



# HEAT AND MASS TRANSFER

A Practical Approach

Third Edition

**YUNUS A. ÇENGEL**

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EDITION

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A PRACTICAL APPROACH

**YUNUS A. ÇENGEL**  
*University of Nevada,  
Reno*



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HEAT AND MASS TRANSFER: A PRACTICAL APPROACH, THIRD EDITION

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# PREFACE

## BACKGROUND

**H**eat and mass transfer is a basic science that deals with the rate of transfer of thermal energy. It has a broad application area ranging from biological systems to common household appliances, residential and commercial buildings, industrial processes, electronic devices, and food processing. Students are assumed to have an adequate background in calculus and physics. The completion of first courses in thermodynamics, fluid mechanics, and differential equations prior to taking heat transfer is desirable. However, relevant concepts from these topics are introduced and reviewed as needed.

## OBJECTIVES

This book is intended for undergraduate engineering students in their sophomore or junior year, and as a reference book by practicing engineers. The objectives of this text are

- To cover the *basic principles* of heat transfer.
- To present a wealth of real-world *engineering examples* to give students a feel for how heat transfer is applied in engineering practice.
- To develop an *intuitive understanding* of heat transfer by emphasizing the physics and physical arguments.

It is our hope that this book, through its careful explanations of concepts and its use of numerous practical examples and figures, helps the students develop the necessary skills to bridge the gap between knowledge and the confidence for proper application of that knowledge.

In engineering practice, an understanding of the mechanisms of heat transfer is becoming increasingly important since heat transfer plays a crucial role in the design of vehicles, power plants, refrigerators, electronic devices, buildings, and bridges, among other things. Even a chef needs to have an intuitive understanding of the heat transfer mechanism in order to cook the food “right” by adjusting the rate of heat transfer. We may not be aware of it, but we already use the principles of heat transfer when seeking thermal comfort. We insulate our bodies by putting on heavy coats in winter, and we minimize heat gain by radiation by staying in shady places in summer. We speed up the cooling of hot food by blowing on it and keep warm in cold weather by cuddling up and thus minimizing the exposed surface area. That is, we already use heat transfer whether we realize it or not.

## GENERAL APPROACH

This text is the outcome of an attempt to have a textbook for a practically oriented heat transfer course for engineering students. The text covers the



- Communicates directly to the minds of tomorrow's engineers in a *simple yet precise* manner.
- Leads students toward a clear understanding and firm grasp of the *basic principles* of heat transfer.
- Encourages *creative thinking* and development of a *deeper understanding* and *intuitive feel* for heat transfer.
- Is *read* by students with *interest* and *enthusiasm* rather than being used as an aid to solve problems.

Special effort has been made to appeal to students' natural curiosity and to help them explore the various facets of the exciting subject area of heat transfer. The enthusiastic response we received from the users of prior editions—from small colleges to large universities all over the world—indicates that our objectives have largely been achieved. It is our philosophy that the best way to learn is by practice. Therefore, special effort is made throughout the book to reinforce material that was presented earlier.

Yesterday's engineer spent a major portion of his or her time substituting values into the formulas and obtaining numerical results. However, now formula manipulations and number crunching are being left mainly to the computers. Tomorrow's engineer will have to have a clear understanding and a firm grasp of the *basic principles* so that he or she can understand even the most complex problems, formulate them, and interpret the results. A conscious effort is made to emphasize these basic principles while also providing students with a perspective at how computational tools are used in engineering practice.

## NEW IN THIS EDITION

All the popular features of the previous edition are retained while new ones are added. With the exception of the coverage of the theoretical foundations of transient heat conduction and moving the chapter "Cooling of Electronic Equipment" to the Online Learning Center, the main body of the text remains largely unchanged. The most significant changes in this edition are highlighted below.

### A NEW TITLE

The title of the book is changed to *Heat and Mass Transfer: A Practical Approach* to attract attention to the coverage of mass transfer. All topics related to mass transfer, including mass convection and vapor migration through building materials, are introduced in one comprehensive chapter (Chapter 14).

### EXPANDED COVERAGE OF TRANSIENT CONDUCTION

The coverage of Chapter 4, Transient Heat Conduction, is now expanded to include (1) the derivation of the dimensionless Biot and Fourier numbers by nondimensionalizing the heat conduction equation and the boundary and initial

conditions, (2) the derivation of the analytical solutions of a one-dimensional transient conduction equation using the method of separation of variables, (3) the derivation of the solution of a transient conduction equation in the semi-infinite medium using a similarity variable, and (4) the solutions of transient heat conduction in semi-infinite mediums for different boundary conditions such as specified heat flux and energy pulse at the surface.

### FUNDAMENTALS OF ENGINEERING (FE) EXAM PROBLEMS

To prepare students for the Fundamentals of Engineering Exam (that is becoming more important for the outcome-based ABET 2000 criteria) and to facilitate multiple-choice tests, about 250 *multiple-choice problems* are included in the end-of-chapter problem sets. They are placed under the title "Fundamentals of Engineering (FE) Exam Problems" for easy recognition. These problems are intended to check the understanding of fundamentals and to help readers avoid common pitfalls.

### MICROSCALE HEAT TRANSFER

Recent inventions in micro and nano-scale systems and the development of micro and nano-scale devices continues to pose new challenges, and the understanding of the fluid flow and heat transfer at such scales is becoming more and more important. In Chapter 6, microscale heat transfer is presented as a Topic of Special Interest.

### THREE ONLINE APPLICATION CHAPTERS

The application chapter "Cooling of Electronic Equipment" (Chapter 15) is now moved to the Online Learning Center together with two new chapters "Heating and Cooling of Buildings" (Chapter 16) and "Refrigeration and Freezing of Foods" (Chapter 17).

### CONTENT CHANGES AND REORGANIZATION

With the exception of the changes already mentioned, minor changes are made in the main body of the text. Nearly 400 new problems are added, and many of the existing problems are revised. The noteworthy changes in various chapters are summarized here for those who are familiar with the previous edition.

- The title of Chapter 1 is changed to "Introduction and Basic Concepts." Some artwork is replaced by photos, and several review problems on the first law of thermodynamics are deleted.
- Chapter 4 "Transient Heat Conduction" is revised greatly, as explained previously, by including the theoretical background and the mathematical details of the analytical solutions.
- Chapter 6 now has the Topic of Special Interest "Microscale Heat Transfer" contributed by Dr. Subrata Roy of Kettering University.
- Chapter 8 now has the Topic of Special Interest "Transitional Flow in Tubes" contributed by Dr. Afshin Ghajar of Oklahoma State University.
- Chapter 13 "Heat Exchangers" is moved up as Chapter 11 to succeed "Boiling and Condensation" and to precede "Radiation."
- In the appendices, the values of some physical constants are updated, and Appendix 3 "Introduction to EES" is moved to the enclosed CD and the Online Learning Center.

## SUPPLEMENTS

The following supplements are available to the adopters of the book.

### **ENGINEERING EQUATION SOLVER (EES) CD-ROM**

(Limited Academic Version packaged free with every new copy of the text) Developed by Sanford Klein and William Beckman from the University of Wisconsin–Madison, this software combines equation-solving capability and engineering property data. EES can do optimization, parametric analysis, and linear and nonlinear regression, and provides publication-quality plotting capabilities. Thermodynamic and transport properties for air, water, and many other fluids are built in, and EES allows the user to enter property data or functional relationships. Some problems are solved using EES, and complete solutions together with parametric studies are included on the enclosed CD-ROM. To obtain the full version of EES, contact your McGraw-Hill representative or visit [www.mhhe.com/ees](http://www.mhhe.com/ees).

### **ONLINE LEARNING CENTER ([www.mhhe.com/cengel](http://www.mhhe.com/cengel))**

Web support is provided for the text on our Online Learning Center. Visit this web site for general text information, errata, and author information. The site also includes resources for students including a list of helpful web links. The instructor side of the site includes the solutions manual, the text's images in PowerPoint form, and more!

### **COSMOS CD-ROM**

(Available to instructors only)

The instructor CD provides electronic solutions delivered via our database management tool. McGraw-Hill's COSMOS (Complete Online Solutions Manual Organization System) allows instructors to streamline the creation of assignments, quizzes, and tests by using problems and solutions from the textbook—as well as their own custom material. Contact your McGraw-Hill representative to obtain a copy.

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**Yunus A. Çengel**



# INTERNAL FORCED CONVECTION

Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications. The fluid in such applications is forced to flow by a fan or pump through a flow section that is sufficiently long to accomplish the desired heat transfer. In this chapter we pay particular attention to the determination of the *friction factor* and *convection coefficient* since they are directly related to the *pressure drop* and *heat transfer rate*, respectively. These quantities are then used to determine the pumping power requirement and the required tube length.

There is a fundamental difference between external and internal flows. In *external flow*, considered in Chapter 7, the fluid has a free surface, and thus the boundary layer over the surface is free to grow indefinitely. In *internal flow*, however, the fluid is completely confined by the inner surfaces of the tube, and thus there is a limit on how much the boundary layer can grow.

We start this chapter with a general physical description of internal flow, and the *average velocity* and *average temperature*. We continue with the discussion of the *hydrodynamic* and *thermal entry lengths*, *developing flow*, and *fully developed flow*. We then obtain the velocity and temperature profiles for fully developed laminar flow, and develop relations for the friction factor and Nusselt number. Finally we present empirical relations for developing and fully developed flows, and demonstrate their use.

## OBJECTIVES

When you finish studying this chapter, you should be able to:

- Obtain average velocity from a knowledge of velocity profile, and average temperature from a knowledge of temperature profile in internal flow,
- Have a visual understanding of different flow regions in internal flow, such as the entry and the fully developed flow regions, and calculate hydrodynamic and thermal entry lengths,
- Analyze heating and cooling of a fluid flowing in a tube under constant surface temperature and constant surface heat flux conditions, and work with the logarithmic mean temperature difference,
- Obtain analytic relations for the velocity profile, pressure drop, friction factor, and Nusselt number in fully developed laminar flow, and
- Determine the friction factor and Nusselt number in fully developed turbulent flow using empirical relations, and calculate the pressure drop and heat transfer rate.

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Then the logarithmic mean temperature difference and the rate of heat loss from the air become

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln \frac{T_s - T_e}{T_s - T_i}} = \frac{80 - 71.3}{\ln \frac{60 - 71.3}{60 - 80}} = -15.2^\circ\text{C}$$

$$\dot{Q} = hA_s \Delta T_{\ln} = (13.5 \text{ W/m}^2 \cdot ^\circ\text{C})(6.4 \text{ m}^2)(-15.2^\circ\text{C}) = -1313 \text{ W}$$

Therefore, air will lose heat at a rate of 1313 W as it flows through the duct in the attic.

**Discussion** The average fluid temperature is  $(80 + 71.3)/2 = 75.7^\circ\text{C}$ , which is sufficiently close to  $80^\circ\text{C}$  at which we evaluated the properties of air. Therefore, it is not necessary to re-evaluate the properties at this temperature and to repeat the calculations.

## TOPIC OF SPECIAL INTEREST

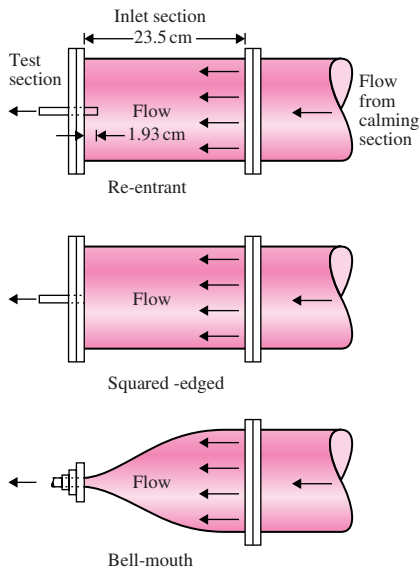
### Transitional Flow in Tubes\*

An important design problem in industrial heat exchangers arises when flow inside the tubes falls into the transition region. In practical engineering design, the usual recommendation is to avoid design and operation in this region; however, this is not always feasible under design constraints. The usually cited transitional Reynolds number range of about 2300 (onset of turbulence) to 10,000 (fully turbulent condition) applies, strictly speaking, to a very steady and uniform entry flow with a rounded entrance. If the flow has a disturbed entrance typical of heat exchangers, in which there is a sudden contraction and possibly even a re-entrant entrance, the transitional Reynolds number range will be much different.

Ghajar and coworkers in a series of papers (listed in the references) have experimentally investigated the inlet configuration effects on the fully developed transitional pressure drop under isothermal and heating conditions; and developing and fully developed transitional forced and mixed convection heat transfer in circular tubes. Based on their experimental data, they have developed practical and easy to use correlations for the friction coefficient and the Nusselt number in the transition region between laminar and turbulent flows. This section provides a brief summary of their work in the transition region.

### Pressure Drop in the Transition Region

Pressure drops are measured in circular tubes for fully developed flows in the transition regime for three types of inlet configurations shown in Fig. 8–32: re-entrant (tube extends beyond tubesheet face into head of



**FIGURE 8–32**

Schematic of the three different inlet configurations.

\*This section is contributed by Professor Afshin J. Ghajar of Oklahoma State University.

distributor), square-edged (tube end is flush with tubesheet face), and bell-mouth (a tapered entrance of tube from tubesheet face) under isothermal and heating conditions, respectively. The widely used expressions for the friction factor  $f$  (also called the Darcy friction factor) or the friction coefficient  $C_f$  (also called the Fanning friction factor) in laminar and turbulent flows with heating are

$$f_{\text{lam}} = 4C_{f,\text{lam}} = 4 \left( \frac{16}{\text{Re}} \right) \left( \frac{\mu_b}{\mu_s} \right)^m \quad (8-79)$$

$$f_{\text{turb}} = 4C_{f,\text{turb}} = 4 \left( \frac{0.0791}{\text{Re}^{0.25}} \right) \left( \frac{\mu_b}{\mu_s} \right)^m \quad (8-80)$$

where the factors at the end account for the wall temperature effect on viscosity. The exponent  $m$  for laminar flows depends on a number of factors while for turbulent flows the most typically quoted value for heating is  $-0.25$ . The transition friction factor is given as (Tam and Ghajar, 1997)

$$f_{\text{trans}} = 4C_{f,\text{trans}} = 4 \left[ 1 + \left( \frac{\text{Re}}{A} \right)^B \right]^C \left( \frac{\mu_b}{\mu_s} \right)^m \quad (8-81)$$

where

$$m = m_1 - m_2 \text{Gr}^{m_3} \text{Pr}^{m_4} \quad (8-82)$$

and the Grashof number (Gr) which is a dimensionless number representing the ratio of the buoyancy force to the viscous force is defined as  $\text{Gr} = g\beta D^3(T_s - T_b)/\nu^2$  (see Chapter 9 for more details). All properties appearing in the dimensionless numbers  $C_f$ ,  $f$ , Re, Pr, and Gr are all evaluated at the bulk fluid temperature  $T_b$ . The values of the empirical constants in Eqs. 8-81 and 8-82 are listed in Table 8-5. The range of application of Eq. 8-81 for the transition friction factor is as follows:

*Re-entrant:*  $2700 \leq \text{Re} \leq 5500$ ,  $16 \leq \text{Pr} \leq 35$ ,  $7410 \leq \text{Gr} \leq 158,300$ ,  
 $1.13 \leq \mu_b/\mu_s \leq 2.13$

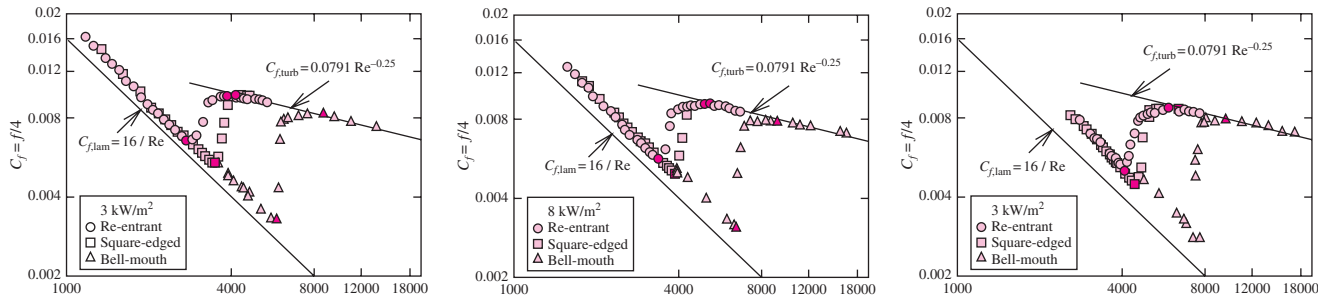
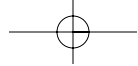
*Square-edged:*  $3500 \leq \text{Re} \leq 6900$ ,  $12 \leq \text{Pr} \leq 29$ ,  $6800 \leq \text{Gr} \leq 104,500$ ,  
 $1.11 \leq \mu_b/\mu_s \leq 1.89$

*Bell-mouth:*  $5900 \leq \text{Re} \leq 9600$ ,  $8 \leq \text{Pr} \leq 15$ ,  $11,900 \leq \text{Gr} \leq 353,000$ ,  
 $1.05 \leq \mu_b/\mu_s \leq 1.47$

**TABLE 8-5**

Constants for transition friction coefficient correlation

Inlet Geometry	A	B	C	$m_1$	$m_2$	$m_3$	$m_4$
Re-entrant	5840	-0.0145	-6.23	-1.10	0.460	-0.133	4.10
Square-edged	4230	-0.1600	-6.57	-1.13	0.396	-0.160	5.10
Bell-mouth	5340	-0.0990	-6.32	-2.58	0.420	-0.410	2.46



**FIGURE 8-33** Fully developed friction coefficients for three different inlet configurations and heat fluxes (filled symbols designate the start and end of the transition region for each inlet). (From Tam and Ghajar, 1997.)

These correlations captured about 82% of measured data within an error band of  $\pm 10\%$ , and 98% of measured data with  $\pm 20\%$ . For laminar flows with heating, Tam and Ghajar give the following constants for determining the exponent  $m$  in Eq. 8-79:  $m_1 = 1.65$ ,  $m_2 = 0.013$ ,  $m_3 = 0.170$ , and  $m_4 = 0.840$ , which is applicable over the following range of parameters:

$$1100 \leq Re \leq 7400, 6 \leq Pr \leq 36, 17,100 \leq Gr \leq 95,600,$$

$$\text{and } 1.25 \leq \mu_b/\mu_s \leq 2.40.$$

The fully developed friction coefficient results for the three different inlet configurations shown in Fig. 8-33 clearly establish the influence of heating rate on the beginning and end of the transition regions, for each inlet configuration. In the laminar and transition regions, heating seems to have a significant influence on the value of the friction coefficient. However, in the turbulent region, heating did not affect the magnitude of the friction coefficient. The significant influence of heating on the values of friction coefficient in the laminar and transition regions is directly due to the effect of secondary flow.

The isothermal friction coefficients for the three inlet types showed that the range of the Reynolds number values at which transition flow exists is strongly inlet-geometry dependent. Furthermore, heating caused an increase in the laminar and turbulent friction coefficients and an increase in the lower and upper limits of the isothermal transition regime boundaries. The friction coefficient transition Reynolds number ranges for the isothermal and nonisothermal (three different heating rates) and the three different inlets used in their study are summarized in Table 8-6.

**TABLE 8-6**

Transition Reynolds numbers for friction coefficient

Heat Flux	Re-entrant	Square-Edged	Bell-Mouth
0 kW/m <sup>2</sup> (isothermal)	2870 < Re < 3500	3100 < Re < 3700	5100 < Re < 6100
3 kW/m <sup>2</sup>	3060 < Re < 3890	3500 < Re < 4180	5930 < Re < 8730
8 kW/m <sup>2</sup>	3350 < Re < 4960	3860 < Re < 5200	6480 < Re < 9110
16 kW/m <sup>2</sup>	4090 < Re < 5940	4450 < Re < 6430	7320 < Re < 9560

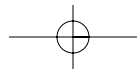




Figure 8–34 shows the influence of inlet configuration on the beginning and end of the isothermal fully developed friction coefficients in the transition region.

Note that the isothermal fully developed friction coefficients in the laminar, turbulent, and transition regions can be obtained easily from Eqs. 8–79, 8–80, and 8–81, respectively, by setting the exponent on the viscosity ratio correction to unity (i.e. with  $m = 0$ ).

### EXAMPLE 8–7 Nonisothermal Fully Developed Friction Coefficient in the Transition Region

A tube with a bell-mouth inlet configuration is subjected to  $8 \text{ kW/m}^2$  uniform wall heat flux. The tube has an inside diameter of  $0.0158 \text{ m}$  and a flow rate of  $1.32 \times 10^{-4} \text{ m}^3/\text{s}$ . The liquid flowing inside the tube is ethylene glycol-distilled water mixture with a mass fraction of  $0.34$ . The properties of the ethylene glycol-distilled water mixture at the location of interest are  $\text{Pr} = 11.6$ ,  $\nu = 1.39 \times 10^{-6} \text{ m}^2/\text{s}$  and  $\mu_b/\mu_s = 1.14$ . Determine the fully developed friction coefficient at a location along the tube where the Grashof number is  $\text{Gr} = 60,800$ . What would the answer be if a square-edged inlet is used instead?

**SOLUTION** A liquid mixture flowing in a tube is subjected to uniform wall heat flux. The friction coefficients are to be determined for the bell-mouth and square-edged inlet cases.

**Assumptions** Steady operating conditions exist.

**Properties** The properties of the ethylene glycol-distilled water mixture are given to be  $\text{Pr} = 11.6$ ,  $\nu = 1.39 \times 10^{-6} \text{ m}^2/\text{s}$  and  $\mu_b/\mu_s = 1.14$ .

**Analysis** For the calculation of the nonisothermal fully developed friction coefficient, it is necessary to determine the flow regime before making any decision regarding which friction coefficient relation to use. The Reynolds number at the specified location is

$$\text{Re} = \frac{(\dot{V}/A_c)D}{\nu} = \frac{[(1.32 \times 10^{-4} \text{ m}^3/\text{s})/(1.961 \times 10^{-4} \text{ m}^2)](0.0158 \text{ m})}{1.39 \times 10^{-6} \text{ m}^2/\text{s}} = 7651$$

since

$$A_c = \pi D^2/4 = \pi(0.0158 \text{ m})^2/4 = 1.961 \times 10^{-4} \text{ m}^2$$

From Table 8–6, we see that for a bell-mouth inlet and a heat flux of  $8 \text{ kW/m}^2$  the flow is in the transition region. Therefore, Eq. 8–81 applies. Reading the constants  $A$ ,  $B$ , and  $C$  and  $m_1$ ,  $m_2$ ,  $m_3$ , and  $m_4$  from Table 8–5, the friction coefficient is determined to be

$$\begin{aligned} C_{f,\text{trans}} &= \left[ 1 + \left( \frac{\text{Re}}{A} \right)^B \right]^C \left( \frac{\mu_b}{\mu_s} \right)^m \\ &= \left[ 1 + \left( \frac{7651}{5340} \right)^{-0.099} \right]^{-6.32} (1.14)^{-2.58 - 0.42 \times 60,800^{-0.41} \times 11.6^{2.46}} = \mathbf{0.010} \end{aligned}$$

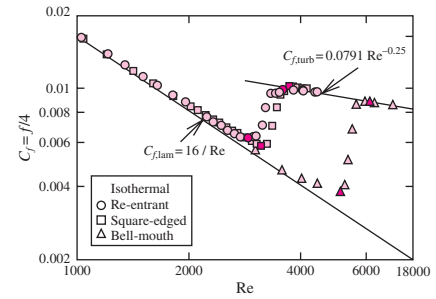


FIGURE 8–34

Influence of different inlet configurations on the isothermal fully developed friction coefficients (filled symbols designate the start and end of the transition region for each inlet).

(From Tam and Ghajar, 1997.)

**Square-Edged Inlet Case** For this inlet shape, the Reynolds number of the flow is the same as that of the bell-mouth inlet ( $Re = 7651$ ). However, it is necessary to check the type of flow regime for this particular inlet with  $8 \text{ kW/m}^2$  of heating. From Table 8–6, the transition Reynolds number range for this case is  $3860 < Re < 5200$ , which means that the flow in this case is turbulent and Eq. 8–80 is the appropriate equation to use. It gives

$$C_{f,\text{turb}} = \left( \frac{0.0791}{Re^{0.25}} \right) \left( \frac{\mu_b}{\mu_s} \right)^\mu = \left( \frac{0.0791}{7651^{0.25}} \right) (1.14)^{-0.25} = \mathbf{0.0082}$$

**Discussion** Note that the friction factors  $f$  can be determined by multiplying the friction coefficient values by 4.

## Heat Transfer in the Transition Region

Ghajar and coworkers also experimentally investigated the inlet configuration effects on heat transfer in the transition region between laminar and turbulent flows in tubes for the same three inlet configurations shown in Fig. 8–32. They proposed some prediction methods for this regime to bridge between laminar methods and turbulent methods, applicable to forced and mixed convection in the entrance and fully developed regions for the three types of inlet configurations, which are presented next. The local heat transfer coefficient in transition flow is obtained from the transition Nusselt number,  $Nu_{\text{trans}}$ , which is calculated as follows at a distance  $x$  from the entrance:

$$Nu_{\text{trans}} = Nu_{\text{lam}} + \{ \exp[(a - Re)/b] + Nu_{\text{turb}}^c \}^c \quad (8-83)$$

where  $Nu_{\text{lam}}$  is the laminar flow Nusselt number for entrance region laminar flows with natural convection effects,

$$Nu_{\text{lam}} = 1.24 \left[ \left( \frac{Re Pr D}{x} \right) + 0.025 (Gr Pr)^{0.75} \right]^{1/3} \left( \frac{\mu_b}{\mu_s} \right)^{0.14} \quad (8-84)$$

and  $Nu_{\text{turb}}$  is the turbulent flow Nusselt number with developing flow effects,

$$Nu_{\text{turb}} = 0.023 Re^{0.8} Pr^{0.385} \left( \frac{x}{D} \right)^{-0.0054} \left( \frac{\mu_b}{\mu_s} \right)^{0.14} \quad (8-85)$$

**TABLE 8–7**

Constants for transition heat transfer correlation

Inlet Geometry	$a$	$b$	$c$
Re-entrant	1766	276	–0.955
Square-edged	2617	207	–0.950
Bell-mouth	6628	237	–0.980

The physical properties appearing in the dimensionless numbers  $Nu$ ,  $Re$ ,  $Pr$ , and  $Gr$  all are evaluated at the bulk fluid temperature  $T_b$ . The values of the empirical constants  $a$ ,  $b$ , and  $c$  in Eq. 8–83 depend on the inlet configuration and are given in Table 8–7. The viscosity ratio accounts for the temperature effect on the process. The range of application of the heat transfer method based on their database of 1290 points (441 points for re-entrant

inlet, 416 for square-edged inlet and 433 points for bell-mouth inlet) is as follows:

$$\begin{aligned}
 \text{Re-entrant:} \quad & 3 \leq x/D \leq 192, 1700 \leq Re \leq 9100, 5 \leq Pr \leq 51, \\
 & 4000 \leq Gr \leq 210,000, 1.2 \leq \mu_b/\mu_s \leq 2.2 \\
 \text{Square-edged:} \quad & 3 \leq x/D \leq 192, 1600 \leq Re \leq 10,700, 5 \leq Pr \leq 55, \\
 & 4000 \leq Gr \leq 250,000, 1.2 \leq \mu_b/\mu_s \leq 2.6 \\
 \text{Bell-mouth:} \quad & 3 \leq x/D \leq 192, 3300 \leq Re \leq 11,100, 13 \leq Pr \leq 77, \\
 & 6000 \leq Gr \leq 110,000, 1.2 \leq \mu_b/\mu_s \leq 3.1
 \end{aligned}$$

These correlations capture about 70% of measured data within an error band of  $\pm 10\%$ , and 97% of measured data with  $\pm 20\%$ , which is remarkable for transition flows. The individual expressions above for  $Nu_{\text{lam}}$  and  $Nu_{\text{turb}}$  can be used alone for developing flows in those respective regimes. The lower and upper limits of the heat transfer transition Reynolds number ranges for the three different inlets are summarized in Table 8–8. The results shown in this table indicate that the re-entrant inlet configuration causes the earliest transition from laminar flow into the transition regime (at about 2000) while the bell-mouth entrance retards this regime change (at about 3500). The square-edged entrance falls in between (at about 2400), which is close to the often quoted value of 2300 in most textbooks.

Figure 8–35 clearly shows the influence of inlet configuration on the beginning and end of the heat transfer transition region. This figure plots the local average peripheral heat transfer coefficients in terms of the Colburn  $j$  factor ( $j_H = St Pr^{0.67}$ ) versus local Reynolds number for all flow regimes at the length-to-diameter ratio of 192, and  $St$  is the Stanton number, which is also a dimensionless heat transfer coefficient (see Chapter 6 for more details), defined as  $St = Nu/(Re Pr)$ . The filled symbols in Fig. 8–35 represent the start and end of the heat transfer transition region for each inlet configuration. Note the large influence of natural convection superimposed on the forced convective laminar-flow heat transfer process ( $Nu = 4.364$  for a fully developed laminar flow with a uniform heat-flux boundary condition without buoyancy effects), yielding a mixed convection value of about  $Nu = 14.5$ . Equation 8–84 includes this buoyancy effect through the Grashof number.

In a subsequent study, Tam and Ghajar (1998) experimentally investigated the behaviour of local heat transfer coefficients in the transition region for a tube with a bell-mouth inlet. This type of inlet is used in some heat exchangers mainly to avoid the presence of eddies which are believed to be one of the causes for erosion in the tube inlet region. For the

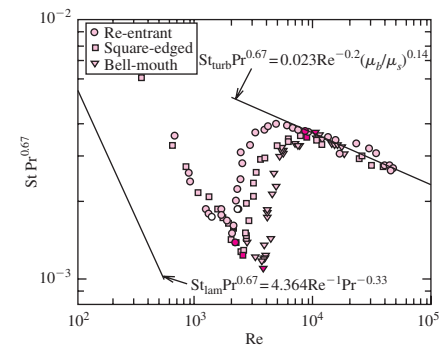


FIGURE 8–35

Influence of different inlets on the heat transfer transition region at  $x/D = 192$  (filled symbols designate the start and end of the transition region for each inlet) between limits of Dittus–Boelter correlation ( $Nu = 0.023 Re^{0.8} Pr^n$ ) for fully developed turbulent flow (using  $n = 1/3$  for heating) and  $Nu = 4.364$  for fully developed laminar flow with a uniform heat flux boundary condition.

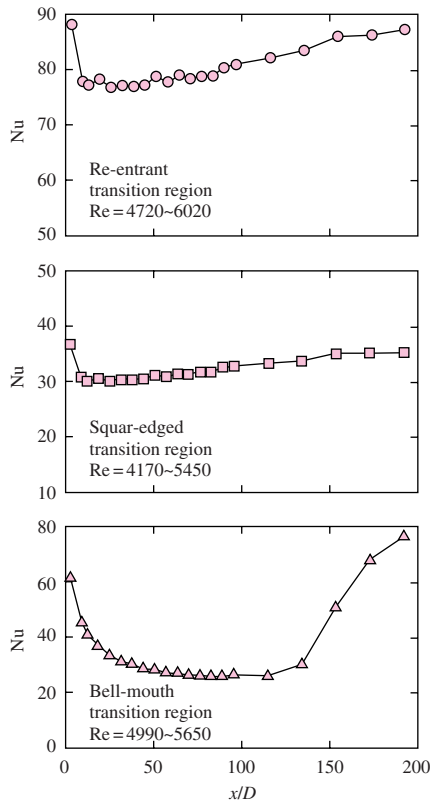
Note buoyancy effect on the laminar flow data giving the much larger mixed convection heat transfer coefficient.

(From Ghajar and Tam, 1994.)

TABLE 8–8

The lower and upper limits of the heat transfer transition Reynolds numbers

Inlet Geometry	Lower Limit	Upper Limit
Re-entrant	$Re_{\text{lower}} = 2157 - 0.65[192 - (x/D)]$	$Re_{\text{upper}} = 8475 - 9.28[192 - (x/D)]$
Square-edged	$Re_{\text{lower}} = 2524 - 0.82[192 - (x/D)]$	$Re_{\text{upper}} = 8791 - 7.69[192 - (x/D)]$
Bell-mouth	$Re_{\text{lower}} = 3787 - 1.80[192 - (x/D)]$	$Re_{\text{upper}} = 10481 - 5.47[192 - (x/D)]$

**FIGURE 8-36**

Variation of local Nusselt number with length for the re-entrant, square-edged, and bell-mouth inlets in the transition region.  
(From Tam and Ghajar, 1998.)

bell-mouth inlet, the variation of the local heat transfer coefficient with length in the transition and turbulent flow regions is very unusual. For this inlet geometry, the boundary layer along the tube wall is at first laminar and then changes through a transition to the turbulent condition causing a dip in the Nu versus  $x/D$  curve. In their experiments with a fixed inside diameter of 15.84 mm, the length of the dip in the transition region was much longer ( $100 < x/D < 175$ ) than in the turbulent region ( $x/D < 25$ ). The presence of the dip in the transition region causes a significant influence in both the local and the average heat transfer coefficients. This is particularly important for heat transfer calculations in short tube heat exchangers with a bell-mouth inlet. Figure 8-36 shows the variation of local Nusselt number along the tube length in the transition region for the three inlet configurations at comparable Reynolds numbers.

### EXAMPLE 8-8 Heat Transfer in the Transition Region

Ethylene glycol-distilled water mixture with a mass fraction of 0.6 and a flow rate of  $2.6 \times 10^{-4} \text{ m}^3/\text{s}$  flows inside a tube with an inside diameter of 0.0158 m subjected to uniform wall heat flux. For this flow, determine the Nusselt number at the location  $x/D = 90$  if the inlet configuration of the tube is: (a) re-entrant, (b) square-edged, and (c) bell-mouth. At this location, the local Grashof number is  $Gr = 51,770$ . The properties of ethylene glycol-distilled water mixture at the location of interest are  $Pr = 29.2$ ,  $\nu = 3.12 \times 10^{-6} \text{ m}^2/\text{s}$  and  $\mu_b/\mu_s = 1.77$ .

**SOLUTION** A liquid mixture flowing in a tube is subjected to uniform wall heat flux. The Nusselt number at a specified location is to be determined for three different tube inlet configurations.

**Assumptions** Steady operating conditions exist.

**Properties** The properties of the ethylene glycol-distilled water mixture are given to be  $Pr = 29.2$ ,  $\nu = 3.12 \times 10^{-6} \text{ m}^2/\text{s}$  and  $\mu_b/\mu_s = 1.77$ .

**Analysis** For a tube with a known diameter and volume flow rate, the type of flow regime is determined before making any decision regarding which Nusselt number correlation to use. The Reynolds number at the specified location is

$$Re = \frac{(\dot{V}/A_c)D}{\nu} = \frac{[(2.6 \times 10^{-4} \text{ m}^3/\text{s})(1.961 \times 10^{-4} \text{ m}^2)](0.0158 \text{ m})}{3.12 \times 10^{-6} \text{ m}^2/\text{s}} = 6714$$

since

$$A_c = \pi D^2/4 = \pi(0.0158 \text{ m})^2/4 = 1.961 \times 10^{-4} \text{ m}^2$$

Therefore, the flow regime is in the transition region for all three inlet configurations (thus use the information given in Table 8-8 with  $x/D = 90$ ) and therefore Eq. 8-83 should be used with the constants  $a$ ,  $b$ ,  $c$  found in Table 8-7. However,  $Nu_{\text{lam}}$  and  $Nu_{\text{turb}}$  are the inputs to Eq. 8-83 and they need to be evaluated first from Eqs. 8-84 and 8-85, respectively. It should be mentioned that the correlations for  $Nu_{\text{lam}}$  and  $Nu_{\text{turb}}$  have no inlet dependency.



From Eq. 8–84:

$$\begin{aligned} \text{Nu}_{\text{lam}} &= 1.24 \left[ \left( \frac{\text{RePr}D}{x} \right) + 0.025(\text{GrPr})^{0.75} \right]^{1/3} \left( \frac{\mu_b}{\mu_s} \right)^{0.14} \\ &= 1.24 \left[ \left( \frac{(6714)(29.2)}{90} \right) + 0.025[(51,770)(29.2)]^{0.75} \right]^{1/3} (1.77)^{0.14} = 19.9 \end{aligned}$$

From Eq. 8–85:

$$\begin{aligned} \text{Nu}_{\text{turb}} &= 0.023\text{Re}^{0.8}\text{Pr}^{0.385} \left( \frac{x}{D} \right)^{-0.0054} \left( \frac{\mu_b}{\mu_s} \right)^{0.14} \\ &= 0.023(6714)^{0.8}(29.2)^{0.385}(90)^{-0.0054}(1.77)^{0.14} = 102.7 \end{aligned}$$

Then the transition Nusselt number can be determined from Eq. 8–83,

$$\text{Nu}_{\text{trans}} = \text{Nu}_{\text{lam}} + \{ \exp[(a - \text{Re})/b] + \text{Nu}_{\text{turb}}^c \}^c$$

Case 1: For re-entrant inlet:

$$\text{Nu}_{\text{trans}} = 19.9 + \{ \exp[(1766 - 6714)/276] + 102.7^{-0.955} \}^{-0.955} = \mathbf{88.2}$$

Case 2: For square-edged inlet:

$$\text{Nu}_{\text{trans}} = 19.9 + \{ \exp[(2617 - 6714)/207] + 102.7^{-0.950} \}^{-0.950} = \mathbf{85.3}$$

Case 3: For bell-mouth inlet:

$$\text{Nu}_{\text{trans}} = 19.9 + \{ \exp[(6628 - 6714)/237] + 102.7^{-0.980} \}^{-0.980} = \mathbf{21.3}$$

**Discussion** It is worth mentioning that, for the re-entrant and square-edged inlets, the flow behaves normally. For the bell-mouth inlet, the Nusselt number is low in comparison to the other two inlets. This is because of the unusual behaviour of the bell-mouth inlet noted earlier (see Fig. 8–36); i.e., the boundary layer along the tube wall is at first laminar and then changes through a transition region to the turbulent condition.

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## SUMMARY

*Internal flow* is characterized by the fluid being completely confined by the inner surfaces of the tube. The mean or average velocity and temperature for a circular tube of radius  $R$  are expressed as

$$V_{\text{avg}} = \frac{2}{R^2} \int_0^R u(r)rdr \quad \text{and} \quad T_m = \frac{2}{V_{\text{avg}}R^2} \int_0^R u(r)T(r)rdr$$

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{V_{\text{avg}} D}{\nu} \quad \text{and} \quad D_h = \frac{4A_c}{P}$$

The flow in a tube is laminar for  $\text{Re} < 2300$ , turbulent for about  $\text{Re} > 10,000$ , and transitional in between.

The length of the region from the tube inlet to the point at which the boundary layer merges at the centerline is the *hydrodynamic entry length*  $L_h$ . The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region*. The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length*  $L_t$ . The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region*. The entry lengths are given by

$$\begin{aligned} L_{h, \text{ laminar}} &\approx 0.05 \text{ Re } D \\ L_{t, \text{ laminar}} &\approx 0.05 \text{ Re Pr } D = \text{Pr } L_{h, \text{ laminar}} \\ L_{h, \text{ turbulent}} &\approx L_{t, \text{ turbulent}} = 10D \end{aligned}$$

For  $\dot{q}_s = \text{constant}$ , the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} c_p (T_e - T_i)$$

For  $T_s = \text{constant}$ , we have

$$\begin{aligned} \dot{Q} &= h A_s \Delta T_{\text{ln}} = \dot{m} c_p (T_e - T_i) \\ T_e &= T_s - (T_s - T_i) \exp(-h A_s / \dot{m} c_p) \\ \Delta T_{\text{ln}} &= \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)} \end{aligned}$$

The irreversible pressure loss due to frictional effects and the required pumping power to overcome this loss for a volume flow rate of  $\dot{V}$  are

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{\text{avg}}^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \dot{V} \Delta P_L$$

For *fully developed laminar flow* in a circular pipe, we have:

$$\begin{aligned} u(r) &= 2V_{\text{avg}} \left(1 - \frac{r^2}{R^2}\right) = u_{\text{max}} \left(1 - \frac{r^2}{R^2}\right) \\ f &= \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}} \\ \dot{V} &= V_{\text{avg}} A_c = \frac{\Delta P R^2}{8\mu L} \pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi R^4 \Delta P}{128\mu L} \end{aligned}$$

$$\text{Circular tube, laminar } (\dot{q}_s = \text{constant}): \quad \text{Nu} = \frac{hD}{k} = 4.36$$

$$\text{Circular tube, laminar } (T_s = \text{constant}): \quad \text{Nu} = \frac{hD}{k} = 3.66$$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

$$\text{Circular tube:} \quad \text{Nu} = 3.66 + \frac{0.065(D/L) \text{ Re Pr}}{1 + 0.04[(D/L) \text{ Re Pr}]^{2/3}}$$