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Reference Temperatures for Supercritical Laminar Free Convection on a Vertical Flat Plate

A reference temperature method was developed by which heat transfer to fluids in the supercritical region under variable property conditions in laminar free convection on a vertical flat plate can be readily evaluated. Based on this method three generalized plots for Refrigerant-114, water, and carbon dioxide were developed. The results obtained with the reference temperature scheme showed good agreement with the existing experimental data and theoretical results for these three variable property fluids.

An excellent survey of heat transfer to near critical fluids is given by Hendricks, et al. [1]. It has been common to use the reference temperature expression, given by

$$T_{\mathrm{ref}} = T_w - r\,(T_w - T_\infty) \tag{1}$$

In development of the reference temperature method of this study, five different steps had to be accomplished:

1 Calculation of heat transfer coefficients. With the use of the conservation equations and the numerical methods of [2] and [3] variable property Nusselt numbers were calculated for several cases.

2 Development of a correlation for the calculated dimensionless heat transfer data, using results obtained from step 1.

The following basic form was employed to correlate the heat transfer data for the variable property conditions:

$$\mathrm{Nu}_{x,w} = a(\mathrm{Gr}_{x,w})^b\,(\mathrm{Pr}_\infty)^c \left(\frac{T_\infty}{T_w - T_\infty}\right)^d \tag{2}$$

where the constants a, b, c, and d of equation (2) were obtained using a non-linear least squares code [4].

3 Development of a correlation for constant property results. The slope of the temperature profile at the wall, $\theta'(0)$, used in these analyses was obtained from the solutions of the following constant property momentum and energy equations, and their boundary conditions:

$$F''' + 3FF'' - 2(F')^2 + \theta = 0 \tag{3}$$

$$\theta'' + 3\mathrm{Pr}F\theta' = 0 \tag{4}$$

$$\left.\begin{array}{l} F = 0 \\ F' = 0 \\ \theta = 1 \end{array}\right\} \eta = 0 \qquad \left.\begin{array}{l} F' = 0 \\ \theta = 0 \end{array}\right\} \eta = \infty \tag{5}$$

In equations (3) and (4), Prandtl number is the single parameter of these equations. However F is a stretched coordinate and this needs to be properly considered. For the pressures and temperatures in this study, Prandtl number covered a range from 0 to 40. For this range of Prandtl numbers the following correlation was obtained for $\theta'(0)$ as a function of Prandtl number:

$$[-\theta'(0)]_{c.p.} = a_1 + b_1(\mathrm{Pr})^{c_1} \tag{6}$$

where $a_1 = 0.10$; $b_1 = 0.486$; and $c_1 = 0.331$.

These constants were obtained using the MARQ program [4]. Equation (6) with the constants given above predicts the slope of the temperature profile at the wall for the constant property cases within an average absolute error of 1.3 percent.

Based on the equations presented in [2] and [3], the slope of the temperature profile at wall, $\theta'(0)$, can be expressed in terms of the dimensionless groups Nusselt number and Grashof number as follows:

$$[-\theta'(0)]_{c.p.} = \frac{\mathrm{Nu}_{x,w}}{(\mathrm{Gr}_{x,w}/4)^{1/4}} \tag{7}$$

4 Development of an optimization procedure to predict reference temperature. For this purpose equation (2) was represented as

$$(\mathrm{Nu}_{x,w})_T = a(\mathrm{Gr}_{x,w})^b{}_T\,(\mathrm{Pr})^c{}_T \left(\frac{T_\infty}{T_w - T_\infty}\right)^d \tag{8}$$

where the properties in the Nusselt number, Grashof number, and Prandtl number were evaluated at a characteristic temperature T, which is a complicated function of wall, free stream, and transposed critical temperatures. The value of T was then optimized between the free stream and wall temperatures until the best agreement between the correlated values, $(\mathrm{Nu}_{x,w})_T$ and $[\mathrm{Nu}_{x,w}/(\mathrm{Gr}_{x,w}/4)^{1/4}]_T$, obtained from equation (8) at temperature T, and the calculated values of $(\mathrm{Nu}_{x,w})$ and $[-\theta'(0)_{c.p.} = \mathrm{Nu}_{x,w}/(\mathrm{Gr}_{x,w}/4)^{1/4}]$ at fixed a, b, c, and d values were reached. This optimum temperature is the reference temperature for that case. In this comparison the calculated Nusselt numbers $(\mathrm{Nu}_{x,w})$ were obtained from the variable property analysis for the cases considered (i.e., step 1), and the values of $\theta'(0)_{c.p.}$ were calculated from equation (6).

5 Development of a general plot for reference temperature. The reference temperature for fluids in the supercritical region is a complicated function of wall, free stream, and transposed critical temperatures. Therefore, it was not possible to express this temperature with a single equation that would represent the fluids in the supercritical region under the physical conditions considered.

The parameter r was plotted with the parameter $(T_M - T_\infty)/(T_w - T_\infty)$ in Fig. 1. This plot was generated for Refrigerant 114 at a constant pressure of 3.7×10^3 kPa (540 psia), and ten different wall temperatures. T_M is that temperature at which specific heat assumes its maximum value. With this type of presentation, the influence of wall temperature T_w has been eliminated since all the points for different T_w values align quite well along a single curve.

The next step was to generalize the single curve in Fig. 1 to cover a wide range of supercritical temperatures and pressures for several fluids. For this purpose Refrigerant-114, water, and carbon dioxide were chosen. The ranges of supercritical temperatures and pressures used to develop the reference temperature plots for the three fluids were:

1 Refrigerant-114 ($P_c = 3.3 \times 10^3$ kPa, $T_c = 145.7$ °C)
 (a) Pressure range, $P = 3.4 \times 10^3$, 3.7×10^3, 4.1×10^3 kPa (500, 540, 600 psia)
 (b) Temperature range, T = 149 to 177°C (300 to 350°F)

2 Water ($P_c = 2.2 \times 10^4$ kPa, $T_c = 374.13$ °C)
 (a) Pressure range, $P = 2.3 \times 10^4$, 2.5×10^4, 2.6×10^4 kPa (3400, 3600, 3800 psia)
 (b) Temperature range, T = 379 to 399°C (715 to 750°F)

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3 Carbon Dioxide (P_c = 7.4 × 10^3 kPa, T_c = 31°C)
 (*a*) Pressure range, P = 8.3 × 10^3, 9.7 × 10^3, 1.1 × 10^4 kPa (1200, 1400, 1600 psia)
 (*b*) Temperature range, T = 32 to 60°C (90 to 140°F)

The ranges chosen for the above fluids cover the experimental pressure and temperature ranges reported in the literature (see Ghajar [3]).

The first step in the generalization of Fig. 1 was to obtain the dimensionless heat transfer coefficients. For each of the pressures assumed for the three fluids, ten different wall temperatures were chosen. These temperatures were selected to cover the range of temperatures given above. For each case the wall temperature was kept constant and the free stream temperature was varied to within one degree of the specified wall temperature. The dimensionless heat transfer coefficients were then calculated based on the equations, procedures, and computer programs introduced in [2] and [3]. The variable property heat transfer results for each fluid were then correlated with the same type of correlation given by equation (2). The three sets of constants a, b, c, and d are tabulated in Table 1.

Reference Temperature Results

The results for R-114, water, and carbon dioxide are presented in Figs. 2–4. Before discussing the results of these figures, it would be important to first consider the single curve in Fig. 1 (i.e., Refrigerant-114 at a constant pressure of 3.7 × 10^3 kPa). There are several interesting points to be noted. The curve behaves very nicely and smoothly up to certain values of $(T_M - T_\infty)/(T_w - T_\infty)$ and then there is a sudden abrupt change in the direction and behavior. This abrupt change in the direction of the curve appears to be primarily due to the large influence of closeness of T_w or T_∞ to T_M, the temperature at which specific heat assumes its maximum. This change occurs for the values of $(T_M - T_\infty)/(T_w - T_\infty)$ in the range of −0.25 to 2.00. These values occur where the wall temperature T_w or the free stream temperature T_∞ are within about 2.8°C (5°F) of T_M on both sides of the specific heat peak.

For the cases where both T_w and T_∞ are away from the T_M (on the right side of the specific heat maximum) the direct and strong influence of T_M on the reference temperature vanishes and the variation of r can be represented by a simple curve. In Fig. 1, this simple curve covers the values of $(T_M - T_\infty)/(T_w - T_\infty)$ in the range of −43.0 to 2.0.

The variation of r for Refrigerant-114, water, and carbon dioxide at different supercritical pressures and over a range of temperatures show results similar to the ones presented in Fig. 1. Therefore, the observations and conclusions made about Fig. 1, for a specific fluid at a single pressure, can be employed for Figs. 2–4, for several fluids at several different pressures and temperatures.

Figures 2–4 indicate that it is possible to obtain a single curve for different pressures up to where the influence of closeness to T_M is not significant, that is, when the wall and free stream temperatures were more than about 2.8°C (5°F) to the right of the specific heat peak. For

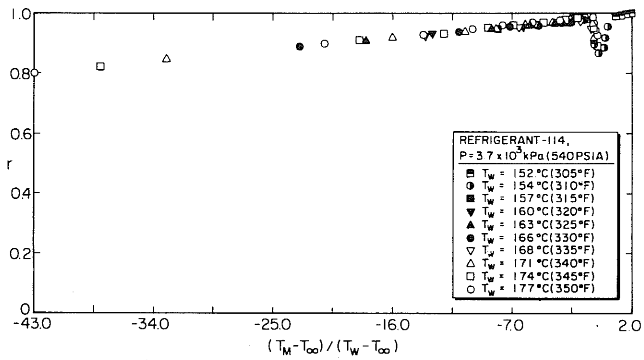

Fig. 1 Variation of r at various temperatures and a single pressure

Table 1 Constants for heat transfer correlation

Fluid	Constants a	b	c	d	Percent Deviation	Range of Application T/T_c	P/P_c
Refrigerant-114	0.3999	0.25	0.3085	0.004972	1.5	1.0076 to 1.0740	1.0638 to 1.2766
Water	0.3153	0.25	0.3921	−0.02230	2.5	1.0080 to 1.0380	1.0598 to 1.1845
Carbon Dioxide	0.3265	0.25	0.4054	−0.02750	2.4	1.0038 to 1.0950	1.1194 to 1.4925

Nomenclature

a,b,c,d = constants used in equations (2) and (8)
a_1,b_1,c_1 = constants used in equation (6)
c_p = specific heat at constant pressure
C = constant, $C = \left[\frac{g}{4\nu^2}\left(\frac{\rho_\infty - \rho_w}{\rho_w}\right)\right]^{1/4}$
$F(\eta)$ = similarity variable,
$$F(\eta) = \left(\frac{1}{4\nu_w C_w}\right)\left(\frac{\psi}{x^{3/4}}\right)$$
Gr = Grashof number,
$$\mathrm{Gr} = \frac{gL^3}{\nu^2}\left(\frac{\rho_\infty - \rho_w}{\rho_w}\right)$$
g = acceleration due to gravity
h = heat transfer coefficient
k = thermal conductivity
L = overall length of plate
Nu = Nusselt number, Nu = hL/k
p = pressure
Pr = Prandtl number
$\dot{q}''$ = heat flux
r = parameter in the reference temperature equation
T = temperature
T_{ref} = reference temperature
T_M = pseudocritical or transposed critical temperature
u = velocity component along the plate
v = velocity component perpendicular to the plate
x = coordinate along the plate
y = coordinate perpendicular to the plate
η = similarity coordinate, $\eta = C_w x^{-1/4}$
$$\int_0^y \frac{\rho}{\rho_w}\,dy$$
μ = absolute viscosity
ν = kinematic viscosity, $\nu = \mu/\rho$
ρ = density
ψ = stream function, $u = \frac{\rho_w}{\rho}\left(\frac{\partial\psi}{\partial y}\right)$ and
$$v = -\frac{\rho_w}{\rho}\left(\frac{\partial\psi}{\partial x}\right)$$
$\theta(\eta)$ = dimensionless temperature,
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Subscripts

c = refers to critical property
$c.p.$ = refers to constant property solution
ref = evaluated at the reference temperature
T = evaluated at a characteristic temperature
$v.p.$ = refers to variable property solution
w = evaluated at the wall temperature
x = evaluated at a particular point along a surface, a local parameter as opposed to a mean parameter
∞ = evaluated at the free stream temperature

the cases where T_w or T_∞ are in the vicinity of T_M, a single curve was not obtainable. This was primarily due to the fact that for each pressure the specific heat assumes a different maximum value, since in these cases being close to T_M has a direct and signficant influence on the prediction of the reference temperature.

However, it is interesting to note that for the cases where T_w and T_∞ were close to T_M, the curves for all three pressures for each fluid branched off from the same line. The amount of departure of these curves from the single curve increases as the pressure is increased, since in this range the higher the pressure the further is the maximum peak of specific heat removed from the critical point. For the cases where the free stream temperature was on the left of the specific heat peak, where $(T_M - T_\infty)/(T_w - T_\infty)$ was positive, as soon as the influence of closeness to specific heat maximum was diminished, the curves behaved smoothly again. This phenomenon can best be observed by considering those curves presented obtained for the highest pressure utilized.

The solid lines drawn thı ough the data points present the best fit obtained for the data generated for the fluids under study. Plots similar to the one given in Fig. 1 were generated for the three fluids at every one of the pressures considered. Then, the best fit through the data points in these plots was demonstrated by a solid curve which expresses the variation of r in Figs. 2–4.

Heat Transfer Results

The validity and universality of this method was accomplished by comparing the results calculated with the predicted reference temperatures with the experimental and theoretical variable property results of Fritsch and Grosh [5] for water, Nishikawa, et al. [6] for carbon dioxide, and Parker and Mullin [7] for Refrigerant-114. In addition, some of the variable property heat transfer results obtained with the use of the equations, procedures, and computer programs introduced in [2] and [3] were compared with the constant property heat transfer results obtained with the predicted reference temperatures.

In order to calculate the constant property results, for the free convective problems considered, the dimensionless reference temperatures were obtained from Figs. 2–4 for different values of $(T_M - T_\infty)/(T_w - T_\infty)$ for the fluids of interest. The reference temperatures were then calculated. These reference temperatures were used to evaluate the value of Prandtl number in the constant property equations. The following constant property equations for momentum and energy were used:

$$F''' = -3FF'' + 2(F')^2 - \theta \tag{9}$$

$$\theta'' = -3\mathrm{Pr}_{\mathrm{ref}} F\theta' \tag{10}$$

where $\mathrm{Pr}_{\mathrm{ref}}$ is Prandtl number evaluated at a reference temperature. The boundary conditions for equations (9) and (10) are identical to those given by equation (5).

Equations (9) and (10) were solved using a Lentini-Pereyra Program [8] (see also [2, 3]). The slope of the dimensionless temperature profile at the wall, $\theta'(0)$, obtained from the solution of the conservation equations was used in the following equations to obtain the local Nusselt number for the constant property problem:

$$(\mathrm{Nu}_{w,x})_{c.p.} = \mathrm{Gr}_{x,\mathrm{ref}}^{1/4}\,[-\theta'(0)]_{\mathrm{ref}} \tag{11}$$

where, from equation (6)

$$-\theta'(0)_{\mathrm{ref}} = \sqrt{2}\,(a_1 + b_1\,\mathrm{Pr}_{\mathrm{ref}}^{c_1}) = \sqrt{2}\,(0.1 + 0.486\,\mathrm{Pr}_{\mathrm{ref}}^{0.331}) \tag{12}$$

The constant property heat flux results were compared with experimental and theoretical results of the three fluids under study, Fig. 5. It can be seen that the constant property heat flux results are within

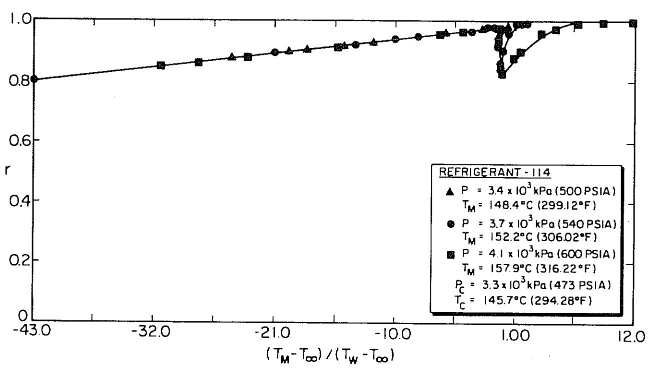

Fig. 2 Variation of *r* at various temperatures and pressures

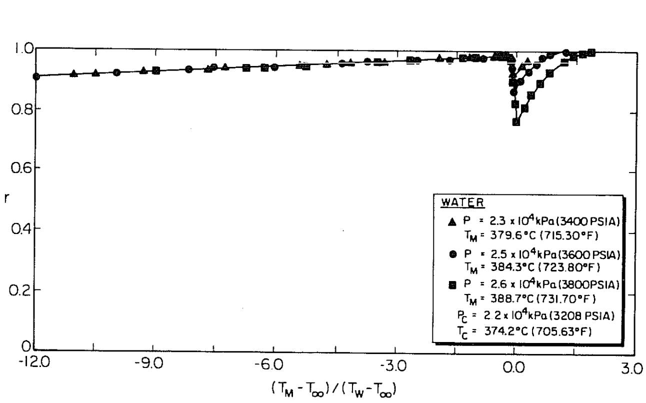

Fig. 3 Variation of *r* at various temperatures and pressures

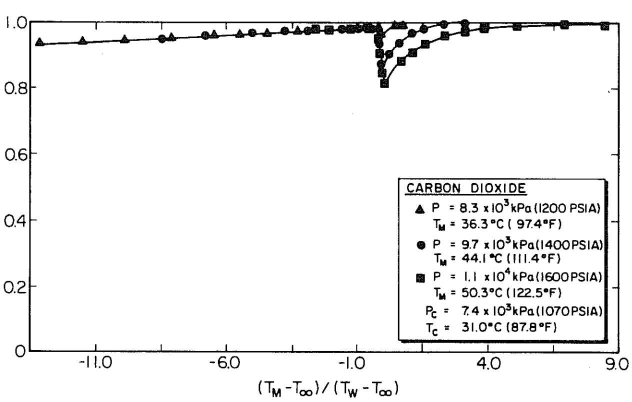

Fig. 4 Variation of *r* at various temperatures and pressures

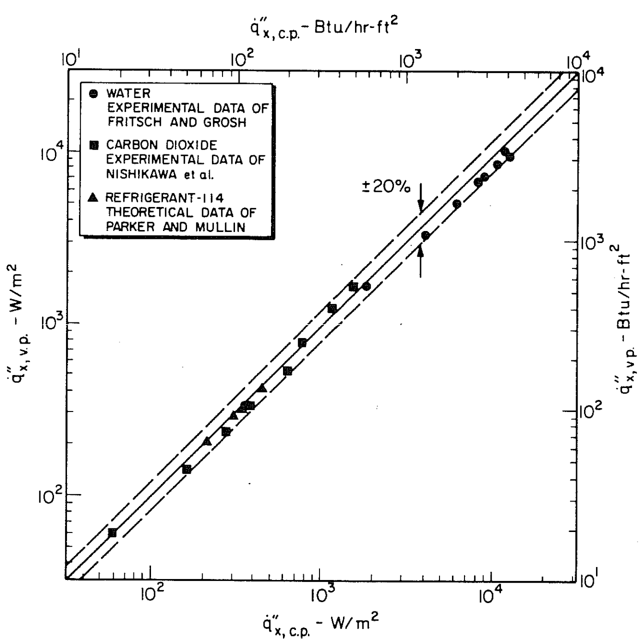

Fig. 5 Comparison of predicted constant property local heat flux with variable property local heat flux

± 20 percent of the variable property experimental and theoretical results.

The variable property results for Refrigerant-114 at 3.7×10^3 kPa (540 psia), based on [2] and [3] were compared with the constant property heat transfer results obtained with the predicted reference temperatures. This was achieved by choosing 37 points on the 3.7×10^3 kPa (540 psia) curve in Fig. 2 for Refrigerant-114 that would best reproduce the curve. Most of the data points were chosen for the wall and free stream temperatures that best represented the abrupt change in the curve. The heat transfer results obtained for these data points cover a temperature range of 149 to 177°C (300 to 350°F). The comparison between the constant property Nusselt numbers obtained from equation (11) and variable property results obtained from [2] and [3] are presented in Fig. 6. The constant property Nusselt numbers predicted by the reference temperature method are within ± 10 percent of the variable property results.

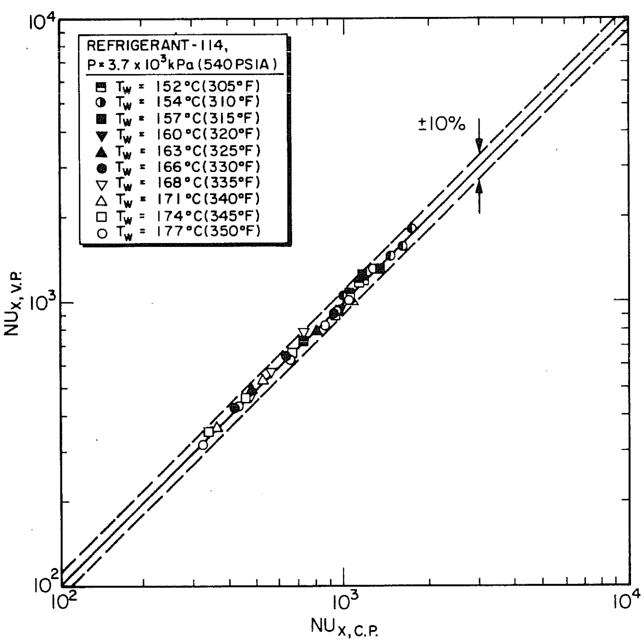

Fig. 6 Comparison of predicted constant property Nusselt numbers with variable property Nusselt numbers

Discussion

One single curve for prediction of reference temperature can be established where T_w, the wall temperature, or T_∞, the free stream temperature were not close to T_M, the temperature at which specific heat assumes its maximum value. The heat transfer results obtained using the predicted reference temperatures with the constant property equations compared well with the experimental and theoretical variable property results for Refrigerant-114, water, and carbon dioxide. For the regions where closeness to T_M was significant, it was still possible to predict reference temperature and heat transfer coefficients with good accuracy, but the curve used for these predictions were pressure dependent.

For a specified problem the free convective heat transfer coefficients to fluids in the supercritical region can be directly calculated from equation (2). The constants a, b, c, and d of equation (2) for Refrigerant-114, water, and carbon dioxide along with its range of application are tabulated in Table 1. Equation (2) required information on physical properties, which can be obtained from the equations and computer programs developed in [3] and [9].

The utility of the reference temperature lies in the fact that it allows the more easily obtained constant property solutions to be used to compute solutions where variable properties occur.

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