transfer coefficient
i	= enthalpy
k	= thermal conductivity
Nu	= Nusselt number, $= hD/k$
P	= pressure
Pr	= Prandtl number, $= \mu C_p / k$
R	= gas constant
Re	= Reynolds number, $= \rho VD / \mu$
T	= temperature
v	= specific volume
V	= velocity
X_9, X_{10}, X_{11}	= constants, Eqs. (3) and (4)
α, γ	= constants, Eqs. (2-4)
μ	= absolute viscosity
ρ	= density
Subscripts	
b	= evaluated at the bulk temperature
c	= critical property

pc	= pseudocritical temperature, temperature at which specific heat assumes its maximum
r	= reduced property, as $P_r = P/P_c$
w	= evaluated at the wall temperature
x	= evaluated at a reference temperature
0	= zero- or low-pressure property

Introduction

HEAT transfer to fluids in the near-critical region has become important due to high heat-transfer rates possible in this region. Applications include the use of supercritical steam boilers in power stations, supercritical hydrogen as a working fluid for both chemical and nuclear rockets, methane as a coolant and fuel for the supersonic transport, and supercritical carbon dioxide as the working fluid in an entirely supercritical turbine power cycle.^{1,2}

For heat-transfer calculations, one needs reliable information concerning a variety of physical properties, e.g., density, specific heat, enthalpy, thermal conductivity, and viscosity. Intense variation of physical properties in the near-critical region for the slightest temperature or pressure change is characteristic of fluids in this region. Figures 1 and 2 illustrate typical variations of specific heat and thermal conductivity of carbon dioxide in the near-critical region. Such changes in fluid properties can effect large changes in the rates of convective heat transfer. Experimental investigations in this region have confirmed that anticipated high rates of heat transfer exist in both laminar and turbulent free and forced convection conditions. For example, experimental data of Yamagata et al.¹⁰ (Fig. 3) illustrates the typical behavior of turbulent forced convective heat-transfer coefficients in the near-critical region.

A variety of correlations have been developed and used for prediction of forced convective heat-transfer rates³⁻¹⁰ (see Table 1). Discrepancies exist between these correlations when applied in the near-critical region. The most significant sources of discrepancy appear to be the influences of 1) property variations, 2) heat flux and buoyancy, and 3) the differences in values of physical properties used by the various investigators.

The objective of the present study is to identify an appropriate forced convective heat-transfer correlation for the near-critical region in turbulent pipe flows (similar to the ones listed in Table 1) and improve its predictive capability by the use of appropriate physical properties.

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Method of Approach

Identification of an appropriate heat-transfer correlation was based on how well the correlations given in Table 1 could predict the available near-critical heat-transfer experimental data (see Table 2). The numerical constants in these correlations were based on different sources of physical property inputs which, in most cases, did not properly represent the variations in the near-critical region. Therefore, for a meaningful comparison, it was necessary to compare the correlations based on the same physical property inputs. For this purpose, the constants in the heat-transfer correlations were determined by curve-fitting the equations to the experimental data based on the best values available for physical property inputs. Then, the predicted results were compared not only with the experimental data of those authors who developed the specific correlations, but also with the experimental data of others. After sufficient study, it was decided that Eq. (16) of Table 1 predicted all of the available experimental data better than the rest of the correlations given in the table. For instance, the authors suggested correlation (15) for experimental data of Ref. 8 for carbon dioxide. This correlation predicted 62 experimental data points with an average absolute error of 8.9% and a maximum deviation of 31%, while Eq. (16) gave 6.4% average absolute error and 24% maximum deviation. Also, the rest of the correlations (see Table 1) were tested with experimental data of Ref. 8, and none of the correlations predicted heat-transfer coefficients as accurately as Eq. (16). For experimental data of Ref. 10 for water, the authors suggested correlation (17). This correlation predicted 67 experimental data points with an average absolute error of 15% and a maximum deviation of 89%, while correlation (16) had 7.1% average absolute error and a maximum deviation of 22%. Some of these results are also shown in Table 2. Further details may be found in Ref. 18. Therefore, based on the findings of this study, the following forced convective heat-transfer correlation is proposed for turbulent pipe flows:

$$Nu_b = aRe_b^c Pr_b^d \left(\frac{\rho_w}{\rho_b}\right)^d \left(\frac{\bar{C}_p}{C_{pb}}\right)^n \quad (1)$$

where *a*, *b*, *c*, and *d* are curve-fitted constants and, for *n*, the following criterion proposed by Jackson and Fewster⁹ was used:

For $T_b < T_w \leq T_{pc}$ and $T_w > T_b \geq 1.2T_{pc}$, $n = 0.4$

For $T_b \leq T_{pc} < T_w$, $n = 0.4 + 0.2(T_w/T_{pc} - 1)$

For $T_{pc} \leq T_b \leq 1.2T_{pc}$ and $T_b < T_w$,

$$n = 0.4 + 0.2(T_w/T_{pc} - 1)[1 - 5(T_b/T_{pc} - 1)]$$

Physical Properties

The expressions used to evaluate thermodynamic and transport properties of water and carbon dioxide in the near-critical region were taken from Refs. 19-22. Also, physical properties of supercritical carbon dioxide beyond the range of Refs. 19-21 were evaluated from different expressions given by Refs. 23-25.

The appropriate thermodynamic and transport property inputs required by the proposed heat-transfer correlation were determined by the following expressions.

Equation of State

In order to describe the thermodynamic surface in the near-critical region, a simple Benedict-Webb-Rubin-type equation of state proposed by Ref. 19 was used. This equation is of the form

$$P = RT/v + (B_0RT - A_0 - C_0/T^2)/v^2 + (bRT - a)/v^3 + a\alpha/v^6 + [c(1 + \gamma/v^2)\exp(-\gamma/v^2)]/(v^3T^2) \quad (2)$$

Table 1 Types of forced convective heat-transfer correlations

Correlation	Eq. no.	Ref. no.
$Nu_a = aRe_a^b Pr_a^c$	(11)	3
$Nu_b = aRe_b^c Pr_{\min}^d$	(12)	4, 5
where		
$Pr_{\min} = Pr_b, Pr_b < Pr_w$ $= Pr_w, Pr_w < Pr_b$		
$Nu_w = aRe_w^b Pr_w^c \left(\frac{\rho_w}{\rho_b}\right)^d$	(13)	6
where		
$Pr_w = \mu_w \bar{C}_p / k_w$		
$Nu_b = Nu_b' (\mu_b/\mu_w)^a (k_w/k_b)^b (\bar{C}_p/C_{pb})^c$	(14)	7
where		
$Nu_b' = 0.125 \xi Re_b Pr_b / [12.7(\xi/8)^{0.5} (Pr_b^{2/3} - 1) + 1.07]$ $\xi = 1 / (1.82 \log_{10} Re_b - 1.64)^2$ $\bar{C}_p = (i_w - i_b) / (T_w - T_b)$		
$Nu_b = Nu_b' \left(\frac{\rho_w}{\rho_b}\right)^a \left(\frac{\bar{C}_p}{C_{pb}}\right)^b$	(1.5)	8
where Nu_b' and \bar{C}_p are the same as defined for Eq. (14), and		
$b = 0.4,$ at $T_w/T_{pc} \leq 1$ or $T_b/T_{pc} \geq 1.2$		
$= b_1 = 0.22$ $+ 0.18(T_w/T_{pc}),$ at $1 \leq T_w/T_{pc} \leq 2.5$		
$= b_1 + (5b_1 - 2)$ $\times (1 - T_b/T_{pc}),$ at $1 \leq T_b/T_{pc} \leq 1.2$		
$= 0.7,$ at $T_b \leq T_{pc}$ and $T_w/T_{pc} = 2.5$		
$Nu_b = aRe_b^c Pr_b^d \left(\frac{\rho_w}{\rho_b}\right)^d \left(\frac{\bar{C}_p}{C_{pb}}\right)^n$	(16)	9
and		
$n = 0.4,$ for $T_b < T_w \leq T_{pc}$ and $1.2T_{pc} \leq T_b < T_w$		
$= 0.4 + 0.2(T_w/T_{pc} - 1),$ for $T_b \leq T_{pc} < T_w$		
$= 0.4 + 0.2(T_w/T_{pc} - 1)$ $\times (1 - 5(T_b/T_{pc} - 1)),$ for $T_{pc} \leq T_b \leq 1.2T_{pc}$ and $T_b < T_w$		
$Nu_b = aRe_b^c Pr_b^d Fc$	(17)	10
where		
$E = \frac{T_{pc} - T_b}{T_w - T_b}$		
$Fc = 1.0,$ for $E > 1$		
$= 0.67 Pr_{pc}^{-0.05} (\bar{C}_p/C_{pb})^{n_1},$ for $0 \leq E \leq 1$		
$= (\bar{C}_p/C_{pb})^{n_2},$ for $E < 0$		
$n_1 = -0.77(1 + 1/Pr_{pc}) + 1.49$		
$n_2 = 1.44(1 + 1/Pr_{pc}) - 0.53$		

The eight parameters B_0 , A_0 , C_0 , a , b , c , α , and γ are numerical constants, which are different for different substances. The units and constants to be used in the equation of state for carbon dioxide and water, as reported by Ref. 19, are given in Table 3. This table also gives the equation's range of application.

Because the equation of state is explicit only in pressure, it was necessary to use a half-interval technique to obtain values for density. Further details may be found in Ref. 21.

The same equation of state (2) with different coefficients was used for supercritical carbon dioxide ($T_r = 1.0-3.5$), ($P_r = 0.70108-1.7752$). The coefficients were fitted by Hsieh.²⁵ The units, constants, and range of application are given in Table 4.

It should be pointed out that, in this study, the input density and specific heat values of water in the critical region ($\rho = 200-420 \text{ kg/m}^3$ and $T = 644-693 \text{ K}$) were not based on Eq. (2). A computer program developed by Levelt Sengers et al.,²² for thermodynamic properties of steam in the critical region, was used. Their expressions proved to provide more accurate density and specific heat values in the critical region.

Specific Heat at Constant Pressure

Specific heat at constant pressure in the near-critical region was obtained based on the expression proposed in Ref. 19.

The form of the expression is as follows:

$$C_p = X_9 + X_{10}T + X_{11}T^2 + \frac{6C_0\rho}{T^3} + \frac{6c}{T^{3\gamma}} \exp(-\gamma\rho^2) + \frac{3c}{T^3} \rho^2 \exp(-\gamma\rho^2) - \frac{6c}{T^{3\gamma}} + T \left(\frac{\partial P}{\partial T} \right)_\rho / \rho^2 \left(\frac{\partial P}{\partial \rho} \right)_T \quad (3)$$

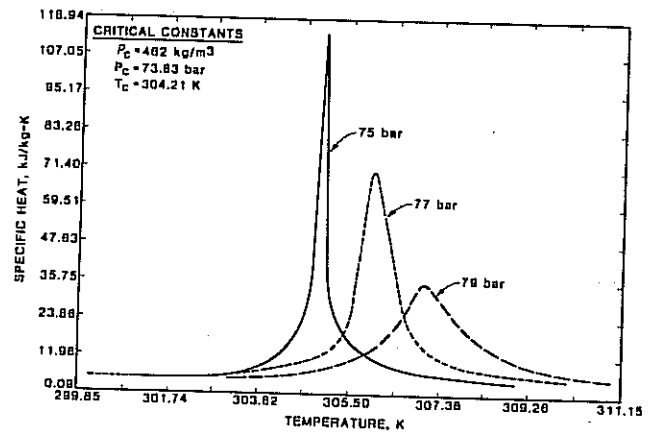


Fig. 1 Variation of specific heat vs temperature for carbon dioxide at different pressures.

Table 2 Forced convection—experimental pools

Fluid	Year	Ref. no.	Range of application		Heat flux, W/cm^2	Mass velocity, $\text{g/cm}^2\text{-s}$	Eq. no.	No. of points	Constants	Max./avg. dev., %
			P/P_c	T_h/T_c						
CO ₂	1957	3 ^b	1.12	0.97-1.06	3.2-32	54-220	(11)	—	$a = 0.0266, b = 0.77, c = 0.55$	—
CO ₂	1961	11 ^a	1.00-1.03	0.96-1.06	6.3-63	26-135	—	—	—	—
CO ₂	1961	7	1.20-1.46	0.98-1.15	3.4-12.0	68	(14)	—	$a = 0.455, b = 0.24, c = 0.65$	22/7.3
							(16) ^d	28	—	—
CO ₂	1966	8	1.06-1.33	0.96-1.26	4.6-260	—	(15)	—	$a = 0.0246, b = 0.80, c = 0.41, d = 0.257$	17/6
							(16)	62	$a = 0.32$	31/8.9
CO ₂	1967	12 ^b	1.06	0.987-1.02	1.43-5.20	50-100	—	—	$a = 0.0245, b = 0.79, c = 0.41, d = 0.22$	24/6.4
CO ₂	1969	13 ^a	1.003-1.03	—	18.7-73.5	45-65	—	—	—	—
CO ₂	1970	14 ^a	1.008-1.398	0.956-1.17	0.8-35	31-170	—	—	—	—
CO ₂	1972	15	1.35-1.42	0.93-1.19	310-1100	2000-2500	(15)	—	—	—
							(16) ^d	35	$a = 0.32$	27/10.6
H ₂ O	1957	4	1.04-1.19	—	18.7-73.5	45-65	(12)	—	$a = 0.061, b = 0.782, c = 0.456, d = 0.6$	13/5.1
							(16) ^d	23	$a = 105, b = 0.016, c = 0.68$	25/14.2
H ₂ O	1957	5 ^c	0.018-1.29	0.43-1.07	29-2320	17-3000	(12)	—	$a = 0.0345, b = 0.75, c = 0.25, d = -0.3$	23/11.2
H ₂ O	1958	16	1.09-1.41	0.58-1.25	870-1820	220-340	(11)	—	$a = 0.023, b = 0.8, c = 0.8$	—
							(16) ^d	25	$a = 0.017, b = 0.85, c = 0.015$	83/10.2
H ₂ O	1965	6 ^c	1.03-1.88	0.54-1.31	20-182	54-218	(13)	—	$a = 0.0055, b = 0.53, c = 0.4, d = 0.4$	14/6.6
H ₂ O	1966	17	1.006-1.08	0.916-1.01	55.5-165	115-525	—	—	$a = 0.00459, b = 0.923, c = 0.613, d = 0.231$	—
							(16) ^d	35	—	—
H ₂ O	1972	10	1.035-1.36	0.77-1.12	11.6-93.0	31-183	(17)	—	$a = 0.00147, b = 1.02, c = 0.99, d = -0.23$	21/9.2
							(16) ^d	67	$a = 0.002, b = 1.0, c = 0.33$	89/15
							(16) ^d	—	$a = 0.0064, b = 0.91, c = 0.48, d = 0.6$	22/7.1

^aData affected by high heat flux or buoyancy. ^bExperimental data not available. ^cInsufficient information given. ^dUsed in this work.

Table 3 Units and constants for equations of state and specific heat in the near-critical region

Parameter	Description	Units	Carbon dioxide	Water
C_p	Specific heat	$\frac{\text{cm}^3 - \text{atm}}{\text{gmol} - \text{k}}$	—	—
T	Temperature	K	—	—
P	Pressure	atm	—	—
v	Specific volume	cm^3/gmol	—	—
R	Gas constant	$\frac{\text{cm}^3 - \text{atm}}{\text{gmol} - \text{k}}$	82.05	82.05
B_0	Constant	—	6.348170×10^1	4.366083×10^1
A_0	Constant	—	2.978731×10^6	3.455640×10^6
C_0	Constant	—	1.411484×10^{11}	1.403741×10^{12}
b	Constant	—	2.675216×10^3	-5.285869×10^2
a	Constant	—	1.050408×10^8	6.437270×10^6
α	Constant	—	8.947460×10^4	2.537106×10^5
c	Constant	—	1.446193×10^{13}	6.846662×10^{13}
γ	Constant	—	4.586636×10^3	1.660691×10^3
X_9	Constant	—	1.441406×10^2	2.140282×10^2
X_{10}	Constant	—	5.460022×10^{-1}	9.095701×10^{-2}
X_{11}	Constant	—	-2.206477×10^{-4}	2.085410×10^{-5}
Range of application			Carbon dioxide	Water
$T_r = T/T_c$			0.9801–1.1450	0.9779–1.1944
$P_r = P/P_c$			0.7018–1.7752	0.8107–1.3122

The appropriate constants are given in Table 3. Also, different coefficients were used for supercritical carbon dioxide in the preceding equation. The coefficients were obtained from Ref. 25. The deviation of density and specific heat values, in this region, from tabulated data of Ref. 26, did not exceed 1 and 5%, respectively. The units, constants, and range of application are given in Table 4.

Enthalpy

In order to evaluate \bar{C}_p —integrated mean specific heat—first an expression for enthalpy had to be developed. Following the developments of Ref. 21, the final expression for enthalpy, based on the equation of state and specific heat equation, is of the form

$$\begin{aligned}
 i = & X_9(T - T_0) + \frac{X_{10}}{2}(T^2 - T_0^2) + \frac{X_{11}}{3}(T^3 - T_0^3) + R(T - T_0) \\
 & + (Pv - P_0v_a) - \left(A_0 + \frac{3C_0}{T^2}\right)(\rho - \rho_a) \\
 & - \frac{a}{2}(\rho^2 - \rho_a^2) + \frac{a\alpha}{5}(\rho^5 - \rho_a^5) \\
 & - \frac{3}{2} \frac{c}{\gamma T^2} [\exp(-\gamma\rho^2) - \exp(-\gamma\rho_a^2)] \\
 & - \frac{3}{2} \frac{c}{T^2} \left[\left(\rho^2 + \frac{1}{\gamma}\right) \exp(-\gamma\rho^2) \right. \\
 & \left. - \left(\rho_a^2 + \frac{1}{\gamma}\right) \exp(-\gamma\rho_a^2) \right] + i_0
 \end{aligned} \quad (4)$$

where v_a or $1/\rho_a$ is specific volume evaluated at (P_0, T) , and i_0 is the enthalpy at the reference state (P_0, T_0) .

In order to evaluate i_0 , the reference state (P_0, T_0) for carbon dioxide was based on the value of zero for the perfect crystal at the absolute zero of temperature.²⁵ At $P_0 = 1$ atm and $T_0 = 300$ K, the reference enthalpy i_0 is 193.5 kcal/kg. For water, the reference state was chosen such that the saturated liquid enthalpy would be zero at the datum temperature,

$T_0 = 273.16$ K.²⁷ The value of reference enthalpy i_0 for water, from steam tables at $T_0 = 273.16$ K and $P_0 = 6.03 \times 10^{-3}$ atm, is 597.35 kcal/kg. The enthalpy values calculated from Eq. (4) were in very close agreement with tabulated data given in Ref. 26. The units and constants for Eq. (4) are given in Tables 3 and 4.

Thermal Conductivity

Thermal conductivity in the near-critical region was evaluated based on an expression developed in Ref. 20. The expression follows:

$$k = a_1 + b_1\rho + c_1\rho^2 + d_1\rho^3 + e_1[T^*(1 + 0.9\rho^{*2.86}/T^*)]^{-0.6}\rho^{0.5} \quad (5)$$

where

$$T^* = \left| \frac{T - T_c}{T_c} \right| \quad \rho^* = \left| \frac{\rho - \rho_c}{\rho_c} \right|$$

Thermal conductivity of supercritical carbon dioxide, where $(P_r = 1.05 - 1.775)$ and $(T_r > 1.1)$, was evaluated using the

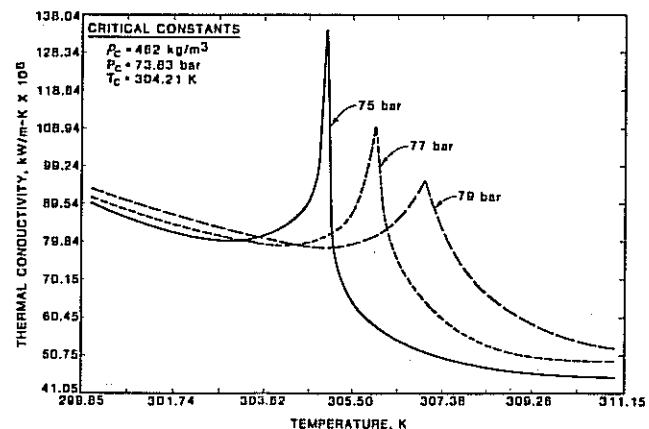


Fig. 2 Variation of thermal conductivity vs temperature for carbon dioxide at different pressures.

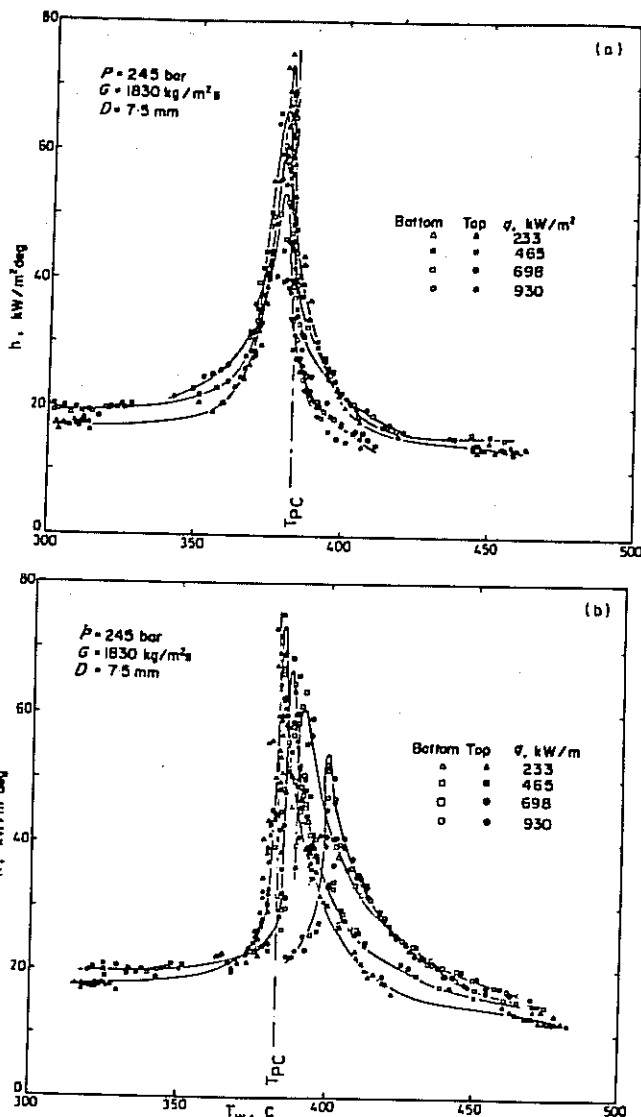


Fig. 3 Heat transfer to supercritical water flowing in horizontal tube a) in relation to bulk temperature, and b) in relation to wall temperature. Taken from Ref. 10.

Table 4 Units and constants for equations of state and specific heat of carbon dioxide in the supercritical region

Parameter	Description	Units	Carbon dioxide
C_p	Specific heat	$\frac{\text{dm}^3 - \text{atm}}{\text{gmol} - \text{k}}$	—
T	Temperature	K	—
P	Pressure	atm	—
v	Specific volume	dm^3/gmol	—
R	Gas constant	$\frac{\text{dm}^3 - \text{atm}}{\text{gmol} - \text{k}}$	0.08205
B_0	Constant	—	0.499101×10^{-1}
A_0	Constant	—	0.273742×10^1
C_0	Constant	—	1.385670×10^{-7}
b	Constant	—	0.721045×10^{-2}
a	Constant	—	1.368140×10^{-1}
α	Constant	—	0.847000×10^{-4}
c	Constant	—	0.149180×10^{-1}
γ	Constant	—	0.053940×10^{-1}
X_9	Constant	—	1.441406×10^{-1}
X_{10}	Constant	—	5.460022×10^{-4}
X_{11}	Constant	—	-2.206477×10^{-7}
Range of application			Carbon dioxide
$T_r = T/T_c$			1.0-3.5
$P_r = P/P_c$			0.70108-1.7752
$\rho_r = \rho/\rho_c$			$\rho_r < 1.8$

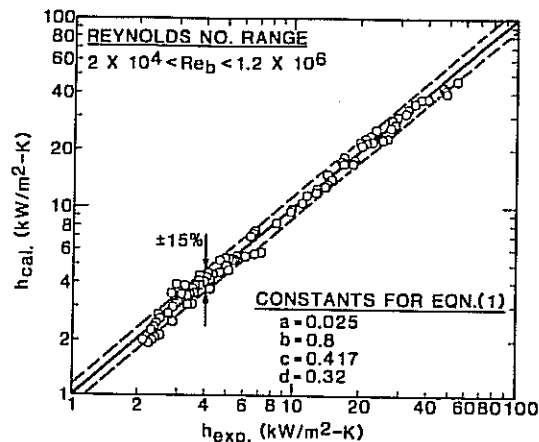


Fig. 4 Comparison of predicted heat-transfer coefficients with experimental data of Refs. 7, 8, and 15 for carbon dioxide.

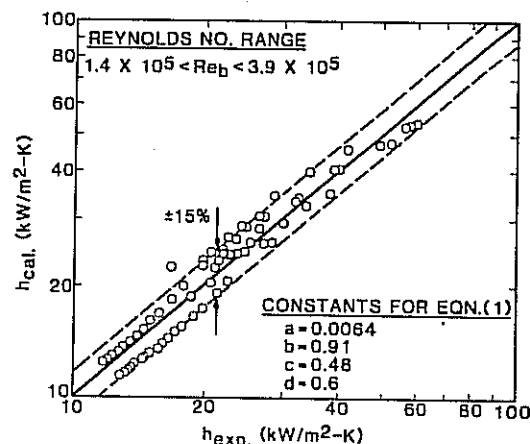


Fig. 5 Comparison of predicted heat-transfer coefficients with experimental data of Ref. 10 for water.

following expression from Ref. 23:

$$k = k_t + 10^{-4} \times (A_1 \rho + A_2 \rho^2 + A_3 \rho^3 + A_4 \rho^4) \quad (6)$$

where k_t is obtained from

$$k_t = k_{t0}(\theta)^{n_1}$$

and

$$k_{t0} = 355.7 \times 10^{-7} \text{ cal/cm-s-deg}$$

$$\theta = T/273.15 \text{ (K)}$$

$$n_1 = A_5 + A_6 \theta + A_7 \theta^2 + A_8 \theta^3$$

According to Vakulovich and Altunin,²³ the accuracy of Eq. (6) is within $\pm 5\%$ of the experimental data. The units and constants to be used in Eqs. (5) and (6), as well as their range of application, are given in Tables 5 and 6.

Viscosity

Viscosity in the near-critical region was evaluated based on an expression similar to Eq. (5) developed in Ref. 20. The resulting equation is

$$\mu = a_2 + b_2 \rho + c_2 \rho^2 + d_2 \rho^3 + e_2 [T_r (1 + 0.9 \rho^{2.86} / T_r^*)]^{-0.6} \rho_r^{0.5} \quad (7)$$

Viscosity of supercritical carbon dioxide, where ($P_r = 1.05-1.775$) and ($T_r \geq 1.062$), was evaluated by the following expression from Ref. 24:

$$\ln \frac{\mu}{\mu_t} = \sum_{i=1}^4 \sum_{j=0}^1 A_{i,j} \rho_r \quad (8)$$

where

$$\mu_t = \sqrt{T_r} (A_3 - B_3/T_r + C_3/T_r^2) \times 10^{-7}$$

According to Altunin and Sakhabetdinov,²⁴ the constant parameters in the equation were obtained based on the most reliable experimental data, and the deviation of the data of the

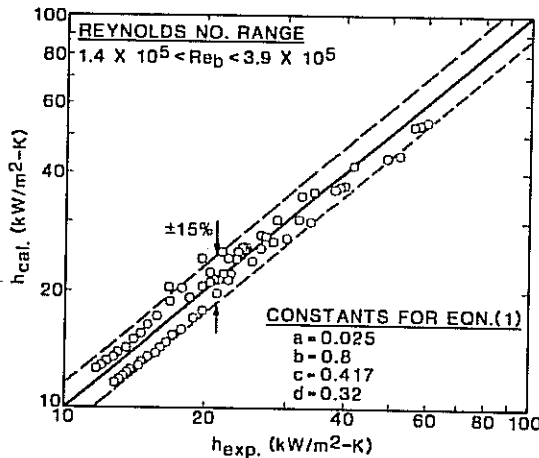


Fig. 6 Comparison of predicted heat-transfer coefficients (using correlation developed for carbon dioxide) with experimental data of Ref. 10 for water.

various authors from the values predicted by Eq. (8) did not exceed 1.5-2%. The units and constants to be used in Eqs. (7) and (8), as well as their range of application, are given in Tables 7 and 8.

Experimental Data Used

An intensive literature survey was carried out to obtain the available forced convective experimental data in the near-critical region.¹⁸ In this study, water and carbon dioxide were chosen for the development of heat-transfer correlations. These fluids were chosen because of the availability of reliable experimental heat-transfer data and accurate physical property expressions.

The available experimental studies for the fluids under study are summarized in Table 2. This table lists, for each substance under study, the year the experiments were reported, the source references, the range of reduced pressure and temperature, the heat flux, the mass velocity, the investigators' developed heat-transfer correlations, the number of experimental heat-transfer data points, the accuracy of their correlations based on physical property expressions of Refs. 19-25 and, finally, the proposed heat-transfer correlations along with their accuracy.

Determination of the Numerical Constants

The constants in the proposed heat-transfer correlation [Eq. (1)] were obtained by curve-fitting the expression to the experimental heat-transfer data of carbon dioxide^{7,8,15} and water.^{4,10,16,17} For this purpose, a nonlinear least-squares fit was used. The constants to be used in the proposed heat-transfer correlation are given in Table 2.

It should be pointed out that, in the determination of the constants of the heat-transfer correlation, the experimental data which might have been affected by high flux relative to

Table 5 Units and constants for thermal conductivity in the near-critical region

Parameter	Description	Units	Carbon dioxide	Water
k	Thermal conductivity	W/m-K	—	—
M	Molecular weight	kg/kg-mole	44.016	18.016
T_c	Critical temperature	K	304.21	647.05
P_c	Critical pressure	kPa	7.383×10^3	2.209×10^4
ρ_c	Critical density	kg/m ³	4.64×10^2	3.22×10^2
a_1	Constant	—	-4.406180×10^{-2}	1.822742×10^{-2}
b_1	Constant	—	5.981447×10^{-4}	8.095208×10^{-4}
c_1	Constant	—	-1.331925×10^{-6}	-1.340863×10^{-6}
d_1	Constant	—	1.022006×10^{-9}	1.995175×10^{-9}
e_1	Constant	—	2.739334×10^{-3}	1.581311×10^{-2}
Range of application			Carbon dioxide	Water
$T_r = T/T_c$			0.98-1.1	0.98-1.1
$\rho_r = \rho/\rho_c$			0.38-2.0	0.27-1.99

Table 6 Units and constants for thermal conductivity of carbon dioxide in the supercritical region

Parameter	Description	Units	Carbon dioxide
ρ	Density	kg/m ³	—
k	Thermal conductivity	cal/cm-s-C	—
A_1	Constant	—	0.42163×10^0
A_2	Constant	—	1.17385×10^{-3}
A_3	Constant	—	-1.77174×10^{-6}
A_4	Constant	—	1.29644×10^{-9}
A_5	Constant	—	1.58960×10^0
A_6	Constant	—	-0.22458×10^0
A_7	Constant	—	3.80755×10^1
A_8	Constant	—	-2.52350×10^{-3}
Range of application			Carbon dioxide
$T_r = T/T_c$			1.1-3.2
$\rho_r = \rho/\rho_c$			0.0-2.0

Table 7 Units and constants for viscosity in the near-critical region

Parameter	Description	Units	Carbon dioxide	Water
μ	Viscosity	N-s/m ²	—	—
M	Molecular weight	kg/kg-mole	44.016	18.016
T_c	Critical temperature	K	304.21	647.05
P_c	Critical pressure	kPa	7.383×10^3	2.209×10^4
ρ_c	Critical density	kg/m ³	4.64×10^2	3.22×10^2
a_2	Constant	—	1.232416×10^{-5}	1.705897×10^{-5}
b_2	Constant	—	3.097380×10^{-8}	5.650212×10^{-8}
c_2	Constant	—	$-3.265285 \times 10^{-12}$	3.508322×10^{-11}
d_2	Constant	—	6.713438×10^{-14}	3.228291×10^{-14}
e_2	Constant	—	1.449306×10^{-8}	2.577068×10^{-8}
Range of application			Carbon dioxide	Water
$T_r = T/T_c$			1.0–1.062	0.999–1.05
$\rho_r = \rho/\rho_c$			0.38–1.6	0.4–1.8

Table 8 Units and constants for viscosity of carbon dioxide in the supercritical region

Parameter	Description	Units	Carbon dioxide
μ	Viscosity	g/cm-s	—
ρ	Density	g/cm ³	—
T	Temperature	K	—
X_{10}	Constant	—	0.248566120
X_{11}	Constant	—	0.004894942
X_{20}	Constant	—	-0.37330066
X_{21}	Constant	—	1.22753488
X_{30}	Constant	—	0.363854523
X_{31}	Constant	—	-0.774229020
X_{40}	Constant	—	0.0639070755
X_{41}	Constant	—	0.142507049
A_3	Constant	—	2.72246461×10^3
B_3	Constant	—	1.66346068×10^3
C_3	Constant	—	4.66920556×10^2
Range of application			Carbon dioxide
$T_r = T/T_c$			1.0–6.57
$\rho_r = \rho/\rho_c$			0.014–40.0

the mass velocity (where deterioration in heat transfer was observed) or buoyancy was excluded. The criterion used for no buoyancy effect was that of Refs. 28 and 29.

Results and Discussion

The fit of the proposed expression for heat transfer to the available experimental data was good for both substances. The results of comparison with seven different experimental works are summarized in Table 2. Some of these results are also shown in Figs. 4–6, along with the appropriate constants for the heat-transfer correlations and their Reynolds number ranges. As indicated in Table 2, for carbon dioxide, the proposed correlation predicts the 28 experimental data points of Ref. 7 with an average absolute error of 6%. The majority of the data points (93%) were predicted with less than 15% deviation. The Reynolds number range was $2 \times 10^4 < Re_b < 8.6 \times 10^5$. For the 62 experimental data points of Ref. 8, the average absolute error was 6.4%. The majority of data points (92%) had deviations less than 15% and 98.5% less than 20%. The range of Reynolds number was $8 \times 10^4 < Re_b < 5 \times 10^5$. Finally, the 35 experimental data points of Ref. 15 for carbon dioxide were predicted with an average absolute error of 5.1%. The majority of data points (95%) deviated by less than 15%. The range of Reynolds number was $6 \times 10^5 < Re_b < 1.2 \times 10^6$. Since experimental data of Refs. 7, 8, and 15 were obtained at about the same range of Reynolds numbers, and were not affected by high heat flux or buoyancy, it was decided to combine these data. Figure 4 compares the predicted heat-transfer coefficients with 125 data points of Refs. 7, 8, and 15. The correlation predicts 86% of

the data with less than 15% deviation and 97.6% with less than 20%. The maximum deviation was 32%, with an average absolute error of 6.8%.

For water, Eq. (1) predicts the 23 experimental data points of Ref. 4 with an average absolute error of 11.2%. Eighty-seven percent of the data had deviations of less than 20%. The range of Reynolds number was $7 \times 10^4 < Re_b < 1.7 \times 10^5$. For 25 experimental data points of Ref. 16, the average absolute error was 6.6%. For this case, 72% of the data had less than 10% deviation and the rest had less than 15%. The Reynolds number range was $3.7 \times 10^6 < Re_b < 1.1 \times 10^7$. The 35 experimental data points of Ref. 17 were predicted with an average absolute error of 9.2%. The equation correlates 82% of the data points with less than 15% deviation and 97% with less than 20%. The experimental Reynolds number was $1.5 \times 10^5 < Re_b < 4.7 \times 10^5$. Finally, Fig. 5 compares the predicted heat-transfer coefficients with the 67 experimental data points of Ref. 10. The proposed correlation predicts the experimental data with an average absolute error of 7.1%. The majority of the data (95%) had deviations of less than 15% and 97% less than 20%. The range of Reynolds number was $1.4 \times 10^5 < Re_b < 3.9 \times 10^5$.

Comparison of heat fluxes and mass velocities of experimental data of water reveals that data of Refs. 4, 16 and 17 were obtained at higher heat fluxes (compared to mass velocities) than the experiments in Ref. 10. For instance, experimental data of Dickinson and Welch¹⁶ were obtained at heat fluxes three times higher than the experiments in Ref. 10 and at higher mass velocities. The criterion for buoyancy effect was not violated in any of the experiments. Therefore, the correlation obtained for the data of Ref. 10 is recommended for water (see Fig. 5).

In order to generalize the outcome of this study, the correlation that was curve-fitted to the experimental data of Refs. 7, 8, and 15 for carbon dioxide (see Fig. 4 for the curve-fitted constants) was used to predict the experimental data of Yamagata et al.¹⁰ for water. The preferential use of this correlation was due to the fact that more confidence was felt with the carbon dioxide data. As shown in Fig. 6, the 67 experimental data points of Ref. 10 were predicted with a maximum deviation of 19.3% and an average absolute error of 9.3%. The equation correlates 88% of the data points with less than 15% deviation.

Conclusions

The proposed heat-transfer correlation based on best available values for physical property inputs used in this study showed that the predicted values are in good agreement with the experimental data of turbulent forced convection to water and carbon dioxide. The average absolute errors between the predicted results and the experimental data of carbon dioxide and water were below 6.5 and 11.5%, respectively. This single correlation also shows better accuracy when compared to

other correlations available in the literature (see Table 2 and Figs. 4-6).

The correlation proposed for carbon dioxide (see Fig. 4) proved to be general, so as to predict heat transfer to water in the near-critical region with good accuracy (see Fig. 6).

The important result of this study was that turbulent forced convective heat transfer to fluids in the near-critical region could be predicted by Dittus-Boelter-type heat-transfer correlations ($Nu_b = 0.023 Re_b^{0.8} Pr_b^{0.4}$) while using the property ratio method to account for large variations of physical properties in the near-critical region.

References

- ¹Hendricks, R.C. and Simoneau, R.J., "Survey of Heat Transfer to Near-Critical Fluids," NASA TN D-5886, 1970, pp. 1-102.
- ²Green, J.R. and Hauptmann, E.G., "Forced Convective Heat Transfer from a Cylinder in Carbon Dioxide Near the Thermodynamic Critical Point," *ASME Transactions, Journal of Heat Transfer*, Vol. 93, Aug. 1971, pp. 290-296.
- ³Bringer, R.P. and Smith, J.M., "Heat Transfer in Critical Region," *AIChE Journal*, Vol. 3, No. 1, March 1957, pp. 49-55.
- ⁴Armand, A.A., Tarasova, N.V., and Kankov, A.S., "An Investigation of Heat Transfer from Wall to Steam Near the Critical Region," NASA TTF-11282, 1967.
- ⁵Miropolskii, L. and Shitsman, M.E., "Heat Transfer to Water and Steam at Variable Specific Heat (in Near Critical Region)," *Soviet Physics-Technical Physics*, Vol. 2, No. 10, Oct. 1957, pp. 2196-2208.
- ⁶Swenson, H.S., Carver, J.R., and Kakarala, C.R., "Heat Transfer to Supercritical Water in Smooth-Bore Tubes," *ASME Transactions, Journal of Heat Transfer*, Vol. 87, No. 4, Nov. 1965, pp. 447-484.
- ⁷Petukhov, B.S., Krasnoshchekov, E.A., and Protopopov, V.S., "An Investigation of Heat Transfer to Fluids Flowing in Pipes Under Supercritical Conditions," *Proceedings of the 2nd International Heat Transfer Conference*, 1961, pp. 569-578.
- ⁸Krasnoshchekov, E.A. and Protopopov, V.S., "Experimental Study of Heat Exchange in Carbon Dioxide in the Supercritical Range at High Temperature," *High Temperature*, Vol. 4, No. 3, May-June 1966, pp. 375-382.
- ⁹Jackson, J.D. and Fewster, J., AERE-R8158, Harwell, 1975.
- ¹⁰Yamagata, K., Nishikawa, K., Hasegawa, S., Fujii, T., and Yoshida, S., "Forced Convective Heat Transfer to Supercritical Water Flowing in Tubes," *International Journal of Heat and Mass Transfer*, Vol. 15, 1972, pp. 2575-2593.
- ¹¹Koppel, L.B. and Smith, J.M., "Turbulent Heat Transfer in the Critical Region," *International Development in Heat Transfer*, ASME, New York, 1961, pp. 585-590.
- ¹²Tanaka, H., Nishiwaki, N., and Hirata, M., "Turbulent Heat Transfer to Supercritical Carbon Dioxide," Japan Society of Mechanical Engineers 1967 Semi-International Symposium, Tokyo, Japan, Sept. 1967.
- ¹³Schnurr, N.M., "Heat Transfer to Carbon Dioxide in Immediate Vicinity of the Critical Point," *ASME Transactions, Journal of Heat Transfer*, Vol. 91, No. 1, Feb. 1969, pp. 16-20.
- ¹⁴Bourke, P.J., Pulling, D.J., Gill, L.E., and Denton, W.H., "Forced Convective Heat Transfer to Turbulent Carbon Dioxide in the Supercritical Region," *International Journal of Heat and Mass Transfer*, Vol. 13, 1970, pp. 1339-1348.
- ¹⁵Krasnoshchekov, E.A., Protopopov, V.S., Parkhovich, I.A., and Silin, V.A., "Some Results of an Experimental Investigation of Heat Transfer to Carbon Dioxide at Supercritical Pressure and Temperature Heads of Up to 850°C," *High Temperature*, Vol. 9, No. 5, 1972, pp. 992-995.
- ¹⁶Dickinson, N.L. and Welch, C.P., "Heat Transfer to Supercritical Water," *Transactions of ASME*, Vol. 80, No. 3, April 1958, pp. 746-752.
- ¹⁷Touba, R.F. and McFadden, P.W., "Combined Turbulent Convection Heat Transfer to Near-Critical Water," Purdue Research Foundation, Purdue Univ., West Lafayette, IN, Tech. Rept. 18, Jan. 1966.
- ¹⁸Asadi, A., "Improved Forced Convective Heat Transfer Correlations in the Near-Critical Region," M.S. Report, Mechanical and Aerospace Engineering Department, Oklahoma State University, Stillwater, OK, 1984.
- ¹⁹Najjar, T.S. and Ghajar, A.J., "Prediction of Density and Constant Pressure Specific Heat for Several Fluids in the Near-Critical Region," AIAA Paper 83-1476, June 1983.
- ²⁰Asgeirsson, L.S. and Ghajar, A.J., "Prediction of Thermal Conductivity and Viscosity for Some Fluids in the Near-Critical Region," *Chemical Engineering Communications*, Vol. 43, Nos. 1-3, 1986, pp. 165-173; also, AIAA Paper 83-1475, June 1983.
- ²¹Ghorbani-Tari, S. and Ghajar, A.J., "Improved Free Convective Heat Transfer Correlations in the Near-Critical Region," *AIAA Journal*, Vol. 23, Oct. 1985, pp. 1647-1649.
- ²²Levelt Sengers, J.M.H., Kamgar-Parsi, B., Balfour, F.W., and Sengers, J.V., "Thermodynamic Properties of Steam in the Critical Region," *Journal of Physics Chemical Reference Data*, Vol. 12, No. 1, 1983, pp. 1-28.
- ²³Vakulovich, M.P. and Altunin, V.V., *Thermophysical Properties of Carbon Dioxide*, United Kingdom Atomic Energy Press, London, England, 1968.
- ²⁴Altunin, V.V. and Sakhabetdinov, M.A., "The Viscosity of Liquid and Gaseous Carbon Dioxide at Temperatures of 220-1300°K and Pressure Up to 1200 Bar," *Thermal Engineering*, Vol. 19, No. 8, 1972, pp. 124-129.
- ²⁵Hsieh, J.S., *Principles of Thermodynamics*, McGraw Hill, New York, 1975.
- ²⁶Vargaftik, N.B., *Tables on Thermophysical Properties of Liquids and Gases*, Translated by Y.S. Touloukian, Purdue Univ., 1975.
- ²⁷Reynolds, W.C., *Thermodynamic Properties in SI*, Department of Mechanical Engineering, Stanford Univ., Stanford, CA, 1979.
- ²⁸Hall, W.B. and Jackson, J.D., "Laminarization of Turbulent Pipe Flow by Buoyancy Forces," ASME Paper 69-HT-55, 1969.
- ²⁹Brassington, D.J. and Cairns, D.N.H., Central Electric Research Lab., Note RD/L/N 156, 1975.