

## **a new heat transfer correlation in the transition region for a horizontal pipe with a reentrant inlet -- using artificial neural network**

afshin j. ghajar<sup>a</sup>, lap mou tam<sup>b</sup>, sik chung tam<sup>b</sup>

*a) school of mechanical and aerospace engineering, oklahoma state university, stillwater, oklahoma 74078, usa*  
*b) faculty of science and technology, university of macau, p. o. box 3001, taipa, macau, china*

Local heat transfer (under uniform wall heat flux condition) was measured along a 6.1 m long stainless steel horizontal circular straight tube with a reentrant inlet configuration. For the heat transfer measurements the Reynolds number varied from about 790 to 49940. In these experiments the test fluid was either distilled water or a mixture of ethylene glycol and distilled water, which gave a Prandtl number range from about 4 to 100. From the experimental data, it can be observed that the transition region for the reentrant inlet ranged from 1700 to 9100. Ghajar and Tam (1994) proposed heat transfer correlations for the laminar and turbulent flow regimes with excellent accuracy. The absolute average deviations are 5.8% and 3.7%, respectively. For the correlation in the transition region, the accuracy is not as accurate since more than 30% of the data were predicted with more than 10% deviation. The reason is due to the abrupt change in heat transfer characteristics in this flow regime. Since the value of heat transfer coefficient has direct impact on the size of heat exchangers, a more accurate correlation is developed using the artificial neural network (ANN). It has been shown that multilayer feedforward ANN's have universal approximation property according to Hornik et al. (1990) and Hornik (1991). The accuracy of the new correlation is excellent with 95.5% of the data points (421 data points) predicted with less than 10% deviation. The ANN method can also be used to establish the most and least important variables according to the form of the correlation.

### **1. introduction**

An important design problem in industrial heat exchangers arises when flow inside the tubes falls into the transition region. In practical engineering design, the usual recommendation is to avoid design and operation in this regime; however, this is not always feasible under design constraints. The usually cited transition Reynolds number of about 2100 applies, strictly speaking, to a very steady and uniform entry flow with a rounded entrance. If the flow has a disturbed entrance typical of heat exchangers, in which there is a sudden contraction and possibly even a reentrant entrance, the transition Reynolds number will be less. Experimental, numerical, and analytical studies are available for forced and mixed convection heat transfer in horizontal tubes with a rounded entrance in the laminar, transitional, and turbulent flow regimes. These works have been reviewed by Shah and London (1978), Shah and Johnson (1981), and Kakac and coworkers (1981 and 1987). However, very little information that is of immediate use to design engineer (i.e., correlation) is available to predict the developing and fully developed transitional forced and mixed convection heat transfer coefficients in a tube with a disturbed entrance. The correlation developed by Ghajar and Tam (1994) is the only one that is available in the open literature for the above-mentioned case. Since the value of heat transfer coefficient has direct impact on the size of heat exchangers, an accurate correlation is necessary. Because the traditional least-squares method cannot provide good accuracy, an alternative method, namely the artificial neural network (ANN) is employed. ANN has been successfully used in the analysis of heat transfer data and evaluation of heat transfer coefficient by Thibault and Grandjean (1991) and Jambunathan et al. (1996). The applications of ANN in different areas in thermal engineering are well described in the comprehensive work done by Sen and Yang (1999). The purpose of this study was to create an accurate correlation using an unconventional method to correlate experimental data for a wide range of Reynolds, Prandtl, and Grashof numbers in the entrance and fully developed regions of a circular horizontal electrically heated straight tube fitted with a reentrant inlet configuration.

## 2. experiments

A schematic diagram of the overall experimental setup used for heat transfer experiments is shown as Fig. 1. The heat transfer test section was a horizontal seamless 316 stainless steel circular tube with an inside diameter of 1.58 cm and an outside diameter of 1.90 cm. The total length of the test section was 6.10 m, providing a maximum length-to-diameter ratio of 385. The end connections of the test section consisted of copper plates that were arc soldered with silver to the ends of the test section to secure a well-defined electric circuit through the end plates. The test section was insulated from the environment by using fiberglass pipe insulation and vaporproof pipe tape. The total thickness of the insulation materials was about 3.18 cm. To ensure a uniform fluid bulk temperature at the exit of the test section, a mixing well that consisted of several baffles was used. A one-shell, two tube pass heat exchanger was used to cool the fluid from the test section to a desirable inlet bulk temperature. Calming and inlet sections were used to obtain uniform velocity distribution. The reentrant entrance was simulated by sliding 1.91 cm of the tube entrance length into the inlet section. A uniform wall heat flux boundary condition was maintained by a Lincoln DC-600 welder. The thermocouples were placed on the outer surface of the tube wall at close intervals near the entrance and at greater intervals further downstream. Thirty-one thermocouple stations were designated with four thermocouples at each station. The thermocouples were placed 90° apart around the periphery. In the data reduction, axial conduction was assumed negligible ( $RePr > 4,200$  in all cases), but peripheral and radial conduction of heat in the tube wall were included. The test fluid was a mixture of ethylene glycol and distilled water producing a transition Prandtl number ranging from 5 to 51. The uniform wall average heat flux for the experiments ranged from about 4 to 670 kW/m<sup>2</sup>. The uncertainty analyses of the overall experimental procedures showed there is a maximum of 9% uncertainty for heat transfer coefficient calculations. Moreover, the heat balance error for each experimental run indicates that in general, the heat balance error is less than 5%. For Reynolds numbers lower than 2500 where the flow is strongly influenced by secondary flow, the heat balance error is relatively higher (5 to 8%) for that particular Reynolds number range. More detailed descriptions of the experimental apparatus and procedures may be found in Ghajar and Tam (1994).

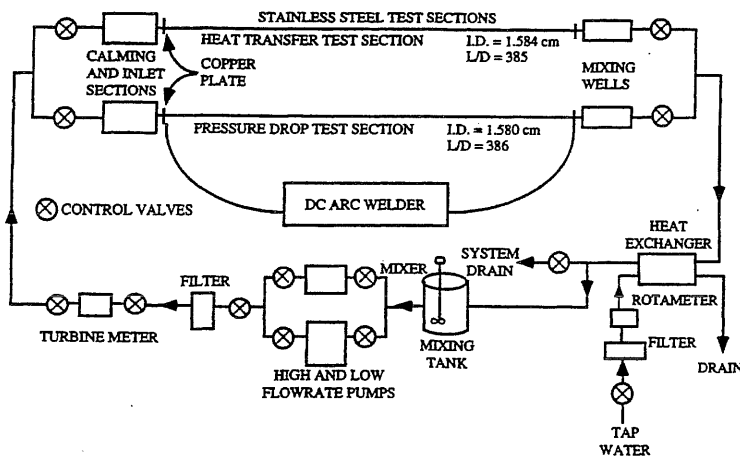


Figure 1. Schematic diagram of the experimental setup.

## 3. correlation using artificial neural network

Figure 2 shows the heat transfer data for the transition region for the reentrant inlet configuration. This figure plots the local average peripheral heat transfer coefficients in terms of the Colburn  $j$  factor ( $StPr^{0.6}$ ) versus local bulk Reynolds number for all flow regimes at the length-to-diameter ratio of 192. The filled symbols represent the start and end of the heat transfer transition region. Figure 2, for comparison purposes, also shows the typical fully developed pipe flow forced convection heat transfer correlations for turbulent (Sieder and Tate, 1936) and

laminar ( $Nu=4.364$ ) flows under the uniform wall heat flux boundary condition. As seen in this figure, the abrupt change of heat transfer characteristic in the transition region is obvious. Ghajar and Tam (1994) used the asymptotic method similar to Churchill (1977) to develop a correlation in the transition region since the variation of heat transfer coefficient, as indicated by Figure 2 is between two asymptotes. The correlation is

$$Nu_{tr} = Nu_l + \{ \exp[(a-Re)/b] + Nu_t^c \}^c \quad (1)$$

Where  $Nu$  is the local Nusselt number ( $=hD/k$ ),  $Re$  is the local bulk Reynolds number ( $=\rho V D/\mu$ ),  $Gr$  is the local bulk Grashof number [ $=g\beta\rho^2 D^3(T_w-T_b)/\mu^2$ ],  $Pr$  is the local bulk Prandtl number ( $=\mu c_p/k$ ),  $x/D$  is the dimensionless location where  $x$  is the local distance and  $D$  is the inside diameter of the tube, and  $(\mu_b/\mu_w)^{0.14}$  is the viscosity ratio for viscosity of the test fluid at the bulk temperature over the viscosity of the test fluid evaluated at the inside wall temperature. The subscripts  $tr$ ,  $l$ , and  $t$  stand for transition, laminar, and turbulent, respectively. The definition of the parameters in the dimensionless numbers  $Nu$ ,  $Re$ ,  $Gr$ , and  $Pr$  used in Equation (1) are:  $h$  = local average or fully developed peripheral heat transfer coefficient,  $k$  = thermal conductivity of the test fluid at the bulk temperature,  $\rho$  = density of the test fluid at the bulk temperature,  $V$  = average velocity in the test section,  $\mu$  = absolute viscosity of the test fluid at the bulk temperature,  $g$  = acceleration of gravity,  $\beta$  = coefficient of thermal expansion of the test fluid at the bulk temperature,  $T_w$  = local average peripheral tube inside wall temperature,  $T_b$  = local bulk temperature of the test fluid, and  $c_p$  = specific heat of the test fluid at the bulk temperature. The constants and the related sub-correlations for Equation (1) are

$$a = 1766, b = 276, c = -0.955$$

$$Nu_l = 0.61[(RePrD/x) + 0.025(GrPr)^{0.8}]^{0.4} (\mu_b/\mu_w)^{0.14} \quad (2)$$

$$Nu_t = 0.023Re^{0.8} Pr^{0.385} (x/D)^{-0.0054} (\mu_b/\mu_w)^{0.14} \quad (3)$$

The range of independent variables used in Equation (1) are:  $3 \leq x/D \leq 192$ ,  $1700 \leq Re \leq 9100$ ,  $5 \leq Pr \leq 51$ ,  $4000 \leq Gr \leq 2.1 \times 10^5$ , and  $1.2 \leq (\mu_b/\mu_w)^{0.14} \leq 2.2$ , respectively.

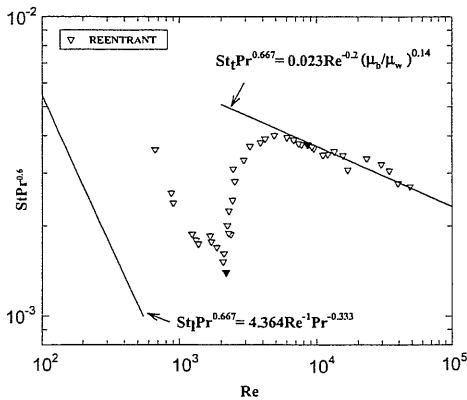


Figure 2. Heat transfer transition region.

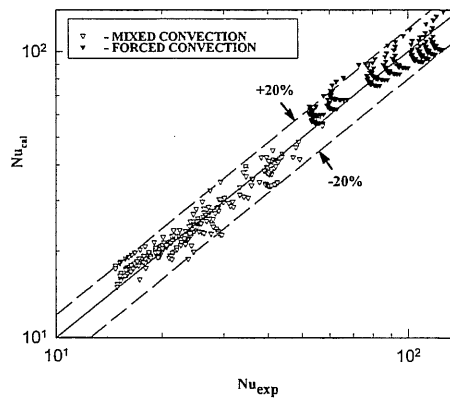


Figure 3. Accuracy of Ghajar and Tam's (1994) correlation.

Four hundred and forty one (441) data points were used. This correlation gave a representation of the experimental data to within +25.1% and -23% and had an absolute average deviation of 8%. Three percent of the data (13 data points) were predicted with more than  $\pm 20\%$  deviation. 29% (129 data points) with  $\pm 10$ -20%

deviation and 68% (299 data points) with less than  $\pm 10\%$  deviation. Figure 3 plots the predicted results using the correlation above versus the experimental values.

As seen in Figure 3, the correlation could not with good accuracy, predict the experimental data for the forced convection in the upper transition region and the mixed convection data with low Nusselt number. Since the accuracy of the heat transfer coefficient will make a direct impact to the size of the heat exchanger, therefore, a correlation with high accuracy using artificial neural network (ANN) is introduced.

The ANN employed in this paper is called a three layer feedforward neural network. Its graphical representation is shown in Figure 4. In the input layer, the independent variables  $p_1, p_2, \dots, p_5$  are taken and the  $s^{\text{th}}$  net input, which is defined as:

$$s^{\text{th}} \text{ net input} = \sum_{r=1}^5 w_{sr}^1 p_r + b_s^1$$

is fed to the  $s^{\text{th}}$  neuron of the hidden layer where  $s=1,2,\dots,6$ . Then the neuron converts the net input to a neuron output by utilizing a sigmoid transfer function  $f(s) = [1-\exp(s)]^{-1}$ . In the output layer, the  $s^{\text{th}}$  neuron output is weighted by  $w_{i,s}^2$ . Finally the unique neuron in the output layer sums all the weighted neuron outputs from the hidden layer and the bias  $b_i^2$  and transforms this sum linearly to produce a number which is the network output. This type of ANN can be used as a high dimensional nonlinear regressor. Precisely, Hornik (1991) has shown that a smooth function defined on any closed and bounded domain of  $R^n$  can be smoothly approximated by the type of neural network within a given error bound if we are allowed to increase the number of hidden neurons of the network. Practically, we will fix the number of hidden neurons and identify the other parameters by using a supervised learning algorithm called backpropagation. Basically, backpropagation proposed by Rumelhart et al. (1986) is an algorithm to find the gradient of the square sums of the difference between the network output and the corresponding experimental reading. So the optimum values of parameters  $w$  and  $b$  can be reached by using a classical gradient based method like steepest descent method. In our application, the number of hidden neurons is 6 and the learning rate needed for backpropagation is 0.01. When the epoch reaches 1000, the training stops. After the backpropagation has been completed and the postprocessing of the network output has been performed, the resulted neural network which is the proposed new correlation is given in a simple matrix form.

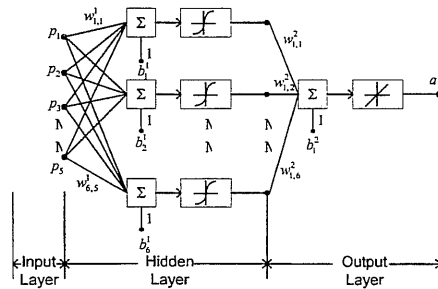


Figure 4. A three layer network with six neurons in its hidden layer.

The proposed new correlation is as follows.

$$Nu = u^3 \left\{ (u^2)^T f(u^1 \Phi + v^1) + v^2 \right\} + v^3 \quad (4)$$

where

$$f(s) = [1-\exp(s)]^{-1}$$

$$\Phi = \begin{bmatrix} Re_{normal} \\ Pr_{normal} \\ Gr_{normal} \\ X/D_{normal} \\ \left(\frac{\mu_b}{\mu_w}\right)^{0.14}_{normal} \end{bmatrix} = \begin{bmatrix} 2(Re - Re_{min}) / (Re_{max} - Re_{min}) - 1 \\ 2(Pr - Pr_{min}) / (Pr_{max} - Pr_{min}) - 1 \\ 2(Gr - Gr_{min}) / (Gr_{max} - Gr_{min}) - 1 \\ 2(x/D - x/D_{min}) / (x/D_{max} - x/D_{min}) - 1 \\ 2\left(\left(\frac{\mu_b}{\mu_w}\right)^{0.14} - \left(\frac{\mu_b}{\mu_w}\right)_{min}^{0.14}\right) / \left(\left(\frac{\mu_b}{\mu_w}\right)_{max}^{0.14} - \left(\frac{\mu_b}{\mu_w}\right)_{min}^{0.14}\right) - 1 \end{bmatrix}$$

The entries of the vector  $\Phi$  represent the normalized Reynolds number, Prandtl number, Grashoff number,  $x/D$ , and  $\mu$  ratio, respectively. The  $u^1, u^2, u^3, v^1, v^2, v^3$  terms used in Equation (4) are constant matrices or scalars. Their numerical values are:

$$u^1 = \begin{bmatrix} 9.20 & -0.81 & -16.69 & 0.35 & -0.29 \\ -14.49 & 11.86 & -1.50 & 3.59 & 9.13 \\ 2.51 & 0.78 & -1.06 & 0.03 & -0.52 \\ 9.14 & -0.83 & -16.39 & 0.33 & -0.26 \\ -2.62 & 12.79 & 5.22 & 0.67 & -1.53 \\ 0.58 & 2.44 & 7.05 & 6.97 & -0.49 \end{bmatrix}, \quad u^2 = \begin{bmatrix} -18.00 \\ 0.44 \\ 1.15 \\ 18.70 \\ -0.78 \\ -46.37 \end{bmatrix}, \quad u^3 = 55.76, \quad v^1 = \begin{bmatrix} -6.93 \\ -11.14 \\ -1.80 \\ -6.75 \\ -1.07 \\ 16.67 \end{bmatrix}$$

$$v^2 = 46.71, \quad v^3 = 14.74$$

The proposed ANN correlation is applicable to the developing and fully developed transition regions. The equation gives a representation of the experimental data to within +18% to -16.5%. Four hundred and forty one (441) data points were used. The absolute deviation is 2.79%. About 4.5% of the data (20 data points) were predicted with more than  $\pm 10\%$  deviation, and 13.4% of the data (59 data points) were predicted within  $\pm 5$  to 10% deviation, and 82.1% of the data (362 data points) with less than  $\pm 5\%$  deviation. As compared to the previous correlation, significant improvement is observed. Figure 5 shows the deviations. The small set of data with more than  $\pm 10\%$  deviation correspond to the data influenced by secondary flow were, as discussed before, the heat balance error was higher. It is also worth to discuss that the least and the most important variables can also be observed according to the coefficient matrices. Let's consider the vector  $u^2$ . We observe that the absolute values of second and fifth entries are much smaller than the rest of the entries of the vector. Therefore, in order to measure the contribution of each row of  $f(u^1\Phi + v^1)$  to the network output, the second and the fifth rows make relatively small contributions. To estimate the involvement of the  $k^{th}$  normalized factor, the index would be the absolute sum of the first, third, fourth and the last entries of the  $k^{th}$  column of the matrix  $u^1$ . For the first normalized factor  $Re_{normal}$ , the index is  $|9.2| + |2.51| + |9.14| + |0.58| = 21.43$ . So the index for  $Pr_{normal}$ ,  $Gr_{normal}$ ,  $X/D_{normal}$  and  $(\mu_b/\mu_w)^{0.14}_{normal}$  are 4.86, 41.19, 7.68 and 1.56, respectively. Hence, by comparing the index values for the different normalized factors, we can identify that  $Re$  and  $Gr$  contribute the most. This supports the observations made with regard to the accuracy of the predictions of the correlation of Ghajar and Tam (1994) in predicting the experimental data for the forced convection in the upper transition region and the mixed convection data with low Nusselt number. On the other hand, the dimensionless term  $(\mu_b/\mu_w)^{0.14}_{normal}$  contributes the least and this is directly due to the fact that the range of  $(\mu_b/\mu_w)^{0.14}$  to establish the correlation is only varied from 1.2 to 2.2 (relatively constant) in comparison to  $Re$  and  $Gr$  which were varied from 1700 to 9100 and 4000 to  $2.1 \times 10^5$ , respectively. Therefore, the ANN method could be used as a tool to find out the most and least important variables.

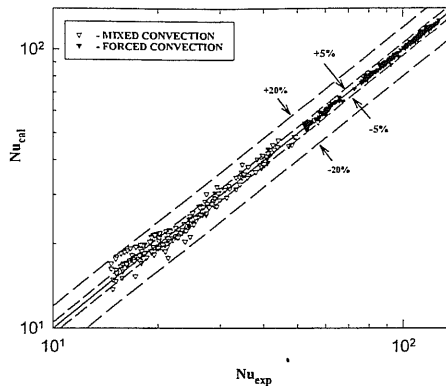


Figure 5. Comparison between experimental and predicted Nusselt numbers using ANN.

#### 4. concluding remarks

This study provided a new correlation for the heat transfer in the transition region for a reentrant inlet using an alternative method, which is the artificial neural network (ANN). The heat transfer correlation developed in this study can be used to assist the heat exchanger designer in predicting the heat transfer coefficient along a horizontal straight circular tube with uniform heat flux boundary condition inside the transition region. The ANN method outperforms the traditional least squares method as indicated by the superb accuracy of the new correlation in comparison to the old one. This method can also be used to determine the importance of the independent variables using the coefficient matrix. Therefore, this method can be used to establish the most and least important variables used in a correlation.

#### References

- Churchill S. W., Comprehensive Correlating Equations for Heat, Mass and Momentum Transfer in Fully Developed Flow in Smooth Tubes, *Ind. Eng. Chem. Fundam.*, Vol. 16, No. 1, pp.109-116, 1977.
- Ghajar A. J., Tam L. M., Heat Transfer Measurements and Correlations in the Transition Region for a Circular Tube with Three Different Inlet Configurations, *Experimental Thermal and Fluid Science*, Vol. 8, No. 1, pp. 79-90, 1994.
- Hornik K., Stinchcombe M. and White H., Universal Approximation of Unknown Mapping and its Derivatives using Multilayer Feedforward Networks, *Neural Networks*, Vol. 3, No. 5, pp. 551-560, 1990.
- Hornik K., Approximation Capabilities of Multilayer Feedforward Networks, *Neural Networks*, Vol. 4, No. 2, pp. 251-257, 1991.
- Jambunathan, K., Hartle, S. L., Ashforth-Frost, S., and Fontama, V. N., Evaluating Convective Heat Transfer Coefficients Using Neural Networks, *Int. J. Heat Mass Transfer.*, Vol. 39, No. 11, pp. 2329-2332, 1996.
- Kakac, S., Shah, R. K., and Bergles, A. E., *Low Reynolds Number Flow Heat Exchanger*, Hemisphere, New York, 1981.
- Kakac, S., Shah, R. K., and Aung, W., *Handbook of Single-Phase Convective Heat Transfer*, Wiley, New York, 1987.
- Rumelhart, D. E., Hinton, G. E., and Williams, R. J., Learning Internal Representations by Error Propagation, *Parallel Distributed Processing: Explorations in the Microstructure of Cognition*, (Eds: D. E. Rumelhart, and J. L. McClelland) Vol. 1, MIT Press, Cambridge, Mass., pp. 318-362, 1986.
- Sen, M., and Yang, K. T., Applications of Artificial Neural Networks and Genetic Algorithms in Thermal Engineering, *CRC Handbook of Thermal Engineering*, F. Kreith, ed., Section 4.24, pp. 620-661, 1999.
- Sieder, E. N., and Tate, G. E., Heat Transfer and Pressure Drop in Liquids in Tubes, *Ind. Eng. Chem.*, 28, 1429-1435, 1936.
- Shah, R. K., and London, A. L., *Laminar Flow Forced Convection in Ducts*, a supplement to *Advances in Heat Transfer*, Academic, New York, 1978.
- Shah, R. K., and Johnson, R. S., Correlations for Fully Developed Turbulent Flow Through Circular and Noncircular Channels, *Sixth Nat. Heat and Mass Transfer Conf.*, Indian Inst. Technology, Madras, India, pp. D-75-D95, 1981.
- Thibault, J., and Grandjean, B. P. A., A Neural Network Methodology for Heat Transfer Data Analysis, *Int. J. Heat Mass Transfer*, Vol. 34, No. 8, pp. 2063-2070, 1991.