AN IMPROVED FALLING SPHERE VISCOMETER FOR
INTERMEDIATE CONCENTRATIONS OF VISCOELASTIC FLUIDS

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ABSTRACT
        It is shown that the falling sphere viscometer can be used to determine
        the steady low shear viscosity of intermediate concentrations of highly
        viscoelastic fluids (polyacrylamide, 1000 and 500 ppm solutions). An
        empirical formula for drag coefficient was used to extend the visco-
        meter's range of applicability to Reynolds numbers greater than unity.
        A new experimental technique, which utilized partially liquid filled
        hollow spheres, was found to be effective in controlling the sphere's
        terminal velocity as it falls through intermediate concentrations of a
        viscoelastic medium.

Introduction

For the analysis of experimental friction and heat transfer data of non-

Newtonian fluids, one needs to know the rheological properties such as the low

shear rate dependent viscosity. This is due to the fact that viscosity at low

shear rate is sensitive to changes in the magnitude of elasticity of a non-

Newtonian fluid, which, in turn, affects the friction factor and heat transfer

behavior [1].

The falling sphere viscometer because of its simple theory and the low

cost to build the equipment has been widely used to determine the viscosity of

Newtonian fluids. The clear advantage of the use of falling sphere viscometer

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for non-Newtonian fluids is its ability to measure the viscosity at low shear rate range. Cho [2] and Cho and Hartnett [3], successfully demonstrated the effectiveness of a falling sphere viscometer as an instrument for determining the steady shear viscosity of high concentrations of viscoelastic fluids at low shear rates. They obtained shear viscosity at low shear rate range for weakly viscoelastic carboxymethyl cellulose (CMC) at 10,000 and 15,000 ppm, and highly viscoelastic polyacrylamide (Separan AP-273) at 5,000 and 10,000 ppm. Solid balls made of different materials with different diameters were used to obtain the desired shear rates. Their proposed analytical technique makes use of the variational principle with improved upper and lower bound solutions in conjunction with the power-law model. Their calculation procedure proved to be reasonably accurate in determination of low shear rate viscosity of high concentrations of CMC and Separan AP-273 solutions directly from the falling sphere viscometer when compared with measurements obtained from the Weissenberg rheogoniometer and capillary viscometer. However, they [2,3], limited their studies to high concentrations of viscoelastic fluids, which have limited practical application in fluid mechanics and heat transfer studies. For example, in a recent review article, Cho and Hartnett [1] revealed that in turbulent pipe flows, 100 and 500 ppm of Separan AP-273 solutions correspond to the maximum drag reduction and heat transfer reduction, respectively. Thus, it can be stated that highly viscoelastic fluids at "intermediate" concentrations (i.e. 500 to 1000 ppm) have far more practical application and economic feasibility in drag and heat transfer reduction than at high concentrations (i.e. 5000 to 10,000 ppm).

To the best of the authors' knowledge, the falling sphere viscometer has not been used for determination of the viscosity of intermediate concentrations of highly viscoelastic fluids. Furthermore, the experimental techniques used by [2,3] were not as advanced as it could have been. Thus,

the primary objectives of this investigation are to improve on the basic design of the falling sphere viscometer and to further extend its ability so that it could be used to obtain the viscosity of intermediate concentrations of highly viscoelastic fluids (Separan AP-273, 1000 and 500 ppm solutions).

Drag Coefficient

For the viscosity measurement of fluids in the falling sphere viscometer, it is necessary to find the viscous drag of a ball falling slowly in an infinite medium of test fluid, which will yield the average shear stress acting on the surface of a sphere. This relates to the problem originally considered by Stokes [4], which was the slowly moving Newtonian flow past a sphere. By completely neglecting the inertia effects in the flow field, Stokes obtained the following equation for the drag coefficient:

$$C_D = \frac{24}{Re_D} \tag{1}$$

Since the Reynolds number is the ratio of the inertia forces to the viscous forces, therefore the restriction of $Re_D < 1$ was imposed by Stokes in order to ensure that the inertia effects are negligible. This restriction presents a dilemma when low Reynolds number cannot be attained during the viscosity measurement of a low viscosity fluid. This is so, because Reynolds number is an inverse function of viscosity; thus, as the viscosity decreases Reynolds number will increase.

In order to extend the falling sphere viscometer's ability so that it could be used to obtain the viscosity of intermediate concentrations of highly viscoelastic fluids, it is necessary to extend its range of applicability to Reynolds numbers greater than unity (i.e. inertia effects should be included). For this purpose, it is proposed to use an empirical formula for the determination of drag coefficient which includes inertia effects. White

[5] used experimental data on drag coefficient versus Reynolds number from many sources and developed the following curve-fit formula:

$$C_D = \frac{24}{Re_D} + \frac{6}{1+(Re_D)^{1/2}} + 0.4 \tag{2}$$

This equation is applicable for Reynolds numbers up to 2×10^5 with an accuracy of $\pm$ 10 percent. However, for Reynolds numbers up to 30 (range of interest for intermediate concentrations), the results of the empirical formula as shown by White [5] are an accurate prediction of the experimental data.

The use of the proposed empirical formula; which includes the inertia effects, in determination of the viscosity, shear rate and shear stress will be presented in the next section.

Analysis

In order to determine the viscosity of non-Newtonian fluids from a falling sphere viscometer, the Newtonian shear rate and shear stress must first be obtained. The procedure outlined here is different in one facet from the previous works [2,3]. That is, the Stokes flow assumption for the case that $Re_D \geq 1$, has been relaxed through the use of an empirical formula for drag coefficient.

Newtonian

From a force-balance equation on a sphere falling at terminal velocity in a viscous medium, the drag equation takes the form:

$$C_D = \frac{4}{3} \frac{gD}{\rho V^2} (\rho_s - \rho) \tag{3}$$

from which the sphere's drag coefficient can be calculated directly. By equating Equations (2) and (3), the Newtonian viscosity μ_N , can now be determined. Since Equation (2) is implicit in μ_N , numerical procedures were

used to achieve convergence between Equations (2) and (3) [6]. As pointed out earlier, use of Equation (2) ensures inclusion of the inertia effects.

The inertia effects which Stokes neglected are transported through the free stream velocity [7]; and since the shear rate is a function of velocity, therefore, it is assumed to be the only variable which is affected by the additional forces. Thus, the Newtonian shear rate $\dot{\gamma}_N$, can be obtained using the definition of viscosity

$$\dot{\gamma}_N = \frac{\tau_N}{\mu_N} \tag{4}$$

where μ_N is determined through the procedures outlined above, and τ_N , the creeping flow Newtonian shear stress is given by

$$\tau_N = \frac{1}{9} \; gD \; (\rho_s - \rho) \tag{5}$$

Although it was realized that the assumption that τ_N is independent of the inertia effects may not be valid for high Reynolds numbers (low concentrations of viscoelastic fluids), nevertheless it was decided to proceed with this assumption in order to calculate the needed information without introducing additional complications due to direct calculation of the actual additional inertia effects on τ_N . This assumption limits our analysis to intermediate and higher concentrations.

The technique described here by-passes the actual calculation of the additional inertia effects, which takes precedence when Reynolds number is greater than unity, through the use of an empirical drag coefficient formula.

Non-Newtonian

The procedure outlined below summarizes the work of Cho [2] and Cho and Hartnett [3], which made use of the variational principle with improved upper and low-bound solutions in conjunction with the power-law model.

To obtain the true average shear rate and shear stress in the falling sphere viscometer for non-Newtonian fluids, $\dot{\gamma}_N$ and τ_N have to be corrected as follows:

$$\dot{\gamma} = C_1 \, \dot{\gamma}_N \tag{6}$$

$$\tau = C_2 \, \tau_N \tag{7}$$

where for the upper bound,

$$C_1 = -1.731 + 41.28n - 116.0n^2 + 123.9n^3 - 46.72n^4 \tag{8}$$

$$C_2 = 0.2827 + 0.8744n + 0.4526n^2 - 0.7486n^3$$

and for the lower bound

$$C_1 = -2.482 + 54.35n - 160.1n^2 + 178.2n^3 - 69.04n^4 \tag{9}$$

$$C_2 = 0.6388 + 0.6418n - 0.4344n^2 + 0.1560n^3$$

In Equations (8) and (9), n is the power-law index and can be calculated from the slope of the $\tau_N - \dot{\gamma}_N$ curve.

For the correction of the wall effect; Faxen's formula [8]

$$V = V_{meas.}/(1-2.104 \, D/D_c) \tag{10}$$

was used to provide a velocity V in the infinite medium of fluid.   This corrected velocity was used in conjunction with Equations (6) and (7) to yield the viscosity of the solutions.

Experiment

The overall schemetic of the falling sphere viscometer used in this study is shown in Figure 1.   It consists of two concentric plexiglass cylinders, a constant temperature bath and circulator (Brinkmann Lauda Circulator Model MT), laser (Spectra-Physics Helium-Neon Laser Model 155), laser optics, electronic infrared detectors and a digital timer. The inside diameter of the "fall-tube" is 5.75 inches. The height of the test section is sufficient (72 inches) to ensure the attainment of terminal velocity for all the test
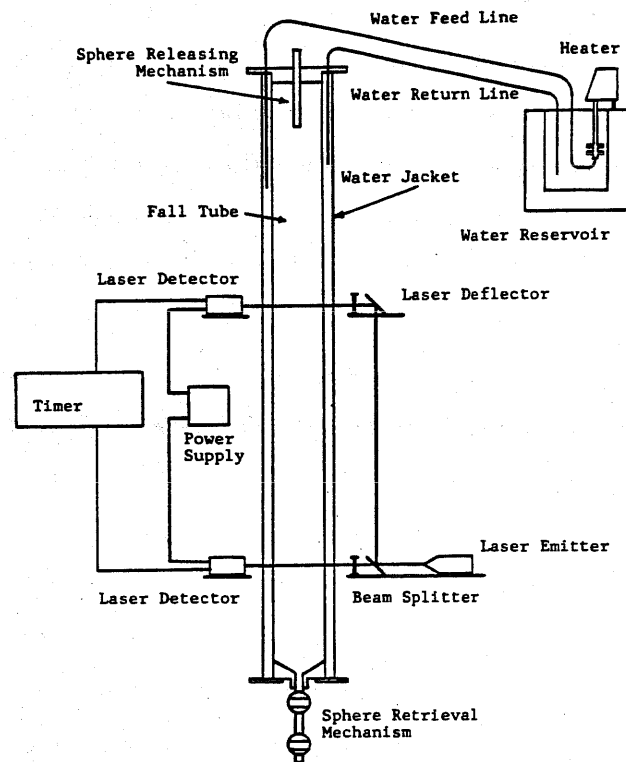
FIG. 1
Schematic of the falling sphere viscometer.

spheres. To ensure that the balls are dropped at the geometric center of the fall-tube a special releasing mechanism made up of a series of 15 brass tubes, each one foot in length with diameters ranging from 3/32 through 17/32 inches incrementing by 1/32 inch were used. For further details on the construction and design of the falling sphere viscometer refer to [6].

In order to use the falling sphere viscometer for intermediate concentrations of highly viscoelastic fluids, hollow aluminum spheres manufactured by Industrial Tectonics, Inc. were used. These spheres were then partially filled with water. This was done by drilling a 0.0225 inch hole in each sphere and then employing an insulin injection syringe, to inject the desired amount of water [6]. The hole was sealed with sillicone rubber adhesive sealant. The surface around the hole was then gently sanded to eliminate any excess sealant material. Extreme caution was exercised in the sanding process, so that scratches or flat spots on the surface did not occur. The use of hollow sphere, which is partially filled with water, allows more control of the sphere's density, which, in turn, controls the sphere's terminal velocity. Thus, the shear rate range can be controlled.

It is to be noted that the viscometer and the experimental procedure described here are different from that of [2,3]. Optical electronics devices, employing laser beams and infrared detectors were incorporated into the design of the falling sphere viscometer for terminal velocity measurements. Whereas, human judgement and a stop watch was used by the previous investigators for this purpose. A new procedure, which utilizes partially liquid filled hollow spheres was developed in order to allow viscosity measurements of intermediate concentrations of highly viscoelastic fluids. Previous investigators used solid spheres, which restricted their measurements to high concentrations of highly viscoelastic fluids. In addition, a longer and larger fall-tube was used in this study (72" and 6" dia. vs 55" and 4" dia.). The increase in the

size of the fall-tube allows a larger sphere diameter to be used and ensures attainment of terminal velocity.

Calibration Experiments

Test runs were conducted with 97% Glycerin, a Newtonian fluid, to establish the accuracy of the experimental procedure. By choosing a Newtonian fluid, the linear relationship between shear stress and shear rate can be observed. Furthermore, the viscosity data will not depend on the shear rate; thus, when employing spheres with different sizes and densities, the viscosity should remain constant.

Aluminum, steel, blackglass, and Teflon solid spheres, with diameters ranging from 3/32 to 9/32 inches were dropped at the center line of the fall-tube filled with the test fluid. The time interval between each drop was 15 minutes. The time required for each sphere to break the top and then the bottom laser plane was recorded by a digital timer. Knowing the distance between the two laser planes and the time, the terminal velocity can be calculated. This measured velocity was then corrected using Faxen's formula, Equation (10), to provide the velocity in the infinite medium of the fluid.

The flow curve τ_N versus $\dot{\gamma}_N$, as defined by Equations (5) and (4) respectively, is shown in Figure 2. The power-law index n calculated from the slope of the $\tau_N - \dot{\gamma}_N$ curve is unity, which is the expected value for Newtonian fluids. To further varify the results, flow curves for the same test fluid were obtained with Fann Model 35 and Brookfield Synchro-Electric Model LVF rotational viscometers, as shown on Figure 2. The agreement between the falling sphere viscometer and the other two viscometers is excellent.

Results and Discussions for Non-Newtonian Fluids

Similar experiments were carried out for polyacrylamide (Separan AP-273) solutions, a highly viscoelastic fluid, at concentrations of 1000 and 500 ppm. In order to determine low shear rate viscosity of these "intermediate"
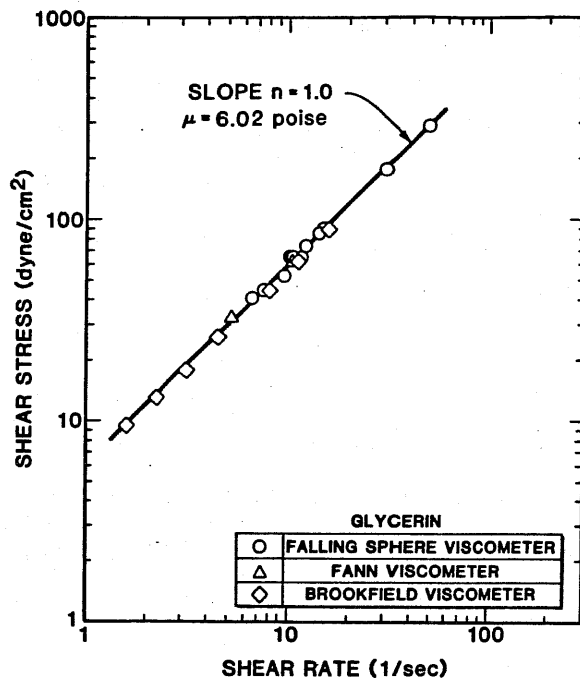
FIG. 2
Flow curve-shear stress vs. shear rate for 97% glycerin at 27°C.

concentration solutions, aluminum hollow spheres with different densities were prepared and dropped at 30 minute time intervals, and correct terminal velocities were determined following the procedures outlined before.

The flow curves (shear stress versus shear rate) are shown in Figure 3. The open circle points and the open square points represent the original experimental data for 1000 and 500 ppm solutions, respectively. These data were obtained using the Newtonian equations (see Equations 5 and 4). The power-law index n was calculated from the slope of the $\tau_N - \dot{\gamma}_N$ curve, and values of 0.642 and 0.723 were determined for the 1000 and 500 ppm solutions, respectively.

The true average shear rate and shear stress in the falling sphere viscometer were obtained by correcting the Newtonian shear rate and shear stress using Equations (6) through (9). Since the difference of the upper and lower bound shear rates and shear stresses obtained from these equations were

small [6], the shear rates and shear stresses will be presented as the average value of the upper and lower bound values. The corrected shear stress and shear rate results are shown in Figure 3 as the closed circles and squares for 1000 and 500 ppm solutions, respectively. Compared with the flow curves obtained in the Weissenberg rheogoniometer (WRG) by Kwack [9], the falling sphere viscometer (FSV), yields reasonably accurate flow curves for the polyacrylamide 1000 and 500 ppm solutions. The data presented on Figure 3 correspnds to Reynolds number ranges from 0.5 to 3 and 6 to 25 for 1000 and 500 ppm solutions, respectively.

Using the definition of an apparent viscosity $\mu = \tau/\dot{\gamma}$, the steady shear viscosity can be calculated as shown in Figure 4, again giving fairly good greement with those obtained by Kwack [9] from Weissenberg rheogoniometer.

Assuming that the viscosity data obtained from Weissenberg rheogoniometer can be used as a standard, the viscosity data obtained from the falling sphere viscometer are within $\pm13\%$, with the majority of data points having less than 10% deviation. This percent difference is within the maximum probable error of $\pm15\%$ computed from the uncertainty analysis [6]. However, some of these differences could be due to the differences in the chemistry of the water, differences in the mixing procedures, differences in batches of polymer, etc. It is to be noted that Kwack [9] did not report the accuracy of his viscosity data, but generally it is believed that Weissenberg rheogoniometer, which is one of the most expensive and sophisticated viscometers commercially available, produces accurate results.

The uncertainty analysis revealed that the viscosity data for intermediate concentrations of highly viscoelastic fluids are very sensitive to both fluid and sphere densities, see Equations (3) and (5). Therefore, accurate measurement of the densities is very crucial for successful use of
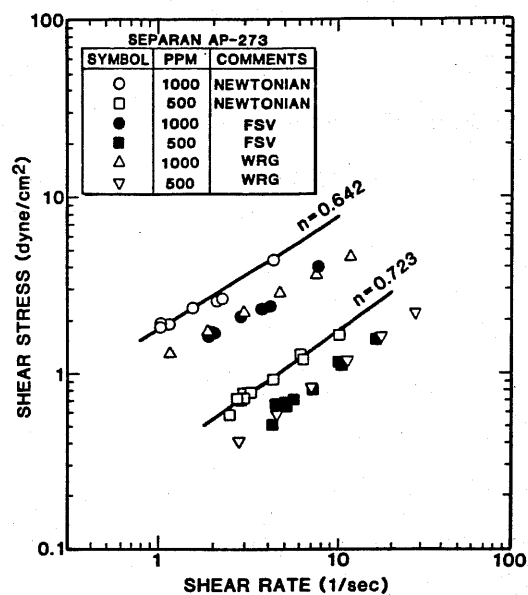
FIG. 3

Flow curve-shear stress vs. shear rate for 1000 and 500 ppm
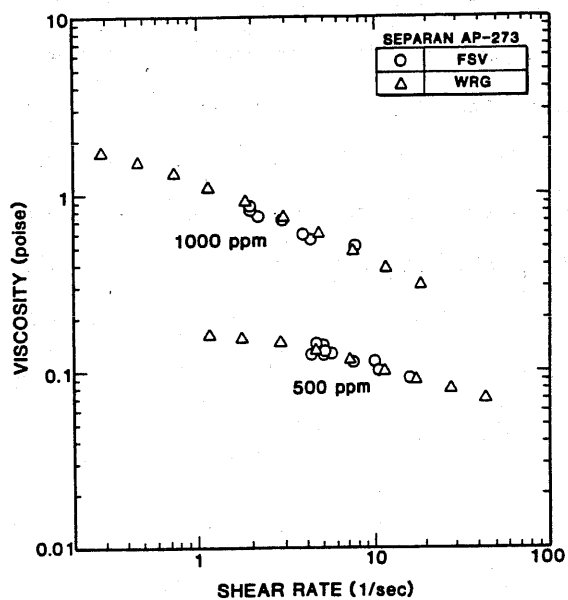Separan AP-273 at 25°C.

FIG. 4

Steady shear viscosity vs. shear rate for 1000 and
500 ppm Separan AP-273 at 25°C.

the falling sphere viscometer for intermediate concentrations.

Conclusions

The steady low shear viscosity measurement of intermediate concentrations of highly viscoelastic fluids (polyacrylamide, 1000 and 500 ppm solutions) can be successfully accomplished in the "upgraded" falling sphere viscometer, using the proposed new experimental and analytical methods. The new experimental technique makes use of hollow spheres which are partially filled with water. This allows more control of the sphere's density, terminal velocity, and shear rate. The proposed analytical method employs an empirical formula for drag coefficient in order to extend the falling sphere viscometer's capability for the case when Stokes law has been violated. However, due to the assumption made, the analytical method is limited to intermediate and higher concentrations.

Nomenclature

C_D	=	drag coefficient
C_1, C_2	=	correction factors in Equations (6) and (7)
D	=	sphere diameter
D_c	=	cylinder diameter
g	=	acceleration of gravity
n	=	power-law index
Re_D	=	Reynolds number, $\rho VD/\mu$
V	=	terminal velocity at infinity
$V_{meas.}$	=	terminal velocity measured at laboratory
$\dot{\gamma}$	=	non-Newtonian shear rate
$\dot{\gamma}_N$	=	Newtonian shear rate
μ	=	apparent viscosity (non-Newtonian)

μ_N =   Newtonian viscosity

ρ =   fluid density

ρ_s =   sphere density

τ =   non-Newtonian shear stress

τ_N =   Newtonian shear stress

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