CHAPTER ONE

Heat transfer and pressure drop in the transition region of smooth horizontal circular tubes with different inlet configurations

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Abstract

This chapter provides an overview of transitional flow in tubes, with particular emphasis on the entrance geometry and the development region. The discussion also deals with flows that may exhibit buoyant motion (secondary flow) and property variations. Practical and easy to use correlations for friction factor and heat transfer coefficient in the transition region as well as the laminar and turbulent regions are recommended. The application of some of the recommended correlations is illustrated with practical solved problems.

Nomenclature

- Cross-sectional area of the tube, m² A_c
- Surface area of the tube, m² A,
- Fully developed friction factor (Fanning friction factor), (= $\Delta PD/2L\rho V^2$), C_{f} dimensionless
- Specific heat of the test fluid evaluated at T_b , J/(kg·K) cp
- Ď Inside diameter of the test section (tube), m
- Gr Local bulk Grashof number $[=g\beta\rho^2D^3(T_w - T_b)/\mu_b^2]$, dimensionless
- Apparent (developing) Fanning friction factor (= $\Delta P_{0-x}D/2x\rho V^2$), dimensionless fapp f Fully developed friction factor (Darcy friction factor), (= $2\Delta PD/L\rho V^2$), dimensionless
- Acceleration of gravity, m/s² g
- Fully developed peripheral heat transfer coefficient, $W/(m^2 \cdot K)$ h
- Local peripheral heat transfer coefficient at bottom of tube, $W/(m^2 \cdot K)$ hb
- $\mathbf{h}_{\mathbf{t}}$ Local peripheral heat transfer coefficient at top of tube, $W/(m^2 \cdot K)$
- Colburn j-factor (= St $Pr^{0.67}$), dimensionless j
- k Thermal conductivity evaluated at T_b , $W/(m \cdot K)$
- L Length of the test section (tube), m
- Exponent of the bulk-to-wall-viscosity ratio; see Eqs. (3) and (6), dimensionless m
- m Mass flow rate, kg/s
- Nu Local average or fully developed peripheral Nusselt number (= hD/k), dimensionless
- Pr Local bulk Prandtl number (= $c_p \mu_b/k$), dimensionless
- Heat flux, W/m² ģ
- Q Q Heat transfer rate, W
- Volume flow rate, m³/s
- Re Local bulk Reynolds number (= $\rho VD/\mu_b$), dimensionless
- Local average or fully developed peripheral Stanton number [= Nu/(PrRe)], St dimensionless
- т Temperature, °C
- Ть Local bulk temperature of the test fluid, °C
- Tw Local inside wall temperature, °C
- v Average velocity in the test section, m/s
- х Local axial distance along the test section from the inlet, m

Greek symbols

| β | Coefficient of thermal expansion of the test fluid evaluated at T _b , K ⁻¹ |
|------------------|--|
| ΔP | Pressure difference, Pa |
| ΔP_{0-x} | Pressure drop from the inlet to a specific location down the tube, Pa |
| μ_b | Absolute viscosity of the test fluid evaluated at T _b , Pa·s |

- Absolute viscosity of the test fluid evaluated at Tw, Pa·s μ_{w}
- Kinematic viscosity of the test fluid evaluated at T_b, m²/s ν
- ρ ζ Density of the test fluid evaluated at T_b, kg/m³
- Dimensionless axial distance (= x/D/Re), dimensionless

Subscripts

| app | Apparent |
|-------|---|
| cal | Refers to the calculated value |
| cr | Critical |
| e | Exit |
| exp | Refers to the experimental value |
| heat | Refers to non-isothermal (or heating) condition |
| i | Inlet |
| iso | Refers to isothermal condition |
| 1 | Refers to laminar |
| lam | Laminar |
| qt | Quasi-turbulent |
| t | Refers to turbulent |
| tr | Refers to transition |
| trans | Transition |
| | |

Superscripts

Average

1. Introduction

For proper design of thermal systems such as compact heat exchangers involving horizontal tubes that are heated by a uniform wall heat flux boundary condition, proper knowledge of the friction factor (pressure drop) and heat transfer coefficient (Nusselt number) in the tube for the entrance and fully developed regions with different inlet geometries under different flow regimes (laminar, transitional, and turbulent) is essential. In practical engineering design, the usual recommendation is to avoid design and operation in this region, but this is not always feasible under design constraints. The usually cited transitional Reynolds number range of about 2300 (onset of turbulence) to 10,000 (fully turbulent condition) applies, strictly speaking, to a very steady and uniform entry flow with a rounded entrance. If the flow has a disturbed entrance typical of heat exchangers in which there is a sudden contraction and possibly even a re-entrant entrance, the transitional Reynolds number range will be very different.

The transition from laminar to turbulent flow has been considered to be one of the most challenging problems in the thermal sciences since it was described by Osborne Reynolds in 1883. However, not a lot of progress has been made since then in the state of the art. Ghajar and co-workers were the first to break the ground to determine the very important relationship between inlet geometry and transition. In a series of studies, they experimentally investigated the inlet configuration and buoyancy (secondary flow) effects on the developing and fully developed transitional pressure drop (friction factor) under isothermal and heating (uniform wall heat flux boundary) conditions and developing and fully developed transitional forced and mixed convection heat transfer in plain circular horizontal tubes. On the basis of their experimental data, they developed practical and easy to use correlations for the isothermal and non-isothermal friction factor (pressure drop) and heat transfer coefficient (Nusselt number) in the transition region as well as the laminar and turbulent flow regions for different inlet configurations.

This article first provides a detailed overview of the pioneering research done by Ghajar and co-workers and then briefly summarizes some of the recent experimental and numerical/analytical work done by other researchers in this area.

2. Effect of inlet configuration and heating on plain tube friction factor

2.1 Fully developed friction factor

To demonstrate the influence of entrance disturbances on the fully developed friction factor (pressure drop), the isothermal and non-isothermal (heating) friction factor data of Tam et al. [1] for re-entrant and squareedged inlets were used. Fig. 1 shows the schematic of the two inlet configurations. In a heat exchanger, the re-entrant inlet represents the tube extending beyond the tube sheet face and into the head of the distributor and the square-edged inlet represents the tube end being flush with the tube sheet face.

Fig. 2 clearly shows the influence of inlet configuration on the beginning and end of the friction factor transition region under isothermal conditions. This figure plots the fully developed friction factors versus the Reynolds numbers for the two inlets in all flow regimes. For purposes of comparison, the figure also shows the classic fully developed laminar pipe flow friction factor equation ($C_f = 16/Re$) and the established Blasius [2] friction factor correlation for fully developed turbulent pipe flow ($C_f = 0.0791Re^{-0.25}$). The solid symbols in the figure represent the start and end of the fully



Fig. 1 Schematic of re-entrant and square-edged inlet configurations.



Fig. 2 Influence of different inlet configurations on the friction factor in the transition region for a plain tube at 200 diameters from the tube entrance under isothermal boundary condition (solid symbols designate the start and end of the transition region for each inlet).

developed transition region for the two different inlets. As shown by the solid symbols, the lower and upper limits of the friction factor (pressure drop) transition Reynolds number range are dependent on the inlet configuration. The Reynolds number for the start of the transition region is defined as the Reynolds number corresponding to the first abrupt change in the friction factor, and the Reynolds number for the end of the transition region corresponds to the Reynolds number of the friction factor that first reaches the fully developed turbulent friction factor line. From these data, the limits for the transition Reynolds number range for these two inlet configurations can be summarized as follows: square-edged (2222 < Re < 3588) and re-entrant (2032 < Re < 3031).

Because the calming section is a common factor for the two inlets (see Fig. 1) and the flow is isothermal, the difference between the transitional Reynolds number ranges is due only to the effect of different inlet configuration. The preceding limits for the friction factor transition Reynolds numbers indicate that the inlet that caused the most disturbance (re-entrant) produced an early transition (Re = 2032) and that the inlet with less disturbance (square-edged) did not experience transition until a Reynolds number of about 2222. From these observations, it can be concluded that the transition Reynolds number range can be manipulated by using different inlet configurations.

Tam et al. [1] used a DC welder to approximate the uniform wall heat flux boundary condition. The results shown in Fig. 3 clearly establish the



Fig. 3 Friction factor characteristics at 200 diameters from the tube entrance under isothermal and heating boundary conditions for different inlet configurations and all flow regimes (laminar, transition, and turbulent).

influence of the heating condition on the friction factor. In the laminar and transition regions, heating seems to have a significant influence on the value of the friction factor. However, in the turbulent region, heating did not affect the magnitude of the friction factor. In the turbulent pipe flow region, the Blasius [2] type fully developed correlation ($C_f = 0.0791 \text{Re}^{-0.25}$) for both isothermal and non-isothermal (heating) conditions could be used. According to Ghajar and Tam [3], the significant influence of heating on the values of the friction factor in the laminar and transition regions is directly due to the effect of secondary flow (the occurrence of the mixed convection). In the turbulent region, the secondary flow effect is suppressed by the turbulent motion (the occurrence of the pure forced convection). Ghajar and Tam [3] stated that the ratio of the heat transfer coefficient of the top and bottom (h_t/h_b) of the heated tube should be close to unity (0.8-1.0) for forced convection and is much less than unity (<0.8) for a case in which mixed convection exists. They established that when h_t/h_b is less than 0.8, a strong buoyancy effect is present. Fig. 4, which is based on the recent experimental data of Tam et al. [1], supports their findings and shows that the buoyancy effect is significant $(h_t/h_b < 0.8)$ when the Reynolds numbers are lower than 2600 (square-edged inlet) and 2400 (re-entrant inlet), respectively.

In their study of the effect of heating on the transition Reynolds numbers, Ghajar and Tam [3] observed that the increase of heat flux delayed the flow transition or stabilized the flow and caused the flow to go into transition at a higher Reynolds number. Table 1 summarizes the start and end of transition of the fully developed friction factor under isothermal and non-isothermal (heating) conditions. For both inlets, the effect of heating is to delay the start and end of the transition. For the square-edged inlet, the start and end of transition are delayed from Re = 2222 to 2316 and from Re = 3588 to 3941, respectively. For the re-entrant inlet, the start and end of transition are delayed from Re = 2032 to 2257 and from Re = 3031 to 3250, respectively. Thus, both entrance disturbance and heating influence the start and end of the transition region of the fully developed friction factor.

2.2 Entrance and fully developed friction factors

To analyze the effects of inlet geometries and heating on the entrance and fully developed friction factors, the friction factor data of Tam et al. [1] are arranged into the *apparent friction factor* plots and are shown in Fig. 5 for the isothermal and non-isothermal conditions. The apparent



Fig. 4 The ratio of heat transfer coefficients at the top and bottom on the tested tube with square-edged and re-entrant inlets and all flow regimes (laminar, transition, and turbulent).

(developing) friction factor accounts for the combined effects of flow acceleration (variation in momentum flux) and surface shear stress. For comparison purposes, the isothermal entrance and fully developed correlation of Shah [4] given by Eq. (1) is also shown in Fig. 5.

| Table 1 | Start | and en | nd of | transition | of th | e fully | developed | friction | factor | at | 200 |
|---------|---------|----------|-------|------------|-------|---------|-----------|----------|--------|----|-----|
| diamete | rs from | n the tu | ube e | ntrance. | | | | | | | |

| | Friction factor | | | | | | | |
|---------------------------|---------------------|----------------|-------------------|----------------|--|--|--|--|
| Condition, inlet | Re _{start} | C _f | Re _{end} | C _f | | | | |
| Isothermal (square-edged) | 2222 | 0.0079 | 3588 | 0.0102 | | | | |
| Heating (square-edged) | 2316 | 0.0074 | 3941 | 0.0100 | | | | |
| Isothermal (re-entrant) | 2032 | 0.0090 | 3031 | 0.0110 | | | | |
| Heating (re-entrant) | 2257 | 0.0079 | 3250 | 0.0105 | | | | |



Fig. 5 Comparison between experimental laminar flow apparent friction factors with square-edged and re-entrant inlets and correlation of Shah [4] under isothermal and heating conditions.

$$f_{\rm app} = \frac{16}{\rm Re} \left(\frac{3.44}{\zeta^{1/2}} + \frac{0.31/\zeta + 16 - 3.44/\zeta^{1/2}}{1 + 0.00021/\zeta^2} \right) \tag{1}$$

where $\zeta = \frac{x/D}{\text{Re}}$.

For the isothermal condition, Fig. 5 displays the laminar apparent friction factors for the two inlets (re-entrant and square-edged). The figure presents the apparent friction factor multiplied by the Reynolds number versus ζ ,

which is the dimensionless axial location of the pressure taps (x/D) divided by the Reynolds number. In Fig. 5, it can be observed that for the squareedged inlet when Re is less 2300, the apparent friction factor data collapse into a single curve (f_{app} Re vs. ζ), and that after a certain length it approaches the fully developed isothermal constant value of the apparent fraction factor $(f_{app}Re = 16)$. For Re values greater than 2300, the apparent friction factor becomes a strong function of the Reynolds number. Similar behavior is shown for the re-entrant inlet. When Re is less than 2100, the apparent friction factor data collapse into a single curve ($f_{app}Re$ vs. ζ), and after a certain length it approaches the fully developed isothermal constant value of the apparent fraction factor ($f_{app}Re = 16$). For Re values greater than 2100, the apparent friction factor becomes a strong function of the Reynolds number. Comparison of the results given in Fig. 5 for the two inlets at comparable Reynolds numbers shows that the length required for the squareedged inlet to reach the fully developed isothermal apparent friction factor $(f_{app}Re = 16)$ is shorter than what is required for the re-entrant inlet. A longer entrance flow region could be attributed to the additional disturbance caused by the re-entrant inlet. In Fig. 5, it also can be observed that the square-edged and re-entrant data do not follow the Shah correlation, Eq. (1). This observation also was made by Ghajar and Madon [5]. Those authors also observed that the Shah correlation was applicable to the bell-mouth inlet (smooth inlet with no appreciable disturbance) but was not applicable to the square-edged and re-entrant inlets.

For the non-isothermal condition, Fig. 5 also provides the laminar apparent friction factors for the two inlets (re-entrant and square-edged), similar to the presentation for the isothermal condition. For comparison purposes, the isothermal entrance and fully developed correlation of Shah [4] given by Eq. (1) is also shown in Fig. 5.

As is shown in Fig. 5, the trend of the apparent friction factor under the non-isothermal condition is similar to that of the isothermal condition. For Re values less than 2400 (square-edged inlet) and 2200 (re-entrant inlet), the apparent friction factor data collapse into a single curve, and after a certain length it approaches the fully developed non-isothermal constant value of the apparent fraction factor. However, compared with the isothermal data, the entire non-isothermal apparent friction factor trend is shifted downward as a result of the effect of heating. The apparent friction factor then approaches a constant line ($f_{app}Re = 14$) that is less than the isothermal value of 16. Therefore, it can also be observed that in the fully developed friction factor region (Fig. 3), the non-isothermal laminar data do not follow

the classic fully developed laminar friction factor ($C_f = 16/Re$). As can be seen in Fig. 5, for Re values greater than 2400 (square-edged inlet) and 2200 (re-entrant inlet), the non-isothermal apparent friction factor becomes a strong function of the Reynolds number. In Fig. 5, it is clear that the Shah isothermal correlation (Eq. 1) predicts friction factor values much higher than do Tam et al. [1] non-isothermal laminar flow apparent friction factor data.

3. Proposed correlations for laminar and transition friction factors

3.1 Laminar region

Tam et al. [1] proposed a friction factor correlation for the entrance region that is similar to the correlation of Shah [4]. They introduced a correction factor in terms of ζ in the classic relation, $C_f = 16/Re$, to account for the variation of the friction factor in the entrance region. In the development of the correlation, they used a total of 223 isothermal experimental data points (130 data points for the square-edged inlet and 93 data points for the re-entrant inlet). For the *isothermal* laminar flow, their proposed correlation is of the following form:

$$f_{\rm app, \ lam, \ iso} = \frac{1}{\rm Re} \left(16 + \frac{0.00314}{0.00004836 + 0.0609 \ \zeta^{1.28}} \right)$$
(2)

where $\zeta = \frac{x/D}{Re}$ and the range of application of Eq. (2) is 799 < Re < 2240 and 3 < x/D < 200.

Eq. (2) gives a representation of the experimental data to within +28.1% to -26.1%. The average deviation between the results predicted by the correlation and the experimental data is 6.4%; 6% (13 data points) were predicted with $\pm 20\%$ to 30% deviation, 17% (38 data points) were predicted with $\pm 10\%$ to 20% deviation, and 77% (172 data points) were predicted with less than $\pm 10\%$ deviation. Fig. 6 compares the predicted entrance and fully developed laminar friction factors obtained from the proposed correlation with measurements. Compared with Shah's correlation, Eq. (2) can predict the isothermal apparent friction factor data with much better accuracy.

For the non-isothermal condition, referring to Fig. 5, the effect of heating influences the entrance and fully developed friction factors in the laminar region. Therefore, to account for the effect of heating, Tam et al. [1]



Fig. 6 Comparison between experimental entrance and fully developed friction factors and the proposed laminar region correlation (Eq. 2) under the isothermal condition.

introduced a correction factor in terms of the bulk-to-wall-viscosity ratio raised to power m in Eq. (2). Similar to the work of Tam and Ghajar [6], the power m is a function of Prandtl and Grashof numbers. The proposed correlation for *non-isothermal (heated)* laminar flow is of the following form:

$$f_{\rm app, \ lam, \ heat} = f_{\rm app, \ lam, \ iso} \left(\frac{\mu_b}{\mu_w}\right)^m$$
 (3)

where $m = -5.06 + 0.84 Pr^{0.23} Gr^{0.09}$ and the range of application of Eq. (3) is 897 < Re < 2189, 7141 < Gr < 18224, $1.27 < \mu_b/\mu_w < 1.56$, and 39 < Pr < 47.

Eq. (3) provides a representation of the experimental data to within +25.2% to -29.0%. In the development of the correlation, Tam et al. [1] used a total of 301 experimental data points (211 data points for the square-edged inlet and 90 data points for the re-entrant inlet). The average deviation between the results predicted by the correlation and the experimental data is 7.6%; 5% (16 data points) were predicted with $\pm 20\%$ to 30% deviation, 24% (73 data points) were predicted with $\pm 10\%$ to 20% deviation, and 70% (212 data points) were predicted with less than $\pm 10\%$



Fig. 7 Comparison between experimental entrance and fully developed friction factors and the proposed laminar region correlation (Eq. 3) under the non-isothermal condition.

deviation. Fig. 7 compares the predicted apparent friction factors obtained from the proposed correlation with measurements for the non-isothermal condition.

3.2 Transition region

As is shown in Fig. 2, the type of inlet configuration influences the beginning and end of the transition region. Thus, a single correlation cannot predict the data, and a specific correlation for each inlet should be developed. The correlation for the fully developed flow is presented first, and then the correlation for the entrance flow is proposed by adding a correction term. Since the fully developed friction factor in the transition region is an asymptotic problem from the laminar region to the turbulent region, under the isothermal condition, the fully developed correlation proposed by Tam et al. [1] was based on the asymptotic correlating technique proposed by Churchill and Usagi [7]. The proposed correlation for the isothermal fully developed flow in the transition region is of the following form:

$$C_{\rm f, tr, iso} = \left(\frac{16}{\rm Re}\right) \left\{ \left[1 + \left(0.0049 {\rm Re}^{0.75}\right)^a\right]^{1/a} + b \right\}$$
(4)

where the coefficients a and b are inlet-dependent and are obtained separately for each inlet. The coefficients, the exponents, and their range of application for each inlet are as follows:

- Re-entrant: a = 0.52, b = −3.47; for 2026 < Re < 3257; 3 < x/D < 200
- Square-edged: a = 0.50, b = -4.0; for 2111 < Re < 4141; 3 < x/D < 200

Eq. (4) is applicable to an isothermal and fully developed transition region and should be used with the a and b values for the correct geometry (re-entrant vs. square-edged). It gives a representation of the experimental data to within +7.9% to -8.6%. In the development of the correlation, Tam et al. [1] used a total of 30 data points for each inlet. The average deviation between the results predicted by the correlation and the experimental data is 3.6%. All the data points were predicted with less than 10% deviation. Fig. 8 compares the predicted fully developed transition region friction factors obtained from the proposed correlation with isothermal measurements.

To account for the entrance effect, Tam et al. [1] introduced a correction factor in terms of length-to-diameter ratio (x/D) into Eq. (4). Therefore, the proposed correlation for the prediction of entrance and fully developed



Fig. 8 Comparison between experimental fully developed friction factors and the proposed transition region correlation (Eq. 4) under the isothermal condition.

flow in the transition region under isothermal conditions is of the following form:

$$f_{\text{app, tr, iso}} = C_{\text{f, tr, iso}} \left[1 + \left(\frac{c}{x/D} \right) \right]$$
 (5)

where the coefficient c is inlet-dependent and is obtained separately for each inlet. The coefficient for each inlet and its range of application are as follows:

- Re-entrant: c = 4.8; for 2019 < Re < 3257; 3 < x/D < 200
- Square-edged: c = 3.0; for 2109 < Re < 4184; 3 < x/D < 200

Eq. (5) is applicable to an isothermal entrance and fully developed transition region and should be used with the c value for the correct geometry (re-entrant vs. square-edged). For the development of the transition region correlation for the re-entrant inlet, Tam et al. [1] used 150 experimental data points. The correlation gave a representation of the experimental data to within +21.9% to -25.9% and had an average deviation of 5.9\%; 3% (5 data points) were predicted with $\pm 20\%$ to 30% deviation, 14% (21 data points) were predicted with $\pm 10\%$ to 20% deviation, and 83% (124 data points) were predicted with less than $\pm 10\%$ deviation. For the square-edged inlet, Tam et al. [1] used 153 experimental data points for the development of the correlation. The equation correlated the experimental data to within +21.4% to -28.7% and had an average deviation of 7.1%; 4% (6 data points) were predicted with $\pm 20\%$ to 30% deviation, 15% (23 data points) were predicted with $\pm 10\%$ to 20% deviation, and 81% (124 data points) were predicted with less than $\pm 10\%$ deviation. Fig. 9 compares the predicted entrance and fully developed friction factors obtained from the proposed Eq. (5) for each inlet with isothermal measurements.

To account for the effect of heating, Tam et al. [1] applied the viscosity ratio correction factor proposed by Tam and Ghajar [6] expressed as a function of the Grashof and Prandtl numbers to the isothermal correlation (Eq. 5). The proposed correlation is of the following form:

$$f_{\text{app, tr, heat}} = f_{\text{app, tr, iso}} \left(\frac{\mu_b}{\mu_w}\right)^m$$
 (6)

where the exponent m is inlet-dependent and is obtained separately for each inlet. The exponent of the viscosity ratio term and its range of application for each inlet are as follows:

 $\begin{array}{l} \mbox{Re-entrant:} \ m = -1.8 + 0.46 \ \mbox{Gr}^{-0.13} \ \mbox{Pr}^{0.41} \mbox{; for } 1883 < \mbox{Re} < 3262 \mbox{,} \\ \mbox{19.1} < \mbox{Pr} < 46.5 \mbox{, } 4560 < \mbox{Gr} < 24339 \mbox{, } 1.12 < \mbox{\mu}_b/\mbox{\mu}_w < 1.54 \end{array}$



Fig. 9 Comparison between experimental entrance and fully developed friction factors and the proposed transition region correlation (Eq. 5) under the isothermal condition.

Square-edged: m = -1.13 + 0.48 Gr^{-0.15} Pr^{0.55}; for 2084 < Re $< 3980, 19.6 < Pr < 47.3, 6169 < Gr < 35892, 1.10 < \mu_b/\mu_w < 1.54$ Eq. (6) is applicable to the non-isothermal entrance and fully developed transition regions and should be used with the m value for the correct geometry (re-entrant vs. square-edged). For the development of the transition region correlation for the re-entrant inlet, Tam et al. [1] used 212 experimental data points. The correlation gave a representation of the experimental data to within +27.6% to -28.0% and had an average deviation of 10.2%; 15% (32 data points) were predicted with $\pm 20\%$ to 30% deviation, 29% (61 data points) were predicted with $\pm 10\%$ to 20% deviation, and 56% (119 data points) were predicted with less than $\pm 10\%$ deviation. For the square-edged inlet, Tam et al. [1] used 226 experimental data points for the development of the correlation. The equation correlated the experimental data to within +18.6% to -29.0% and had an average deviation of 9.2%; 8% (19 data points) were predicted with $\pm 20\%$ to 30% deviation, 31% (69 data points) were predicted with $\pm 10\%$ to 20% deviation, and 61% (138 data points) were predicted with less than $\pm 10\%$ deviation. Fig. 10 compares the predicted entrance and fully developed friction factors obtained from the proposed Eq. (6) for each inlet with non-isothermal measurements.



Fig. 10 Comparison between experimental entrance and fully developed friction factors and the proposed transition region correlation (Eq. 6) under the non-isothermal condition.

4. Effect of inlet configuration on plain tube heat transfer

Ghajar and coworkers also experimentally investigated the inlet configuration effects on heat transfer in the transition region between laminar and turbulent flows in plain circular horizontal tubes. For this investigation, they used the same two inlet configurations (re-entrant and square-edged) used for the friction factor (pressure drop) studies (Fig. 1) and added a third inlet (bell-mouth), as shown in Fig. 11. In a heat exchanger, the bell-mouth inlet represents a tapered entrance of tube from the tube sheet face.



Fig. 11 Schematic of bell-mouth inlet configuration.



Fig. 12 Influence of different inlet configurations on heat transfer in the transition region for a plain tube at 192 tube diameters from the tube entrance (filled symbols designate the start and end of the transition region for each inlet).

Fig. 12 clearly shows the influence of inlet configuration on the beginning and end of the heat transfer transition region. This figure plots the local average peripheral heat transfer coefficients in terms of the Colburn j-factor $(j = \text{St Pr}^{0.67})$ versus the local Reynolds number for all flow regimes at the length-to-diameter ratio of 192. The filled symbols in Fig. 12 represent the start and end of the heat transfer transition region for each inlet configuration. The presented results show how heat transfer in the transition region varies between the limits of the Sieder and Tate [8] correlation $[\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.33} (\mu_b / \mu_w)^{0.14}]$ for fully developed turbulent flow and Nu = 4.364 for fully developed laminar flow with a uniform wall heat flux boundary condition. Note the large influence of natural convection (buoyancy effect) superimposed on the forced convective laminar flow heat transfer process (Nu = 4.364 for a fully developed laminar flow with a uniform heat flux boundary condition without a buoyancy effect), yielding a much larger mixed convection value of about Nu = 14.5.

As shown by the filled symbols in Fig. 12, the lower and upper limits of the heat transfer transition Reynolds number range depend on the inlet configuration. In addition, these transition Reynolds number limits are x/D-dependent, and they linearly increase with an increase in x/D, as reported by Ghajar and Tam [9]. To determine the range of heat transfer transition Reynolds numbers along the tube, Ghajar and Tam [9] used their experimental data and developed figures similar to Fig. 12 for 20 other x/D locations. From those figures, the heat transfer transition Reynolds number range for each inlet was determined to be about 2000-8500 for the reentrant inlet, 2400-8800 for the square-edged inlet, and 3400-10,500 for the bell-mouth inlet. The lower and upper limits of the heat transfer transition Reynolds number ranges for the three different inlets along the tube $(3 \le x/D \le 192)$ are summarized in Table 2. The results in this table indicate that the re-entrant inlet configuration causes the earliest transition from laminar flow into the transition regime (at about 2000) whereas the bell-mouth entrance retards this regime change (at about 3400). The square-edged entrance falls in between (at about 2400), which is close to the frequently quoted value of 2300 in most textbooks.

Application of heat to the tube wall produces a temperature difference in the fluid. The fluid near the tube wall has a higher temperature and a lower density than the fluid close to the centerline of the tube. This temperature difference may produce a secondary flow caused by free convection. In the laminar flow region, the effect of free convection (or buoyancy) on forced convection can be seen clearly in Fig. 12, and it resulted in an upward parallel shift of the Colburn j-factors from their fully developed forced convection laminar values. In the transition region, the effect of mixed convection cannot be seen easily unless the local heat transfer information, which is the ratio of the local peripheral heat transfer coefficient at the top of the tube to the local peripheral heat transfer coefficient at the bottom of the tube (h_t/h_b) , is examined carefully. To account for the effect of mixed convection properly, in their experiments Ghajar and Tam [3] used four thermocouples

Table 2Lower and upper limits of heat transfer transition Reynolds numbers.Inlet geometry Lower limitUpper limit

| Re-entrant | $Re_{lower} = 2157 - 0.65[192 - 0.65]$ | $Re_{upper} = 8475 - 9.28[192 - 0.000]$ |
|--------------|---|---|
| Square-edged | (x/D)] Re _{lower} = 2524 - 0.82[192 - | (x/D)] Re _{upper} = 8791 - 7.69[192 - |
| Bell-mouth | (x/D)] B e ₁ = 3787 - 1.80[192 - | (x/D)] Be = 10481 - 5 47[192 - |
| Den-mouth | (x/D)] | (x/D)] |

around the periphery of the tube $(90^{\circ} \text{ apart})$ at each of the 26 designated axial locations along the tube. The thermocouples were at close intervals near the entrance of the tube and at greater intervals farther downstream.

According to Ghajar and Tam [3] (as mentioned above in the context of the influence of heating on the friction factor; see Fig. 4), h_t/h_b should be close to unity (0.8–1.0) for forced convection and is much less than unity (<0.8) for a case in which mixed convection exists. Fig. 13, which is similar



Fig. 13 Effect of secondary flow on heat transfer coefficient for different inlets and flow regimes.

to Fig. 4, shows the effect of secondary flow on the heat transfer coefficient for three different inlets and all flow regimes. For the re-entrant, squareedged, and bell-mouth inlets when the Reynolds numbers were greater than 2500, 3000, and 8000, respectively, the flows were dominated by forced convection heat transfer and the heat transfer coefficient ratios (h_t / $h_{\rm b}$) did not fall below 0.8–0.9 and at times exceeded unity as a result of rounding-off errors in the property evaluation calculation of their [3] data reduction program. The flows dominated by mixed convection heat transfer had h_t/h_b ratios beginning near 1 but dropping off rapidly as the length-todiameter ratio (x/D) increased. Beyond about 125 diameters from the entrance, the ratio was almost invariant with x/D, indicating a much less dominant role for forced convection heat transfer and increased free convection activity. In Fig. 13, it is interesting to observe that the starting length necessary for the establishment of the free convection effect for low Reynolds number flows is also inlet-dependent. When the secondary flow is established, a sharp decrease in h_t/h_b occurs.

Depending on the type of inlet configuration, for low Reynolds number flows (Re < 2500 for re-entrant, Re < 3000 for square-edged, and Re < 8000 for bell-mouth), the flow is dominated by forced convection over the first 20 to 70 diameters from the entrance to the tube. It should be noted that the reported lower and upper limits of heat transfer transition Reynolds numbers in Table 2 are not influenced by the presence of mixed convection. However, as the flow travels the tube length required for the establishment of secondary flow, the lower transition region for all three inlets will be influenced by the presence of mixed convection. It should be pointed out that for the bell-mouth inlet, the mixed convection effect will influence not only the lower transition region but also the upper part of the transition region, as shown by Tam and Ghajar [10]. These results will be discussed next.

In a focused experimental study, Tam and Ghajar [10] further explored the unusual behavior of the local heat transfer coefficients in the transition region for a tube with a bell-mouth inlet. This type of inlet is used in some heat exchangers mainly to avoid the presence of eddies, which are believed to be one of the causes of erosion in the tube inlet region. Fig. 14 shows the variation of the local Nusselt number along the tube length (x/D) in the transition region for the three inlet configurations at comparable Reynolds numbers. As is shown in Fig. 14, the re-entrant and square-edged inlets show no influence on the local heat transfer coefficients. For these inlets, the local Nusselt number has a minimum value at an x/D



Fig. 14 Variation of local Nusselt number with length for re-entrant, square-edged, and bell-mouth inlets in the transition region.

that is approximately equal to 25 and increases monotonically along the tube rather than staying at a relatively constant value, as is the expected behavior in the fully developed laminar and turbulent flows (Fig. 15). Considering Newton's law of cooling for a uniform wall heat flux boundary condition,



Fig. 15 Variation of local Nusselt number with length for re-entrant, square-edged, and bell-mouth inlets in the laminar and turbulent regions.

the increase in the Nu values with the tube length is associated with the decrease in the difference between the inside wall and fluid bulk temperatures $(T_w - T_b)$ along the tube. The inside wall temperature and the fluid bulk temperature increase monotonically after they have reached their fully

developed values but the fluid bulk temperature has a larger rate of increase. This causes the heat transfer coefficient to increase along the tube.

However, for the bell-mouth inlet, the variation of the local heat transfer coefficient with tube length in the transition (Fig. 14) and turbulent (Fig. 15) flow regions is very unusual. Tam and Ghajar [10] in their work further investigated this unusual behavior of the bell-mouth inlet by looking into the influence of Reynolds number and inlet disturbance on the length of the dip in the transition region. To investigate the effect of inlet disturbance on the bell-mouth entrance, three different screen sizes were used in the last section of the calming section before the fluid entered the bell-mouth entrance (see Fig. 11). The three different plastic mesh screens were referred to as coarse, medium, and fine mesh screens with open area ratios of 0.825, 0.759, and 0.650, respectively. For their experiments, the length of the dip varied from x/D = 100 to 175 depending on the level of disturbance at the inlet and the transition flow Reynolds number.

The results of the study showed that the boundary layer along the tube wall is first laminar and then changes through a transition to the turbulent condition, causing a dip in the Nu versus x/D curve. The length of the dip decreases with the increase of Reynolds number. The length of the dip in the transition region is much longer (100 < x/D < 175) than it is in the turbulent region (x/D < 25), as is shown in Figs. 14 and 15, respectively. Hence, a mixed convection effect is strongly present even in the high Reynolds numbers upper transition region. The presence of the dip in the transition region causes a significant influence in both the local and the average heat transfer coefficients. This is particularly important for heat transfer calculations in short tube heat exchangers with a bell-mouth inlet.

5. Proposed correlations for laminar, transition, and turbulent heat transfer

In developing their transition region heat transfer correlations, Ghajar and Tam [3] took an approach completely different from that of the previous investigators. They first performed very careful experiments in the transition region, paying particular attention to the role of secondary flow (free convection superimposed on the forced convection or mixed convection) and inlet configuration effects on the start and end of the transition region and the magnitude of heat transfer in this region. After careful analysis of their transitional heat transfer data, following the general form of Churchill's [11] correlation, they proposed some prediction methods for this region to bridge between laminar correlations and turbulent correlations, applicable to forced and mixed convection in the entrance and fully developed regions for three types of inlet configurations (re-entrant, square-edged, and bell-mouth), which are presented below. The local heat transfer coefficient in transition flow is obtained from the transition Nusselt number, Nu_{trans}, which is calculated as follows at a distance x from the tube entrance:

$$Nu_{trans} = Nu_{l} + \left\{ \exp[(a - Re)/b] + Nu_{t}^{c} \right\}^{c}$$
(7)

where Nu_l is the laminar flow Nusselt number and Nu_l is the turbulent flow Nusselt number given by Eqs. (8) and (9), respectively.

Ghajar and Tam [3] used a total of 546 experimental data points in the entrance and fully developed laminar regions with natural convection effects and proposed the following correlation for the laminar flow Nusselt number (Nu_l), which represented 86% of experimental data with less than $\pm 10\%$ deviation and 100% of measured data with less than $\pm 17\%$ deviation:

$$Nu_l = 1.24 \left[\left(\frac{\text{RePr}D}{x} \right) + 0.025 (\text{GrPr})^{0.75} \right]^{1/3} \left(\frac{\mu_{\mathbf{b}}}{\mu_w} \right)^{0.14}$$
(8)

where $3 \le x/D \le 192$, $280 \le \text{Re} \le 3800$, $40 \le \text{Pr} \le 160$, $1000 \le \text{Gr} \le 28000$, $1.2 \le \mu_b/\mu_w \le 3.8$.

For the turbulent flow Nusselt number (Nu_t), Ghajar and Tam [3] used a total of 604 experimental data points in the entrance and fully developed turbulent region and developed the following correlation, which correlated 100% of experimental data with less than $\pm 11\%$ deviation and 73% of measured data with less than $\pm 5\%$ deviation:

$$Nu_t = 0.023 \operatorname{Re}^{0.8} \operatorname{Pr}^{0.385} \left(\frac{x}{D}\right)^{-0.0054} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{9}$$

where $3 \le x/D \le 192$, $7000 \le \text{Re} \le 49000$, $4 \le \text{Pr} \le 34$, $1.1 \le \mu_b/\mu_w \le 1.7$.

The physical properties (k, μ , ρ , c_p , β) appearing in the dimensionless numbers (Nu, Re, Pr, Gr) are all evaluated at the bulk fluid temperature (T_b) unless otherwise specified. The values of the empirical constants a, b, and c in Eq. (7) depend on the inlet configuration and are given in Table 3. The viscosity ratio accounts for the temperature effect on the process. The range of application of the heat transfer correlation based on Ghajar and Tam's [3] database of 1290 data points (441 points for re-entrant inlet,

| Inlet geometry | а | b | c |
|----------------|------|-----|--------|
| Re-entrant | 1766 | 276 | -0.955 |
| Square-edged | 2617 | 207 | -0.950 |
| Bell-mouth | 6628 | 237 | -0.980 |

 Table 3 Constants for transition heat transfer correlation (Eq. 7).

416 points for square-edged inlet, and 433 points for bell-mouth inlet) is as follows:

 $\begin{array}{ll} \text{Re-entrant:} & 3 \leq x/D \leq 192, & 1700 \leq \text{Re} \leq 9100, & 5 \leq \text{Pr} \leq 51, \\ 4000 \leq \text{Gr} \leq 210000, \ 1.2 \leq \mu_b/\mu_w \leq 2.2 \\ & \text{Square-edged:} & 3 \leq x/D \leq 192, & 1600 \leq \text{Re} \leq 10700, & 5 \leq \text{Pr} \leq 55, \\ & 4000 \leq \text{Gr} \leq 250000, \ 1.2 \leq \mu_b/\mu_w \leq 2.6 \\ & \text{Bell-mouth:} & 3 \leq x/D \leq 192, & 3300 \leq \text{Re} \leq 11100, & 13 \leq \text{Pr} \leq 77, \\ & 6000 \leq \text{Gr} \leq 110000, \ 1.2 \leq \mu_b/\mu_w \leq 3.1 \end{array}$

These correlations capture about 70% of measured data within a deviation band of $\pm 10\%$ and 97% of measured data within $\pm 20\%$, which is remarkable for transition flows. The individual expressions above for Nu_l and Nu_t can be used alone for developing and fully developed flows in those respective regimes. The lower and upper limits of the heat transfer transition Reynolds number ranges for the three different inlets are summarized in Table 2.

A detailed comparison of the performance of Eq. (7) with the experimental data of Ghajar and Tam [3] is shown in Figs. 16–18 for the reentrant, square-edged, and bell-mouth inlets, respectively. Eq. (7) predicted 96.9%, 99.7%, and 98.9% of the data in the mixed convection region with less than 20% deviation for the re-entrant, square-edged, and bell-mouth inlets, respectively. In the forced convection region, Eq. (7) predicted 97.2% (re-entrant inlet), 90.8% (square-edged inlet), and 100% (bell-mouth) of the experimental data with less than 20% deviation.

In a subsequent study, Tam and Ghajar [12] conducted a detailed study comparing the performance of their proposed correlation (Eq. 7) with all the available correlations in the transition region using the experimental data of Ghajar and Tam [3]. The study showed that all the existing correlations failed to account properly for the existence of the mixed convection in the laminar and transition regions. In addition, none of the correlations could account properly for the effect of inlet configuration on the start and end of the transition region.



Fig. 16 Comparison between predictions of Eq. (7) for the transition region and heat transfer experimental data of Ghajar and Tam [3] for the re-entrant inlet.



Fig. 17 Comparison between predictions of Eq. (7) for the transition region and heat transfer experimental data of Ghajar and Tam [3] for the square-edged inlet.



Fig. 18 Comparison between predictions of Eq. (7) for the transition region and heat transfer experimental data of Ghajar and Tam [3] for the bell-mouth inlet.

6. Flow regime map

Ghajar and Tam [9] used their uniform wall heat flux heat transfer data and their proposed correlations for laminar (Eq. 8), transitional (Eq. 7), and turbulent (Eq. 9) flows for the three different inlets (re-entrant, squareedged, and bell-mouth) and constructed a flow regime map (see Fig. 19), similar to the one developed for uniform wall temperature boundary condition by Metais and Eckert [13]. Their flow regime map is unique in the sense that it was the first attempt to develop such a map for the case of horizontal tube with uniform wall heat flux.

In the development of their flow regime map, they paid particular attention to the influence of inlet configuration on the start and end of the heat transfer transition region and the development of secondary flow along the tube. Their flow regime map determines the boundary between forced and mixed convection in a horizontal circular straight tube with three different inlets under uniform wall heat flux boundary condition. Their map is applicable to all flow regimes and both developing and fully developed flows. From the flow regime map, for any forced flow represented by a given Reynolds number, the value of the parameter Gr Pr at a particular x/D location indicates where it is necessary to consider buoyancy effects (secondary flow). Once the convection heat transfer flow regime (pure forced or mixed) for



Fig. 19 Flow regime map for flow in horizontal tubes with three different inlet configurations and uniform wall heat flux.

any of the three inlets has been established, correlations for calculation of heat transfer coefficient in the laminar, transition, and turbulent regimes are offered. Eq. (8) is recommended for laminar forced and mixed convection in the entrance and fully developed regions and can be used for all three inlets. For turbulent forced convection in the entrance and fully developed regions for all three inlets, Eq. (9) is recommended. Eq. (7) is applicable to transition forced and mixed convection in the entrance and fully developed regions and should be used with the a, b, and c values for the correct inlet geometry (see Table 3).

Ghajar and Tam [9] developed and used the following equation as shown in Fig. 19, to separate the forced and mixed convection regions in their flow regime map:

$$Re = 2674 + 5.35 \times 10^{-13} (GrPr)^{2.5} - 1.85 \times 10^{-16} (GrPr)^3 - 2.64 \times 10^{14} (GrPr)^{-2}$$
(10)

Eq. (10) correlates the 51 data points with a correlation coefficient of 0.96. These data points represented the boundary between laminar and transition forced and mixed convection for the three inlets in their experimental data. In identifying these points, they used the criterion that the ratio of the local peripheral heat transfer coefficient at the top of the tube to the local peripheral heat transfer coefficient at the bottom of the tube (h_t/h_b) should be greater than or equal to 0.8 for the forced convection and less than 0.8 for the mixed convection (see Fig. 13).

7. Simultaneous heat transfer and friction factor analysis

In most studies, heat transfer and friction factor characteristics usually are represented and discussed independently. The simultaneous representation for the heat transfer and friction factor characteristics may not only help in observing the difference of the transition range between the heat transfer and the friction factor but also inspire the researchers to study the relationship between the heat transfer and the friction factor. Therefore, the heat transfer and pressure drop data, which were measured simultaneously in the experiments of Tam and coworkers [1], were plotted together in a single graph (Fig. 20). The figure includes the fully developed heat transfer data (with uniform heat flux boundary condition) and the fully developed



Fig. 20 Simultaneous heat transfer and friction factor characteristics for square-edged and re-entrant inlets at 200 diameters from the tube entrance (experimental data of Tam et al. [1]).

friction factor data (with isothermal and uniform heat flux boundary conditions) for the square-edged and re-entrant inlets. On the basis of Fig. 20, the transition Reynolds numbers for heat transfer and friction factor are summarized in Table 4.

In the laminar region, it can be observed from Fig. 20 that the heating friction factor is slightly lower than the isothermal friction factor as a result of the effect of heating. In the same plot, the laminar heat transfer data have a downward trend similar to that of the laminar friction factor data. When the Reynolds number reaches about 2300, the heat transfer and the friction factor start to move away from the laminar region (start of the transition

| | | Heat t | ransfe | r | Friction factor | | | |
|---|---------------------|---------------------|-------------------|---------------------|----------------------|----------------------------|----------------------|----------------------------|
| Condition, inlet | Re _{start} | StPr ^{0.7} | Re _{end} | StPr ^{0.7} | Re _{start} | C _f | Re _{end} | C _f |
| Isothermal (square-edged) | | | | | 2222 | 0.0079 | 3588 | 0.0102 |
| Heating (square-edged) Isothermal (re-entrant) Heating (re-entrant) | 2298 2001 | 0.0017 0.0020 | 8357 7919 | 0.0041 0.00412 | 2316 2032 2257 | 0.0074 0.0090 0.0079 | 3941 3031 3250 | 0.0100 0.0110 0.0105 |

Table 4 Start and end of transition of the fully developed heat transfer and frictionfactor at 200 diameters from the tube entrance.

region). As was discussed above, the start of transition for the friction factor is inlet-dependent. Moreover, the heating condition also influences the start of transition of the friction factor. As is shown in Table 4, the start of transition for heat transfer is also inlet-dependent. The square-edged inlet (Re = 2298) will delay the start of transition compared with the re-entrant inlet (Re = 2001). For the heating condition (Table 4), it can be observed that the transition Reynolds number for heat transfer (Re = 2298 for square-edged and Re = 2001 for re-entrant inlets) is slightly earlier than that of the heating friction factor (Re = 2316 for square-edged and Re = 2257 for re-entrant inlets).

As is shown in Fig. 20, when the Reynolds number reaches about 3000, all the friction factor data start to decrease and follow the fully developed turbulent pipe flow friction factor line of Blasuis [2] $(C_f = 0.0791 \text{Re}^{-0.25})$. However, the heat transfer data continue to increase until a Reynolds number of about 8000. Around a Reynolds number of 8000, the heat transfer data start to follow the fully developed turbulent pipe flow heat transfer line represented by the Sieder and Tate [8] correlation [St $Pr^{0.67} = 0.023 Re^{-0.2} (\mu_b/\mu_w)^{0.14}$]. It can be observed clearly in Table 4 that the end of transition for the heat transfer (Re = 8357 for squareedged and Re = 7919 for re-entrant inlets) is much later than that of the heating friction factor (Re = 3941 for square-edged and Re = 3250 for re-entrant inlets). On the basis of the start and end of transition, it can be observed that heat transfer has a much wider transition range than does the friction factor. For the turbulent region, the heat transfer and the friction factor data will continue to decrease along the fully developed turbulent heat transfer and the friction factor lines.

So far the pioneering work of Ghajar and co-workers in the transition region have been reviewed in detail. From the presented results it is clear that the shape of inlet configuration, buoyancy effect, and property variation plays a very significant role in the start and end of the transition for both pressure drop (friction factor) and heat transfer coefficient (Nusselt number). In addition, as it was shown that buoyancy effects and property variation significantly influence the values of friction factor and heat transfer coefficient in the laminar and lower transition regions. The only other work in the transition region that is very similar to the work of Ghajar and coworkers is the research work done by Meyer and co-workers which is an extension of the work of Ghajar and co-workers. Their work will be briefly reviewed next.

8. Transitional flow heat transfer works of Meyer and co-workers

The work of Meyer and Oliver [14,15] is an extension of the transition work of Ghajar and co-workers. However, they used a different experimental approach for their transitional work from Ghajar and co-workers with five major differences: (1) they considered cooling and not heating; (2) their experiments were conducted for the case of constant wall temperature, and not uniform wall heat flux, as was done by Ghajar and coworkers; (3) they used an additional type of inlet (hydrodynamically fully developed inlet) in addition to the three inlet configurations used by Ghajar and co-workers; (4) their heat transfer and friction data was averaged over the whole tube length (including the developing part of the flow), while Ghajar and co-workers included local measurements in their work; (5) Ghajar and co-workers used distilled water, as well as different ethylene glycol water mixtures (Prandtl number of up to 160) as the test fluid, while Meyer and Oliver used mainly water (Pr \approx 7) with a limited number of experiments with propylene glycol-water mixtures (Pr \approx 26).

Their experiments were conducted on two smooth horizontal tubes with inner diameters of 15.88 mm and 19.02 mm in which water was cooled. Adiabatic (isothermal) as well as diabetic (heated) experiments were conducted. Their Reynolds numbers ranged between 1000 and 20,000, Prandtl numbers between 4 and 6, and Grashof numbers were on the order of 10^5 . Their results for both friction factor and heat transfer showed that transition from laminar to turbulent was strongly inlet configuration dependent. The bell-mouth inlet delayed the transition the most, followed by the square-edged inlet and the re-entrant inlet, confirming results of Ghajar and co-workers. Their laminar heat transfer and friction factor results were much higher than their theoretically predicted values due to the significant influence of the secondary flow (buoyancy effects), again confirming the findings of Ghajar and co-workers. They did not develop any correlations for prediction of friction factor and heat transfer coefficient in the transition region for the different inlet configurations used.

The recent extensive work of Meyer and Everts [16–19] in the transition region duplicates, complements, and extends the transition work of Ghajar and co-workers. They experimentally investigated the pressure drop and heat transfer characteristics of developing and fully developed flow in smooth tubes in the laminar, transitional, quasi-turbulent and turbulent flow regimes. Their experiments were conducted with water as the working fluid on two smooth circular test sections with inner diameters of 4 mm and 11.5 mm, and a maximum length to diameter ratio of 1373 and 872, respectively. The Reynolds number was varied between 700 and 10,000 to ensure that the transitional and quasi-turbulent flow regimes, as well as sufficient parts of the laminar and turbulent flow regimes, were covered. Heat transfer measurements were taken at different uniform wall heat fluxes.

The broad purpose of their study was to experimentally investigate the heat transfer and pressure drop characteristics of developing and fully developed flow of low Prandtl number fluids (3–7) in smooth horizontal tubes for forced and mixed convection conditions with uniform wall heat flux boundary condition. To address some of the shortcomings in the literature, their specific objectives were: (1) to investigate the mixed convection laminar flow, as well as the effect of free convection on the laminar-turbulent transition along the tube length [16]; (2) to investigate the heat transfer characteristics of developing and fully developed flow in the transitional flow regime [17]; (3) to investigate the relationship between heat transfer and pressure drop in all flow regimes [18]; and (4) to develop flow regime maps for developing and fully developed flows [19]. Some of their findings are briefly presented below.

Analogy between momentum and heat transfer: Everts and Meyer [18] investigated pressure drop and heat transfer simultaneously in smooth horizontal tubes using water as the test fluid. The relationship between pressure drop and heat transfer was investigated by dividing the friction factors by the Colburn *j*-factors. It was found that the f/j-factors in the laminar flow regime were a function of Grashof number (free convection effects), while it was a function of Reynolds number in the other flow regimes. The following correlation was developed for laminar flow:

$$\frac{f}{j} = 109.71 \ Gr^{-0.215} \tag{11}$$

where the Colburn j-factors used in this equation are the average values.

Because the f/j-factors in the transitional, quasi-turbulent and turbulent flow regimes were a stronger function of Reynolds number than Grashof number, a single correlation for these three flow regimes was obtained:

$$\frac{(f/j)}{\Pr^{0.42}} = \frac{3.74\text{Re} - 8066}{\text{Re} - 2320}$$
(12)

| | Eq. | Data points | ±5% [%] | ±10% [%] | ± 20% [%] | Ave [%] | Range |
|------------------|------|----------------|------------|-------------|---------------------|------------|---------------------------------|
| Friction factors | | | | | | | |
| Laminar | (11) | 495 | 92 | 100 | 100 | 2.4 | $467 \le \text{Re} \le 3217$, |
| | | | | | | | $3 \leq Pr \leq 7.4$, |
| | | | | | | | $2.6 \le Gr \le 5589$ |
| Transitional, | (12) | 834 | 85 | 97 | 99.9 | 2.8 | $2483 \le \text{Re} \le 9787$, |
| quasi- | | | | | | | $5.4 \le Pr \le 6.9,$ |
| turbulent and | | | | | | | $8.9 \times 10^2 \le$ |
| turbulent | | | | | | | $Gr \le 3.2 \times 10^4$ |

 Table 5 Summary of the ranges and performance of the correlations [17].

Similar correlations were also developed in terms of the modified Grashof number [18], because the temperature difference is not always known for uniform heat flux applications while the heat flux usually is. Table 5 summarizes the valid ranges and performances of the correlations. The laminar correlation could predict approximately all the data within 5%, while a single correlation for the transitional, quasi-turbulent and turbulent flow regimes could predict almost all the data (97%) within 10%.

Correlations for the start and end of transition region: Everts and Meyer [17] similar to Ghajar and co-workers, based on their experimental data developed correlations for the start and end of the transition region. As the boundaries of the transitional flow regime were not only a function of axial position (x/D), but also free convection effects (*Gr*), the following correlation was developed to predict the start (Re_{cr}) of the transitional flow regime:

$$\operatorname{Re}_{cr} = \left(0.1972\frac{x}{D} + 1156.7\right) Gr^{0.077}$$
(13)

From Eq. (13) it follows that the critical Reynolds number (Re_{cr}) increased linearly along the test section (due to the decreasing viscosity with temperature) and the gradient of the line was influenced by the Grashof number (free convection – secondary flow). They [17] also developed correlations in terms of the modified Grashof number since for uniform heat flux applications; the temperature difference is not always available while the heat flux usually is.

Their experimental results showed that the Reynolds number at which transition ended (Re_{qt}) was not a function of axial position (x/D), since the Grashof number is a function of tube location (temperature difference and

thermal properties vary along the tube length). The following correlation was developed to predict the end (Re_{at}) of the transitional flow regime:

$$\operatorname{Re}_{qt} = 2504 \ Gr^{0.018} \tag{14}$$

where Re_{qt} is the start of quasi-turbulent flow regime. A similar correlation was also developed in terms of the modified Grashof number [17].

The performance of Eqs. (13) and (14) were compared with the experimental results of their study and literature. The Re_{cr} correlation could predict 80% of the data within 10%, and approximately all the data within 20%. The Re_{qt} correlation could predict 85% of the data within 10%, and the average deviation was approximately 6%.

Correlations for average Nusselt numbers and friction factors: Everts and Meyer [18] developed the following correlations to predict the average Nusselt numbers (\overline{Nu}) and friction factors (f) of developing flow in the transitional flow regime:

$$\overline{Nu} = (0.00108 \ Re - 2.49) \ Gr^{-0.04} Pr^2 \tag{15}$$

$$f = \left(\frac{3.74 \ Re - 8066}{Re - 2320}\right) \ \frac{\overline{Nu}}{Re \ Pr^{0.087}} \tag{16}$$

As Eq. (15) is a function of Grashof number, a correlation was also developed in terms of the modified Grashof number [18]. Table 6 summarizes the valid ranges and performances of the average Nusselt number and friction factor correlations for developing and fully developed flow. The average Nusselt number correlation could predict almost all the data within 20%, while the friction factor correlation was able to predict all the data within 10%.

| | Eq. | Data points | ±5% [%] | ±10% [%] | ± 20% [%] | Ave [%] | Range |
|----|------|----------------|------------|-------------|---------------------|---------|--|
| Nu | (15) | 179 | 51 | 76 | 95 | 7.1 | $2520 \le \text{Re} \le 3361,$ $5.4 \le Pr \le 6.8,$ $2.8 \times 10^4 \le$ |
| f | (16) | 834 | 85 | 97 | 99.9 | 2.8 | $Gr \le 3.2 \times 10^{-10}$ $2483 \le \text{Re} \le 9787,$ $5.4 \le Pr \le 6.9,$ $8.9 \times 10^{2} \le$ $Gr \le 3.2 \times 10^{4}$ |

 Table 6
 Summary of the ranges and performance of the average Nusselt number

 and friction factor correlations for developing transitional flow [18].

Flow Regime Maps: Everts and Meyer [19] showed that although the flow regime map of Ghajar and Tam [9] may be very accurate for high Prandtl number fluids, it became inaccurate once the Prandtl number and tube diameter was decreased, for example, when water was used in small diameter tubes. The result was that the flow was predicted to be in the forced convection region, while free convection effects were significant and the flow was actually dominated by mixed convection. In Ref. [19] they developed new flow regime maps that can be used for both high and low Prandtl number fluids, a wide range of tube diameters, and developing and fully developed flows. In the development of the new flow regime maps they used the results of their previous studies [16–18].

The following experimental and numerical/analytical works related to the transition region did not account for the influence of inlet configuration on the start and end of the transition region and the secondary flow effects (buoyancy) on the friction factor and heat transfer results. These works will be briefly reviewed next.

9. Transitional flow heat transfer works of Abraham and co-workers

As mentioned in the Introduction section, there is a regime of fluid flow, which is neither fully laminar nor fully turbulent. Generally, internal flows with Reynolds numbers less than 2300 are considered fully laminar. As the Reynolds number rises, it contains increasingly more turbulent motion until Re ~4000; at this point, it is mostly turbulent. By the time the Reynolds number is increased to ~10,000, it is normally fully turbulent.

In all modern heat transfer textbooks, for example see Cengel and Ghajar [20], the correlation proposed by Gnielinski [21] which is an improved version of the correlation proposed by Petukhov [22], is recommended for fully developed forced convection heat transfer calculations (Nusselt number) in smooth circular horizontal tubes. Gnielinski's correlation is valid over a much lower Reynolds number range including part of the transition region. The Gnielinski correlation is of the form

Nu =
$$\frac{(f/8)(\text{Re} - 1000) \text{ Pr}}{1 + 12.7 (f/8)^{0.5} (\text{Pr}^{2/3} - 1)}$$
 (17)

The correlation is valid for $0.5 \le Pr \le 2000$ and $3000 \le Re \le 5 \times 10^4$. The friction factor *f* can be determined from an appropriate relation

such as the correlation of Petukhov [22], valid for $3000 \le \text{Re} \le 5 \times 10^4$, which is of the form

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} \tag{18}$$

Since the friction factor correlation proposed to be used in Gnielinski's correlation does not cover the lower range of transitional Reynolds number, Abraham et al. [23] proposed a simple approach. The recommendation is to continue to use Eq. (17) along with friction factor values determined from the following expression for horizontal smooth circular tubes. Their friction factor correlation is valid for 2300 < Re < 4500. The correlation is of the form

$$f = 3.03 \times 10^{-12} \text{ Re}^3 - 3.67 \times 10^{-8} \text{ Re}^2 + 1.46 \times 10^{-4} \text{ Re} - 0.151$$
(19)

Abraham et al. [23] in their work provided reliable heat transfer information for the low-Reynolds-number end of the transition region. They used friction factor correlations appropriate to that Reynolds number range and their correlation is based on a new flow-regime model capable of smoothly and automatically bridging between all flow regimes. They also developed friction factor correlations for the transition region in parallel plate channels and straight pipes with conical diffusers. Other related publications in the transition region by Abraham and co-workers [24–26] will be briefly discussed next.

In Ref. [24], Abraham and co-workers numerically predict the breakdown of laminar pipe flow into transitional intermittency. They show that subsequent to transitional intermittency, two types of fully developed flow (intermittent or fully turbulent) may be present. They predict the fully developed friction factors as a function of Reynolds number throughout the intermittent region. These predicted friction factors bridges the gap between the well-established laminar and turbulent friction factors.

In Ref. [25], Abraham and co-workers provide a numerical model that is capable of predicting developing and fully developed heat transfer in all possible flow regimes (laminar, transitional, and turbulent) that can take place when fluid flows in a straight tube. Their model is also capable of predicting the variation of the local heat transfer coefficient along the length of the tube as well as the fully developed values. In their analysis, they also considered the two basic thermal boundary conditions of uniform heat flux (UHF) and uniform wall temperature (UWT). In the developing region, the UHF heat transfer coefficient was 25% higher than the UWT heat transfer coefficient. For the fully developed turbulent region, the respective heat transfer coefficients were almost identical. However, in the intermittent region, they differed by 25%.

Abraham and co-workers in their most recent publication [26] applied a computational turbulent transition model to internal flow problems that can be laminar, turbulent, transitional, or any combination. The model was successfully applied to circular tubes, parallel plate channels, and circular tubes with an abrupt change in diameters in the Reynolds number range of 100 to 100,000. Their model predicted the fully developed friction factors for the entire range of Reynolds numbers for both laminar and turbulent regimes. Based on their results, they concluded that the intermittency model can be used regardless of the status of the flow for both round and non-round duct shapes.

10. Transitional flow heat transfer works of Gnielinski

The calculation of Nusselt number or heat transfer coefficient in the transition region was further studied in detail by Gnielinski [27]. According to his study, the equations used to calculate the forced convection Nusselt number in tubes do not agree in the transition region between laminar and turbulent flow. There is a gap at Re = 2300. He developed a linear interpolation in the transition region between the Nusselt numbers at Re = 2300 and Re = 4000. His proposed linear interpolation connects the laminar flow equation at Re = 2300 with the turbulent flow equation at Re = 4000 and avoids the gap between the equations for laminar and turbulent flows and leads to a method of calculation providing heat transfer coefficients for all flow regimes in tubes. The linear interpolation is based on the following equations:

$$Nu = (1 - \gamma)Nu_{lam,2300} + \gamma Nu_{turb,4000}$$
(20)

with

$$\gamma = \frac{\text{Re} - 2300}{4000 - 2300} \text{ and } 0 \le \gamma \le 1$$
(21)

For the case of "uniform wall heat flux" boundary condition and no influence of free-convection, $Nu_{lam,2300}$ used in Eq. (20) is calculated from the following equations at Re = 2300:

$$Nu_{H} = \left[Nu_{H,1}^{3} + 0.6^{3} + \left(Nu_{H,2} - 0.6 \right)^{3} + Nu_{H,3}^{3} \right]^{1/3}$$
(22)

with $Nu_{H,1}$, $Nu_{H,2}$ and $Nu_{H,3}$ are calculated from the following equations:

$$Nu_{H,1} = 4.354, Nu_{H,2} = 1.953 \sqrt[3]{RePr(\frac{D}{L})},$$

$$Nu_{H,3} = 0.924 \sqrt[3]{Pr} \sqrt{Re(D/L)}$$
(23)

Gnielinski [27] provided equations similar to Eqs. (22) and (23) for the case of "uniform wall temperature" boundary condition.

The value of $Nu_{turb,4000}$ used in Eq. (20) is calculated from the following equation at Re = 4000:

Nu =
$$\frac{(f/8)(Re - 1000) Pr}{1 + 12.7 (f/8)^{0.5} (Pr^{2/3} - 1)} \left[1 + \left(\frac{D}{L}\right)^{\frac{2}{3}} \right] \left(\frac{Pr}{Pr_w}\right)^{0.11}$$
 (24)

To calculate the friction factor (f) in Eq. (24), use Eq. (18). Eq. (24) is a modified version of Eq. (17). Gnielinski [27] modified Eq. (17) by adding the correction term $[1 + (D/L)^{2/3}]$ to account for the effect of pipe length and the ratio $(Pr/Pr_w)^{0.11}$ to account for the variation of the properties of liquids. For gases, the property correction term should be replaced by $(T_b/T_w)^{0.45}$ for $0.5 < \frac{T_b}{T} < 1.5$.

According to Gnielinski [27], no experimental data were found to allow checking the Nusselt numbers obtained by the proposed linear interpolation method between Re = 2300-4000. However, in addition to the earlier experimental data [21], new experimental data by various researchers were used to confirm the validity of Eq. (24) for heat transfer in turbulent flow used for linear interpolation [27]. It should be noted that Gnielinski's transition Reynolds number range does not account for the influence of inlet configuration on the start and end of the transition region and the Nusselt numbers predicted from their proposed method do not account for the influence of secondary flow in the laminar and lower transition regions. Strictly speaking, their assumed transition Reynolds number range of 2300–4000 is applicable to a very steady and uniform entry flow with a rounded entrance.

11. Transitional flow heat transfer work of Taler

Taler [28] developed a correlation for the Nusselt number in terms of the friction factor, Reynolds number $(2300 \leq \text{Re} \leq 10^6)$ and Prandtl number $(0.1 \leq \text{Pr} \leq 1000)$ which is valid for transitional flow and fully

developed turbulent flow. His proposed correlation is valid for both a uniform surface temperature and uniform wall heat flux boundary condition. In the development of his correlation, he assumed Nusselt number in the transitional and turbulent flow regime is the sum of the laminar component and a turbulent component. The form of his correlation was selected in such a way that for Re = 2300, his assumed start of transition from laminar to transitional flow, the Nusselt number should change continuously. His developed heat transfer relationship correlated well with the experimental data he used.

As it was pointed out in the previous section in reference to the heat transfer correlation of Gnielinski for the transition region, Taler's correlation does not account for the significant effect of inlet configuration on the transition Reynolds number range and the fact that Nusselt number values in the laminar and lower transition regions are significantly affected by the secondary flow effect (free convection). As his proposed correlation is not a function of Grashof number, it is more appropriate for forced convection conditions, rather than mixed convection conditions.

12. Application of Ghajar and co-workers recommended friction factor and heat transfer correlations

This section provides two detailed worked out examples that illustrate the use of the transitional friction factor and heat transfer correlations recommended by Ghajar and co-workers.

Example 1: Non-isothermal apparent friction factor in the transition region

A 4-m-long tube is subjected to uniform wall heat flux. The tube has an inside diameter of D = 0.0149 m and a volume flow rate of Q = 7.8×10^{-5} m³/s. The liquid flowing inside the tube is ethylene glycol-distilled water mixture with a mass fraction of 0.5. Determine the apparent (developing) friction factor at the location x/D = 20 if the inlet configuration of the tube is (a) re-entrant and (b) square-edged. At this location, the local Grashof number is Gr = 28,090 and the properties of the mixture of distilled water and ethylene glycol (with a mass fraction of 0.5) are Pr = 20.9, v = 2.33×10^{-6} m²/s, and $\mu_b/\mu_w = 1.25$.

Solution: For the calculation of the non-isothermal apparent (developing) friction factor, it is necessary to determine the flow regime before making a

Example 1: Non-isothermal apparent friction factor in the transition region (*cont'd*)

decision about which friction factor relation to use. The Reynolds number at the specified location is

$$\operatorname{Re} = \frac{(Q/A_c)D}{\nu} = \frac{\left[\left(7.8 \times 10^{-5} \,\mathrm{m}^3/\mathrm{s}\right) / \left(1.744 \times 10^{-4} \,\mathrm{m}^2\right)\right] (0.0149 \,\mathrm{m})}{2.33 \times 10^{-6} \,\mathrm{m}^2/\mathrm{s}} = 2860$$

since $A_c = \pi D^2/4 = \pi (0.0149 \text{ m})^2/4 = 1.744 \times 10^{-4} \text{ m}^2$.

From Table 1, we see that for both inlets, with heating the flow is in the transition region. Therefore, Eq. (6) applies.

$$f_{\mathrm{app, tr, heat}} = f_{\mathrm{app, tr, iso}} \left(\frac{\mu_b}{\mu_W} \right)^m$$

where f_{app, tr, iso} is calculated from Eq. (5) and C_{f, tr, iso} is based on Eq. (4):

$$f_{\text{app, tr, iso}} = C_{\text{f, tr, iso}} \left[1 + \left(\frac{c}{x/D} \right) \right]$$
$$C_{\text{f, tr, iso}} = \left(\frac{16}{\text{Re}} \right) \left\{ \left[1 + \left(0.0049 \text{Re}^{0.75} \right)^a \right]^{1/a} + b \right\}$$

Introducing Eqs. (4) and (5) into Eq. (6), the final equation for the calculation of friction factor in this case becomes

$$f_{\text{app, tr, heat}} = \left\{ \left(\frac{16}{\text{Re}}\right) \left[\left(1 + \left(0.0049 \text{Re}^{0.75}\right)^a \right)^{1/a} + b \right] \left[1 + \left(\frac{c}{x/D}\right) \right] \right\} \left(\frac{\mu_b}{\mu_w}\right)^m$$

Note that the values of m and the constants a, b, and c are inlet-dependent and are provided with the appropriate equations.

(a) Re-entrant inlet:

$$\begin{split} m &= -1.8 + 0.46 \mbox{ Gr}^{-0.13} \mbox{ Pr}^{0.41} = -1.8 + 0.46 \mbox{ (28,090)}^{-0.13} \mbox{ (20.9)}^{0.41} = -1.3776. \\ a &= 0.52, \mbox{ b} = -3.47, \mbox{ c} = 4.8 \end{split}$$

$$f_{\text{app, tr, heat}} = \left\{ \left(\frac{16}{2860}\right) \left[\left(1 + \left(0.0049 \times 2860^{0.75}\right)^{0.52}\right)^{1/0.52} - 3.47 \right] \\ \left[1 + \left(\frac{4.8}{20}\right) \right] \right\} (1.25)^{-1.3776}$$

f_{app, tr, heat} = 0.009820

(b) Square-edged inlet:

$$\begin{split} m &= -1.13 + 0.48 \ \text{Gr}^{-0.15} \ \text{Pr}^{0.55} = -1.13 + 0.48 \ (28,090)^{-0.15} \\ (20.9)^{0.55} &= -0.58041. \end{split}$$

(Continued)

Example 1: Non-isothermal apparent friction factor in the transition region (*cont'd*)

a = 0.50, b = -4.0, c = 3.0

$$f_{\text{app, tr, heat}} = \left\{ \left(\frac{16}{2860}\right) \left[\left(1 + \left(0.0049 \times 2860^{0.75}\right)^{0.5}\right)^{1/0.5} - 4 \right] \right] \\ \left[1 + \left(\frac{3}{20}\right) \right] \right\} (1.25)^{-0.58041}$$

f_{app, tr, heat} = 0.009523

Discussion: If the flow is considered to be isothermal, in the above calculations the viscosity ratio should be set to unity (m = 0) or one should use Eq. (5). The apparent friction factor for the re-entrant inlet would be $f_{app,tr,iso} = 0.01335$ (about a 36% increase), and for the square-edged inlet it would be $f_{app,tr,iso} = 0.01084$ (about a 14% increase). Heating causes a decrease in the friction factor.

Example 2: Mixed convection heat transfer in the transition region

An ethylene glycol-distilled water mixture with a mass fraction of 0.6 and a volume flow rate of Q = 2.6×10^{-4} m³/s flows inside a tube with an inside diameter of D = 0.0158 m with a uniform wall heat flux boundary condition. For this flow, determine the Nusselt number at the location x/D = 90 if the inlet configuration of the tube is (a) re-entrant, (b) square-edged, and (c) bell-mouth. At this location, the local Grashof number is Gr = 51,770. The properties of the ethylene glycol-distilled water mixture (with a mass fraction of 0.6) at the location of interest are Pr = 29.2, v = 3.12×10^{-6} m²/s, and $\mu_b/\mu_w = 1.77$.

Solution: For a tube with a known diameter and volume flow rate, the type of flow regime is determined before any decision about which Nusselt number correlation to use is made.

The Reynolds number at the specified location is

$$\operatorname{Re} = \frac{(Q/A_c) D}{\nu} = \frac{\left[\left(2.6 \times 10^{-4} \text{ m}^3/\text{s} \right) / \left(1.961 \times 10^{-4} \text{ m}^2 \right) \right] (0.0158 \text{ m})}{3.12 \times 10^{-6} \text{ m}^2/\text{s}} = 6713$$

since $A_c = \pi D^2/4 = \pi (0.0158 \text{ m})^2/4 = 1.961 \times 10^{-4} \text{ m}^2$.

From Table 2 with x/D = 90 and Re = 6713, the flow for all three inlet configurations is in the transition region. Therefore, Eq. (7) should be used with the constants a, b, and c given in Table 3. However, Nu_l and Nu_t are the inputs to Eq. (7), and they have to be evaluated first from Eqs. (8) and (9),

Example 2: Mixed convection heat transfer in the transition region (*cont'd*)

respectively. It should be mentioned that the correlations for Nu_l and Nu_t have no inlet dependency.

From Eq. (8), the laminar Nusselt number is

Nu_I = 1.24
$$\left[\left(\frac{\text{RePr}D}{x} \right) + 0.025 \ (\text{GrPr})^{0.75} \right]^{1/3} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

= 1.24 $\left[\left(\frac{(6713)(29.2)}{90} \right) + 0.025 \ [(51770)(29.2)]^{0.75} \right]^{1/3} (1.77)^{0.14} = 19.9$

From Eq. (9), the turbulent Nusselt number is

Nu_t = 0.023Re^{0.8}Pr^{0.385}
$$\left(\frac{x}{D}\right)^{-0.0054} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

= 0.023 (6713)^{0.8} (29.2)^{0.385} (90)^{-0.0054} (1.77)^{0.14} = 102.7

Then the transition Nusselt number can be determined from Eq. (7):

 $Nu_{trans} = Nu_{I} + \{ exp[(a - Re)/b] + Nu_{t}^{c} \}^{c}$

(a) Re-entrant inlet:

$$\mathsf{Nu}_{\mathsf{trans}} = \mathsf{19.9} + \big\{ \; \mathsf{exp}[(\mathsf{1766} - \mathsf{6713})/\mathsf{276}] + \mathsf{102.7}^{-0.955} \big\}^{-0.955} = \mathbf{88.2}$$

(b) Square-edged inlet:

$$Nu_{trans} = 19.9 + \left\{ exp[(2617 - 6713)/207] + 102.7^{-0.950} \right\}^{-0.950} = 85.3$$

(c) Bell-mouth inlet:

$$Nu_{trans} = 19.9 + \left\{ exp[(6628 - 6713)/237] + 102.7^{-0.980} \right\}^{-0.980} = 21.2$$

Discussion: It is worth mentioning that for the re-entrant and square-edged inlets, the flow behaves normally. For the bell-mouth inlet, the Nusselt number is low in comparison to the other two inlets. This is the case because of the unusual behavior of the bell-mouth inlet noted earlier (Fig. 14); that is, the boundary layer along the tube wall is at first laminar and then changes through a transition region to the turbulent condition.

13. Application of Meyer and co-workers transitional flow heat transfer correlations

This section provides two detailed worked out examples that illustrate the use of the transitional flow friction factor and heat transfer correlations recommended by Meyer and co-workers.

Example 3: Analogy between momentum and heat transfer

Water enters a heated smooth tube at 23 °C with a mass flow rate of 217.8 kg/h and exits at 27 °C. The tube is heated at a uniform wall heat flux of 3500 W/m² and the average surface temperature of the tube is measured to be 26.2 °C. The diameter and length of the tube are 11.5 mm and 8 m, respectively. The pressure drop across the tube length was measured to be 4 kPa. Determine the average heat transfer coefficient across the tube length using the relationship between pressure drop and heat transfer. In your calculations use the following properties of water at the bulk mean temperature of 25 °C: $\rho = 997.0 \text{ kg/m}^3$, $\mu = 0.891 \times 10^{-3} \text{ kg/m} \cdot \text{s}$, $k = 0.607 \text{ W/m} \cdot \text{K}$, $c_p = 4180 \text{ J/kg} \cdot \text{K}$, and Pr = 6.14.

Solution: The flow Reynolds number is

$$\operatorname{Re} = \frac{\dot{m}D}{\mu A_c} = \frac{\frac{217.8 \text{ kg/h}}{3600 \text{ s}} \times 0.0115 \text{ m}}{(0.891 \times 10^{-3} \text{ kg/m} \cdot \text{s}) (1.039 \times 10^{-4} \text{ m}^2)} \approx 7515$$

where $A_c = \pi D^2/4 = \pi (0.0115 \text{ m})^2/4 = 1.039 \times 10^{-4} \text{ m}^2$.

According to Table 5, based on the calculated Reynolds number of 7515, Eq. (12) should be used to calculate the Colburn *j*-factor from the friction factor.

The friction factor for this flow is calculated from the given pressure drop as

$$f = \Delta P \left(\frac{D}{L}\right) \left(\frac{\rho V^2}{2}\right) = (4 \text{ kPa}) \left(\frac{0.0115 \text{ m}}{8 \text{ m}}\right) \left(\frac{\left(997.0 \frac{\text{kg}}{\text{m}^3}\right) \left(0.584 \frac{\text{m}}{\text{s}}\right)^2}{2}\right)$$

= 0.0338

where $V = \dot{m}/\rho A_c = \left(\frac{217.8 \text{ kg/h}}{3600 \text{ s}}\right) / \left(997.0 \frac{\text{kg}}{\text{m}^3}\right) \left(1.039 \times 10^{-4} \text{ m}^2\right) = 0.584 \text{ m/s}$

The Colburn *j*-factor is calculated using Eq. (12) that describes the relationship between pressure drop and heat transfer for transitional, quasi-turbulent and turbulent flows as follows

$$\frac{(f/j)}{\Pr^{0.42}} = \frac{3.74\text{Re} - 8066}{\text{Re} - 2320} \rightarrow \frac{(0.0338/j)}{6.14^{0.42}} = \frac{3.74(7515) - 8066}{7515 - 2320} \rightarrow j = 4.089 \times 10^{-3}$$

The Nusselt number is calculated from the definition of Colburn *j*-factor

$$j = \frac{Nu}{\text{RePr}^{1/3}} \rightarrow Nu = j \text{ RePr}^{1/3} = (4.089 \times 10^{-3})(7515)(6.14)^{\frac{1}{3}} = 56.3$$

Example 3: Analogy between momentum and heat transfer (cont'd)

Finally, the heat transfer coefficient is calculated as

$$Nu = \frac{hD}{k} \rightarrow h = \frac{k(Nu)}{D} = \frac{0.607 \text{ W/m} \cdot \text{K} (56.3)}{0.0115 \text{ m}} = 2969.7 \text{ W/m}^2 \text{ K}$$

Discussion: The calculated heat transfer coefficient can be verified by using the Newton's law of cooling as

$$\dot{q} = h(T_w - T_b) \rightarrow h = \dot{q}/(T_w - T_b) = (3500 \text{ W/m}^2)/(26.2^{\circ}\text{C} - 25^{\circ}\text{C})$$

= 2916.7 W/m² K

Therefore, the estimated value of heat transfer coefficient using Eq. (12) is within 1.8% of the heat transfer coefficient obtained from the measured surface and fluid temperatures. Note that the given value of heat flux (3500 W/m²) was calculated from the conservation of energy equation for steady flow of a fluid in a tube expressed as $\dot{Q} = \dot{q}A_s = \dot{m}c_p(T_e - T_i)$.

Example 4: Transitional flow correlations for average Nusselt number and friction factor

Water flows through a 1-m-long tube heated at a uniform wall heat flux with a mass flow rate of 87 kg/h. The diameter of the tube is 11.5 mm and the bulk mean and surface temperatures are 21 °C and 24.8 °C, respectively. Determine the average Nusselt number and friction factor. In your calculations use the following properties of water at the bulk mean temperature of 21 °C: $\rho = 997.8 \text{ kg/m}^3$, $\mu = 0.98 \times 10^{-3} \text{ kg/m} \cdot \text{s}$, $\nu = 9.822 \times 10^{-7} \text{ m}^2/\text{s}$, $k = 0.6 \text{ W/m} \cdot \text{K}$, $\beta = 0.205 \times 10^{-3} \text{ 1/K}$, and Pr = 6.84.

Solution: The flow Reynolds number is

$$\operatorname{Re} = \frac{\dot{m}D}{\mu A_{c}} = \frac{\frac{87 \text{ kg/h}}{3600 \text{ s}} \times 0.0115 \text{ m}}{(0.98 \times 10^{-3} \text{ kg/m} \cdot \text{s}) (1.039 \times 10^{-4} \text{ m}^{2})} \approx 2730$$

where $A_c = \pi D^2 / 4 = \pi (0.0115 \text{ m})^2 / 4 = 1.039 \times 10^{-4} \text{ m}^2$.

The calculated Reynolds number of 2730 is greater than 2300, therefore the flow is most likely in the transitional flow regime. By making use of Eqs. (13) and (14) the transitional flow regime is determined as

$$\operatorname{Re}_{cr} = \left(0.1972\frac{x}{D} + 1156.7\right)Gr^{0.077} = \left(0.1972\frac{1 \text{ m}}{0.0115 \text{ m}} + 1156.7\right)12048^{0.077}$$
$$= 2420$$

and

$$\operatorname{Re}_{qt} = 2504 \ Gr^{0.018} = 2504 \ \times \ 12048^{0.018} = 2965$$

Example 4: Transitional flow correlations for average Nusselt number and friction factor (*cont'd*)

where the Grashof number (Gr) is

$$Gr = \frac{g\beta(T_w - T_b)D^3}{\nu^2} = \frac{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(0.205 \times 10^{-3} \ \frac{1}{K}\right)\left(24.8^{\circ}\text{C} - 21^{\circ}\text{C}\right)\left(0.0115 \ \text{m}\right)^3}{\left(9.822 \times 10^{-7} \ \text{m}^2/\text{s}\right)^2}$$
$$= 12048$$

Therefore, the flow through the tube is in the transitional flow regime ($2420 \le \text{Re} \le 2965$) and according to Table 6, Eqs. (15) and (16) can be used to determine the average Nusselt number and friction factor, respectively.

The average Nusselt number is calculated from Eq. (15) as

$$\overline{Nu} = (0.00108 \text{ Re} - 2.49) \text{ Gr}^{-0.04} \text{Pr}^2$$

$$\overline{Nu} = (0.00108 \times 2730 - 2.49) \times 12048^{-0.04} \times 6.84^2 = 14.73$$

The average friction factor is calculated from Eq. (16) as

$$f = \left(\frac{3.74 \text{ Re} - 8066}{\text{Re} - 2320}\right) \frac{\overline{Nu}}{\text{Re} \text{ Pr}^{0.087}}$$
$$f = \left(\frac{3.74 \times 2730 - 8066}{2730 - 2320}\right) \frac{14.73}{2730 \times 6.84^{0.087}} = 0.024$$

Discussion: The application of Eqs. (15) and (16) for the determination of the average Nusselt number and friction factor depends on the determination of the boundaries of the transitional flow. That is the use of Eqs. (13) and (14) for the establishment of the start and end of the transitional flow regime. These equations show that the boundaries of the transitional flow are not only a function of axial location, but also free convection (secondary flow) effects.

14. Application of Abraham and co-workers and Gnielinski recommended friction factor and heat transfer correlations

The following example of forced convection heat transfer in the transition region will be solved using two different methods. The simple method suggested by Abraham and co-workers [23] and the linear interpolation method recommended by Gnielinski [27].

Example 5: Forced convection heat transfer in the transition region

Water is to be heated from 15 °C to 65 °C as it flows with an average velocity of 0.07 m/s, through a 3-cm-internal-diameter 5-m-long tube. The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. Determine the average Nusselt number for this flow. The properties of water at the bulk mean temperature of 40 °C are: $\nu = 0.658 \times 10^{-6} \text{ m}^2/\text{s}$ and Pr = 4.32.

Solution 1: Method of Abraham and co-workers [23]. The flow Reynolds number is

$$\operatorname{Re} = \frac{VD}{\nu} = \frac{(0.07 \text{ m/s})(0.03 \text{ m})}{0.658 \times 10^{-6} \text{ m}^2/\text{s}} \approx 3192$$

which is between 2300 and 4000. Therefore, the flow is in the transitional region. With the Reynolds number known, calculate the friction factor from Eq. (19). The friction factor for transitional flow in a smooth tube with Re = 3192 is

$$f = 3.03 \times 10^{-12} \text{Re}^3 - 3.67 \times 10^{-8} \text{Re}^2 + 1.46 \times 10^{-4} \text{Re} - 0.151 = 0.0396$$

To calculate the transitional Nusselt number, use Eq. (17). The Nusselt number in the transitional flow with Re = 3192, Pr = 4.32, and f = 0.0396 is

Nu =
$$\frac{(f/8) (\text{Re} - 1000) \text{Pr}}{1 + 12.7(f/8)^{0.5} (\text{Pr}^{2/3} - 1)} = 18.9$$

Solution 2: Method of Gnielinski [27].

To calculate the uniform heat flux Nusselt number (Nu_H) in the transition region, start with Eq. (20)

$$Nu = (1 - \gamma)Nu_{lam,2300} + \gamma Nu_{turb,4000}$$

where from Eq. (21) we have $\gamma=\frac{Re-2300}{4000-2300}=\frac{3192-2300}{4000-2300}\approx 0.525$

Next, for the case of "uniform wall heat flux" boundary condition and no influence of free-convection, $Nu_{lam,2300}$ used in Eq. (20) is calculated from the following equation (Eq. 22) at Re = 2300

$$Nu_{H} = \ \left[Nu_{H,1}^{3} + 0.6^{3} + \left(Nu_{H,2} - 0.6 \right)^{3} + \ Nu_{H,3}^{3} \ \right]^{1/3}$$

where the calculated values $Nu_{H,1}$, $Nu_{H,2}$ and $Nu_{H,3}$ from Eq. (23) with Re = 2300, Pr = 4.32, and D/L = 0.03/5 = 0.006, are

$$\begin{split} \text{Nu}_{\text{H},1} = & 4.354, \text{Nu}_{\text{H},2} = 1.953 \sqrt[3]{\text{RePr}} \left(\text{D}/\text{L}\right) = 7.63, \\ & \text{Nu}_{\text{H},3} = 0.924 \sqrt[3]{\text{Pr}} \sqrt{\text{Re}} \left(\text{D}/\text{L}\right) = 5.59 \end{split}$$

(Continued)

Example 5: Forced convection heat transfer in the transition region (*cont'd*)

Substituting the values for $Nu_{H,1}$, $Nu_{H,2}$ and $Nu_{H,3}$ from Eq. (23) in Eq. (22), we have

$$Nu_{lam,2300} = Nu_{H} = \left[(4.354)^{3} + 0.6^{3} + (7.63 - 0.6)^{3} + (5.59)^{3} \right]^{1/3} = 8.46$$

Next, the value of $Nu_{turb,4000}$ used in Eq. (20) is calculated from Eq. (24) at Re = 4000, with Pr = 4.32, D/L = 0.03/5 = 0.006 and Pr/Pr_w ≈ 1

$$Nu_{turb,4000} = Nu = \frac{(f/8)(Re - 1000) Pr}{1 + 12.7 (f/8)^{0.5} (Pr^{2/3} - 1)} \left[1 + \left(\frac{D}{L}\right)^{\frac{2}{3}}\right] \left(\frac{Pr}{Pr_w}\right)^{0.17}$$

= 27.6

where the friction factor (f) was calculated from Eq. (18) with Re = 4000 to be

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} = 0.04144$$

Substituting the calculated values of γ , $Nu_{lam,2300}$ and $Nu_{turb,4000}$ in Eq. (20), the Nusselt number in the transitional flow is

$$\begin{split} Nu \!=\! (1 - \gamma) Nu_{\text{lam,2300}} + \gamma Nu_{\text{turb,4000}} &= (1 - 0.525)(8.46) + (0.525)(27.6) \\ &= \textbf{18.5} \end{split}$$

Discussion: The simple approach proposed by Abraham and co-workers [23] provided comparable result to the more complicated method of Gnielinski [27]. Assuming Gnielinski's method provides the more accurate results; the two methods provide almost identical results and differ only by 2%.

15. Concluding remarks

The friction factor and heat transfer results of Ghajar and co-workers and Meyer and co-workers presented in this chapter clearly showed that the type of inlet configuration (re-entrant, square-edged, and bell-mouth) can significantly influence the start and end of the transition region. The inlet that caused the most disturbance (re-entrant) produced an early transition, followed by the square-edged inlet, and finally the bell-mouth inlet. Therefore, the transition Reynolds number range can be manipulated by using different inlet configurations. Furthermore, natural convection effect (buoyancy/secondary flow) significantly affected the thermal entrance length, laminar and lower transition along the tube length, as well as the local pressure drop (friction factor) and heat transfer (Nusselt number) characteristics in the laminar and lower transitional flow regime. In addition, the pressure drop and heat transfer characteristics of developing and fully developed flow were considerably different, in particular in the transition region.

Understanding of pressure drop and heat transfer behavior in the transition region is very critical to proper design and operation of heat exchangers. Because of our limited understanding of flow behavior in the transition region, the designers are typically advised not to design the heat transfer equipment that operates in this region. However, this is not always possible as operating conditions change or as erosion and/or scaling occurs, the heat transfer equipment might operate in the transitional flow regime. The lack of experimental data in the transitional flow regime is most probably the main reason for availability of little design information on transitional flow and our limited understanding of this flow regime.

The works reviewed in this chapter showed that it is not necessary to avoid this regime and that some repeatable and reliable design data is available. The friction factor and Nusselt number transitional flow correlations, as well as the flow regime maps introduced in this chapter should improve our fundamental understanding of mixed convection in developing and fully developed flow and enable the heat exchanger designers to optimize their designs. As was shown in this chapter, some aspects of the transitional flow regime have been investigated more than the others. Therefore, more pressure drop and heat transfer work needs to be done to make available a larger database with correlations. The importance of this topic is demonstrated by the number of independent researchers and duration of focus. In fact, this work continues to be the subject of experimental, numerical, and analytical studies [29–33] that are additional to those discussed in this chapter.

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