

SINGLE –PHASE FRICTION FACTOR IN MICRO-TUBES: A CRITICAL REVIEW OF MEASUREMENT, INSTRUMENTATION AND DATA REDUCTION TECHNIQUES FROM 1991-2006

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ABSTRACT

The objective of this paper is to provide a state-of-art review in the field of single-phase pressure drop (friction factor) measurements in micro-tubes. Twenty-three experiments from 1991 to 2006 are reviewed critically in areas of different measurement techniques, instrumentation used and the various data-reduction methods employed. The review confirms that researchers unanimously agree that friction factor in micro-tubes can be predicted by using macro-scale theory and that there is a need to investigate certain issues like (a) the effect of roughness on friction factor and transition and (b) the effect of micro-tube diameter on transition Reynolds number. The state-of-art review thus provides the contemporary experimenters in the field of mini-micro channel fluid flow this vast amount of tabulated data on experimental set-up, results, instrumentation and uncertainties for all twenty-three experiments. The data can be used to investigate how the different parameters affect the fluid flow in these small scales and to validate future numerical and experimental work. Moreover, the review observes that smooth micro-tubes follow classical laws while roughness does seem to play a major role in the dynamics of smaller diameter tubes.

NOMENCLATURE

Symbol	Description	Unit
D	Diameter of micro-tube	m
Re	Reynolds number	No unit
RR	Relative roughness	No unit
f	Darcy friction factor	No unit
ℓ	Length of micro-tube	m
p_1	Inlet pressure	Pa
p_2	Outlet pressure	Pa
u	Flow velocity	m/s
Greek Symbols		
ε	Roughness height	m
ρ	Density of fluid	kg/m ³

Δp	Pressure drop across the micro-tube	Pa
$\sum L$	Sum of minor losses	Pa

INTRODUCTION

The miniaturization of components and devices using advanced fabrication techniques has taken the industry to new heights of advancement (Hoffman et al., 1998). The engineering applications using advanced micro-electro-mechanical systems (MEMS) are opening new avenues in various disciplines of engineering (Ho and Tai, 1998). Present research in the fields of miniature heat-exchangers, micro fluidic devices like pumps and compressors, electronics cooling, fuel-cells, sensor technology and a myriad of new applications in bio-mechanics require a firm and sound understanding of fluid flow and heat transfer at small scales. However, the science behind these advanced technologies seems to be controversial, especially fuelled by the experimental results of the fluid flow and heat transfer at these small scales.

Palm (2001) and Papautsky et al. (2001) were one of the first reviews to analyze pressure-drop and heat transfer in micro-channels. Sobhan and Garimella (2001) and Rostami et al. (2002) provide extensive tabulated data on the nature of study and their results. These studies reported disparity among the results of various researchers. One similarity amongst the previous reviews is that, they contain comparisons of results from the rectangular, circular and trapezoidal geometries. The first reviews to concentrate on micro-tubes were the work of Celata (2004) and Celata et al. (2004). They observed that experimental and theoretical values of friction factor compare well till $Re = 600$ after which higher friction factor values are observed. Moreover, they highlight the importance of diameter measurement in micro-tube experiments. The authors found that it was very difficult to quantify the effect of roughness due to the large number of parameters describing various roughness geometries. The present review analyzes in detail the effect of roughness and diameter on friction factor and transition in micro-tubes. Micro-tubes are chosen in this study

over other non-circular micro-channels to negate the aspect ratio effects on the flow that might affect the fluid at such small scales. Moreover, experimental components and measurements may provide important insight into the complex flow behavior in micro-tubes and channels (Ferguson et al., 2005). Hence the need to look at these experiments chronologically in terms of instrumentation, diameters of micro-tubes, the different working fluids, and surface roughness of the tubes is necessitated in this paper.

The following are the objectives of the review:

- To provide the contemporary researchers, a vast data-bank of tabulated data on instrumentation, measurements, uncertainties and other experimental parameters and results of micro-tube pressure drop experiments from 1991 to 2006.
- To investigate controlled factors like pressure drop, flow rate, diameter and roughness measurements and highlight their significance with respect to pressure drop studies.
- To analyze the disparity of results from the literature and identify the effects of roughness and diameter on friction factor and transition in micro-tubes.

LITERATURE REVIEW

Most of the papers discussed in this section include the results of heat transfer experiments, but this paper will concentrate only on the pressure drop studies. Experiments from 1991–2000 (Choi et al., 1991; Yu et al., 1995; Mala & Li, 1999; Judy et al., 2000), mostly indicate lower values of friction factor than the theoretical predictions; those from 2001–2003 (Kandlikar et al., 2001; Celata et al., 2002; Judy et al., 2002; Bucci et al., 2003; Li et al., 2003; Brutin & Tadrif, 2003), along with results from 1991–2000, showed contradictory observations resulting in widespread disparity, while the most recent ones from 2003–2006 (Yang et al., 2003; Sharp & Adrian, 2004; Lelea et al., 2004; Cui and Silber-Li, 2004; Asako et al., 2005; Hwang & Kim, 2005; Rands et al., 2006; Silber-Li et al., 2006; Yang & Lin, 2006; Zhao & Liu, 2006; Celata et al., 2006a; Celata et al., 2006b) suggest that laminar and turbulent friction factors can be predicted well by macro-scale theory within the experimental uncertainties. A summary of the experiments is presented in Tables 1 and 2.

Early Results (1991–2000): Mostly less than theory

Choi et al. (1991) performed the pressure drop measurements on fused-silica micro-tubes with dry-nitrogen gas as the test fluid. The diameters ranged from 3 to 81 μm and the roughness measurements confirmed that the micro-tubes were essentially smooth. They found the $f\cdot\text{Re}$ value to be around 53, which was considerably less than the theoretical value of 64. Similar results were obtained for the turbulent

flow data. The authors also observed that the readings were not influenced by the roughness of the micro-tubes.

Similar results were obtained by Yu et al. (1995) in their experiment using water and nitrogen gas. The micro-tubes used were from the same manufacturer (Polymicro technologies) as Choi et al. (1991). They found the $f\cdot\text{Re}$ product to be 50.13. Both Choi et al. (1991) and Yu et al. (1995) used compressible flow analysis for nitrogen and the friction factor was calculated using Fanno-line expression.

Mala and Li (1999) analyzed water flowing through fused-silica and stainless steel tubes ranging from 50 to 254 μm . Contrary to the previous researchers, they found friction factor values larger than what the theory predicted. Moreover, they also observed a dependence of the $f\cdot\text{Re}$ product on Reynolds number. Early transition at $\text{Re} = 300$ to 900 was reported and surface roughness was proposed as a significant cause of early flow transition.

Judy et al. (2000) also found results similar to those of Choi et al. (1991) and Yu et al. (1995). They examined various test fluids like water, iso-propanol and hexane through fused-silica tubes. Their $f\cdot\text{Re}$ product remained constant with an increase in Reynolds number but less than 64. They found the largest deviation from theory decreased as the micro-tube diameter increased. Thus, suggesting the effect of diameters on the fluid flow. They investigated various possibilities of reduced friction factor, namely: shear heating, pressure dependence and surface forces. The authors finally concluded that all above mentioned effects have a tendency to increase the friction factor and not the other way around.

Mixed Results (2001 – 2003): higher than theory or agrees with theory

Kandlikar et al. (2001) investigated the effect of roughness on pressure drop in micro-tubes. The roughness is changed by etching the tubes with different acids. They observed that for the larger tube (1067 μm); the effect of roughness is negligible. For smaller tube (620 μm); more roughness results in higher pressure drop accompanied by early transition.

Celata et al. (2002) performed pressure drop tests using R-114 in a 130 μm micro-tube. The Reynolds number ranged from 100 to 8000 while the transition was observed to be in the range of 1880 to 2480. In the laminar region, the experimental values matched well with the theoretical predictions only till $\text{Re} = 585$. For $\text{Re} > 585$, higher friction factor values were observed. The authors attribute this deviation from theory to roughness of the stainless steel micro-tube.

Judy et al. (2002) investigated the laminar flow through round and square channels with distilled water, methanol and iso-propanol. Their study could not detect any deviation from Stokes flow regime for all the test-fluids, pipe material and cross-section. However, the authors do not mention any roughness measurement or effect of roughness on the flow in their paper.

Li et al. (2003) analyzed flow through smooth glass pipes and rough stainless steel pipes. They found that the fRe product for smooth pipes was nearly 64, while the rough pipes (with relative roughness of 3 to 4 %) showed almost 15 to 37 % higher friction factor values. Their experiment clearly indicates the influence of roughness on the friction factors.

Bucci et al. (2003) studied water flow in stainless steel capillary tubes. Their results indicate that in the laminar regime good correlation was observed till $Re = 1000$, after which there was significant increase in experimental friction factor values. They observed the transition to occur in the range of $Re = 1800$ to 3000 . Interestingly, the smallest diameter tube ($172 \mu\text{m}$) showed the largest transition Reynolds number, 3120.

Brutin and Tadrif (2003) analyzed water flow through fused-silica pipes with diameters ranging from 50 to $530 \mu\text{m}$. They conclude that the deviation is primarily due to the ionic composition of the fluid. The deviation from theory decreased with an increase in diameter, but the authors found all the friction factors to be higher than theory.

Recent Results (2003–2006): Mostly agrees with theory

Yang et al. (2003) conducted pressure drop studies in micro-tubes and mini-tubes ranging from $173 \mu\text{m}$ to 4.01 mm . They observed that minor losses used in macro-scale theory can be used for liquids and low speed air flows. However, for high speed air flows compressibility effects hamper their use. In pressure drop tests, both laminar and turbulent regimes were predicted well by macro-scale theory, but for high speed air flow the Blasius equation failed to capture the physics.

Sharp and Adrian (2004) investigated transitional flows through steel micro-tubes from $247 \mu\text{m}$ to $50 \mu\text{m}$. Their results are very similar to Yang et al. (2003), with the conclusion that macro-scale theory is very much valid even for diameters of the order of $50 \mu\text{m}$. Transition for all the micro-tubes was reported between $Re = 1800$ to 2300 . The authors investigated different fluids namely: water, 1-propanol, and 20%-glycerol. They did not observe any effect of polarity or viscosity on the pressure drop characteristics. Again, it is worth mentioning that the authors do not mention any information about the roughness of the micro-tubes.

Lelea et al. (2004) analyzed water flow through 100 , 300 and $500 \mu\text{m}$ stainless steel tubes. The authors reported results only in the laminar regime till $Re = 800$. They found that conventional theories used for macro-tubes can be applied on micro-tubes down to $100 \mu\text{m}$. The effect of surface roughness was not considered by the authors in their paper.

Cui and Silber-Li (2004) examined the effect of high pressures on the viscosities of distilled water, iso-propanol and carbon tetra-chloride (CCl_4). The diameters in the experiment ranged from $10 \mu\text{m}$ to $3 \mu\text{m}$. The authors observed variation in normalized friction factor with pressure for iso-propanol and CCl_4 but for distilled water no inconsistency is detected. Moreover, the authors propose an equation for exponential

function of viscosity (as a function of pressure) to account for the disparity between experimental and theoretical values.

Asako et al. (2005) performed compressible studies for air flow through $150 \mu\text{m}$ fused silica tubes whose roughness was of the order of 5 nm . Hence, their tubes could be considered smooth. They found that the fRe product is a function of Mach number and increases with rising Mach number. These effects show similar trends as observed by Yang et al. (2003).

Hwang and Kim (2005) investigated the pressure drop characteristics of R-134a in stainless steel tubes with diameters: 244 , 430 and $792 \mu\text{m}$. They found that within experimental uncertainty, conventional theories are able to predict the experimental friction factors. The authors found no evidence of early transition and they reported the transition Reynolds number to be slightly less than 2000 . Moreover, no mention about the roughness of the micro-tubes can be found in their paper.

Rands et al. (2006) studied fused silica tubes with diameters from $32.2 \mu\text{m}$ to $16.6 \mu\text{m}$. The authors reported macro-scale behavior in all the tubes and the transition was found in the range of $Re = 2100$ to 2500 . Similar to Bucci et al. (2003), they also observed a slight dependence of critical Reynolds number on the micro-tube diameter. The critical Re increased slightly with a decrease in diameter.

Silber-Li et al. (2006) extended the work of Cui and Silber-Li (2004) by observing a non-linear variation of viscosity and pressure along the axial direction. This was found for most liquids except water. Thus the significance of viscosity varying as a function of pressure (at high pressures $1\text{--}30 \text{ MPa}$) was highlighted in this research.

Yang and Lin (2006) analyzed water flow through stainless steel pipes with diameters ranging from $123 \mu\text{m}$ to $962 \mu\text{m}$. They found that the results correlate well with macro-scale theory and no significant effect of diameter or roughness was observed on the pressure drop characteristics and transition was observed from $Re = 2300$ to 3000 .

Zhao and Liu (2006) conducted pressure drop studies on smooth quartz-glass tubes and rough stainless steel tubes of varying diameters. They observed that in the laminar regime experimental results agree well till $Re = 1100$ to 1500 (for smooth micro-tubes) and early transition is observed. For rough micro-tubes (with $\epsilon/D = 0.08$), laminar theory agrees only till $Re = 800$ and similar early transition is observed.

Celata et al. (2006a) conclude that flow characteristics is not affected by aspect-ratio, nature of fluid and the inclination angle up to diameters of $259 \mu\text{m}$. These experiments were carried out at three different institutes and all the results indicate macro-scale flow behavior.

Celata et al. (2006b) investigated diameters from $300 \mu\text{m}$ to $30 \mu\text{m}$ and found that within the experimental uncertainty no deviation from the classical theory was observed. They attribute the deviation from theory to aspect-ratio effects rather than effects of roughness.

Table 1: Summary of Reynolds number range, Diameters, Test Fluids and Micro-tube materials

Authors (Year)	Re Range	Micro-tube Diameters (μm)	Test Fluid	Micro-tube Material
Choi et al. (1991)	-	3,7,10,53,81	Nitrogen	Polyimide and Fused Silica
Yu et al. (1995)	250-20000	19,52,102	Nitrogen, Distilled water	Fused Silica
Mala and Li (1999)	Up to 2500	50,80,101,150,205 (FS) 63.5,101.6,130,152,203,254 (SS)	De-ionized water	Fused Silica (FS) Stainless Steel (SS)
Judy et al. (2000)	20-2000	20 to 150	Distilled water, Hexane, Iso-Propanol	Fused Silica
Kandlikar et al. (2001)	50-2600(1) 900-3000(2)	1062(1) and 620(2)	Distilled water	-
Celata et al. (2002)	100-8000	130	R-114 (Freon)	AISI 316 Stainless Steel
Judy et al. (2002)	8-2431	15,20,30,40,50,75,100,125,150	Distilled water, Methanol, Iso-Propanol	Stainless Steel (SS) Fused Silica (FS)
Li et al. (2003)	-	79.9,108.3,137.5,155.5,166.3(G) 100.25,150.18,205.3(S) 128.6,136.5,179.8(SS)	De-ionized water	Glass (G) Silicon (S) Stainless Steel (SS)
Bucci et al. (2003)	100-6000	520, 290 and 170	Degassed water	AISI 316L Stainless Steel
Brutin and Tadrst (2003)	Up to 1000	50,100,150,250,320,500	Tap water, Distilled water	Fused Silica (FS)
Yang et al. (2003)	-	173 to 4010 (10 tubes)	Air, Water, R134a	-
Sharp and Adrian (2004)	20-2900	50-250	De-ionized water, 1-propanol, 20%wt. Glycerol	Fused Silica
Lelea et al. (2004)	Up to 800	100,300,500	Distilled water	Stainless Steel SUS304
Cui and Silber-Li (2004)	0.1-24	2.95, 4.99 and 10.02	Distilled water, Iso-Propanol, Carbon-tetrachloride	-
Asako et al. (2005)	-	150	Air	Fused Silica
Hwang and Kim (2005)	200-10000	244, 430, 792	R-134a	Stainless Steel
Rands et al. (2006)	300-3400	16.6, 19.7, 26.3, 32.2	Water	Fused Silica
Silber-Li et al. (2006)	-	10,5,3	Iso-Propanol	-
Yang and Lin (2006)	From 1000	123, 220, 308, 416, 764, 962	Water	Stainless Steel
Zhao and Liu (2006)	Up to 2500	168,242,315,399,520 and 1000	Distilled Water, Ethanol, Tetra-chloromethane	Quartz-glass(smooth) Stainless Steel(rough)
Celata et al. (2006a)	Up to 6200	259 and 325	De-Ionized water	Glass, Stainless Steel
Celata et al. (2006b)	From 300	31, 50,101,259,70, 116, 326, 300, 126, 300	Degassed water	Fused Silica Glass Teflon

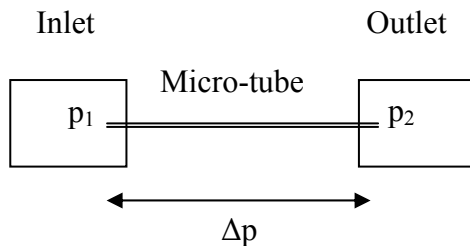


Figure 1: Schematic describing parameters in Equation (1)

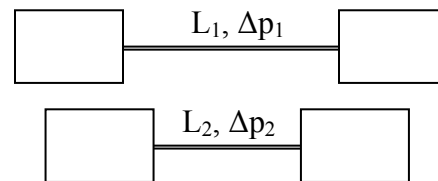


Figure 2: Schematic of pressure measurement using different lengths

Table 2: Summary of Roughness, Critical Re and f-Re results

Authors (Year)	Surface Roughness (RR= Relative Roughness, ϵ/D)	Critical Reynolds Number	f-Re
Choi et al. (1991)	10 nm to 80 nm (RR=0.00017 to 0.0011)	-	↓↓
Yu et al. (1995)	RR=0.0003 for D=50 μ m	-	↓↓
Mala and Li (1999)	1.75 μ m	300-900	↑↑
Judy et al. (2000)	-	-	↓↓ for D < 75 μ m
Kandlikar et al. (2001)	RR 0.00161 to 0.00355	2300 (1062 μ m) < 2300 (for 620 μ m)	↑↑
Celata et al. (2002)	3.45 μ m(RR=0.0265)	1880 to 2480(17 C), 2245 to 2295(33 C)	OK till Re=580, Then ↑↑
Judy et al. (2002)	-	Around 2000 for SS	OK
Li et al. (2003)	Peak to valley: 0.05 μ m for Glass (RR<0.1%) 5.5 μ m for SS (RR = 3% - 4%)	Around 2000	OK for G,S ↑↑ for SS
Bucci et al. (2003)	1.609(520), 2.166(290) and 1.498(170)	1800-3000	OK (Re: 800-1000) ↑↑ for Re>1000
Brutin and Tadrast (2003)	10nm (ϵ/D ratio=0.0002 for D = 50 μ m)	-	↑↑
Yang et al. (2003)	-	1200-3800	OK
Sharp and Adrian (2004)	-	1800-2300	OK
Lelea et al. (2004)	-	-	OK
Cui and Silber-Li (2004)	Absolute Roughness:7.12 nm; RR less than 0.7%	-	↑↑
Asako et al. (2005)	5 nm	-	↑↑
Hwang and Kim (2005)	-	2000	OK
Rands et al. (2006)	-	2100-2500	OK
Silber-Li et al. (2006)	Absolute Roughness :7.12 nm; RR less than 0.7%	-	↑↑
Yang and Lin (2006)	1.4 μ m, RR-0.15% TO 1%	2300-3000	OK
Zhao and Liu (2006)	RR = 8% for SS	1500-1800 for SS	↑↑
Celata et al. (2006a)	Ra < 0.1 μ m(G) ,Ra < 1 μ m (SS)	2000 to 3000	OK
Celata et al. (2006b)	0.2 – 0.7 μ m (RR < 1%)	2000 to 3000	OK

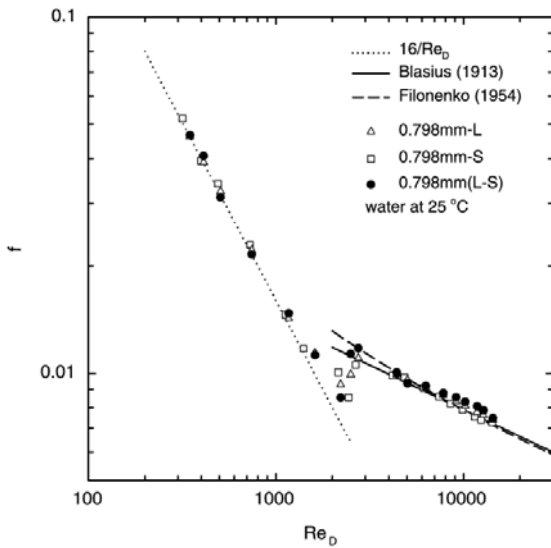


Figure 3: f-Re plot for water from Asako et al. (2005)

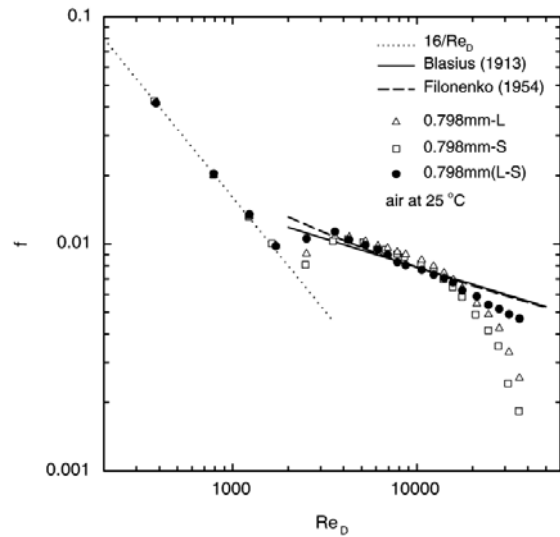


Figure 4: f-Re plot for air from Asako et al. (2005)

Table 3: Summary of Instrumentation Used

Authors (Year)	Method used to pump fluid	Pressure	Flow	Diameter
Choi et al. (1991)	Standard Gas Cylinder (with pressure regulators)	-	Precision Volumetric Flow meter	Computer enhanced interferometric technique and laser interferometry microscope
Yu et al. (1995)	Nitrogen Gas Cylinder	2 Transducers	-	SEM
Mala and Li (1999)	Precision Pump (Ruska Instruments) 2.5-560±0.2 cc/h	Gage and Transducer	(1)read directly from pump (2)flow sensor (3)collecting liquid at known time interval	From manufacturer
Judy et al. (2000)	High Pressure Syringe Pump	Precision transducer	Graduating Cylinder	SEM
Kandlikar et al. (2001)	Bronze Gear Pump	Transducer and Atmosphere	Flow Meter (0-200 cc/min)	SEM
Celata et al. (2002)	Piston Pump	Transducer	-	SEM
Judy et al. (2002)	High Pressure Syringe Pump	Omega PX 202	-	SEM
Li et al. (2003)	Pressurized tank	Transducer and atmosphere	Weighing mass of fluid	SEM
Bucci et al. (2003)	Commercial He gas cylinder	1 Differential 2 Absolute	2 Coriolis Flow Meters	Non-contact surface profiler, laser interferometry microscope
Brutin and Tadrst (2003)	Pressurized tank (using pressostat)	Water Column and Transducer	Mass of fluid measured at exit	Optical μ scope CCD Camera SEM
Yang et al. (2003)	Gear Pump	Differential Transducer	Liquid flow meter and air mass flow controller	Mercury filling to get average diameter
Sharp and Adrian (2004)	Compressed gas Cylinder	CD 15-30 PR transducer	Model 310 balance	SEM
Lelea et al. (2004)	Micro-pump (NS type NP-KX110)	PR transducer	Digital Balance	High Precision Microscope
Cui and Silber-Li (2004)	High Pressure Pump	Transducer	-	SEM
Asako et al. (2005)	Compressed Nitrogen Cylinder	VALCOM, VESX500, and Isuzu fortin barometer	Kofloc 2412T	Atomic Force Microscope
Hwang and Kim (2005)	Syringe pump	Differential Transducer	-	-
Rands et al. (2006)	High Pressure Syringe Pump	Transducer	-	SEM
Silber-Li et al. (2006)	High Pressure Pump	Transducer	-	SEM
Yang and Lin (2006)	Pressurized Nitrogen Reservoir	-	Programming Electronic Microbalance	SEM & OM
Zhao and Liu (2006)	Nitrogen Cylinder and Tank	Two calibrated transducers	Graduating Cylinder	SEM
Celata et al. (2006a)	Gear Pump	Transducer	High Precision Scale	SEM
Celata et al. (2006b)	Gear Pump	Transducer and Differential Manometer	High Precision Scale	SEM

Table 4: Summary of Accuracies (of instruments) and Uncertainties (in measurement)

Authors (Year)	Pressure	Flow	Diameter	Re	f
Choi et al. (1991)	0.5%	2%	3%	-	12.7%
Yu et al. (1995)	-	-	-	-	Laminar 19%, turbulent 5%
Mala and Li (1999)	2%	2%	2%	3 %	9.2%
Judy et al. (2000)	0.1% to 4%	0.1 mL/Δt (2 % in worst case)	1 μm	-	-
Kandlikar et al. (2001)	0.7%	-	-	-	-
Celata et al. (2002)	-	0.7 to 7 %	-	0.1 to 5 %	6 to 7 %
Judy et al. (2002)	0.25 % of maximum reading	0.1 mL	2.5% (FS) 5%(SS)	-	-
Li et al. (2003)	5%	2%	2%	-	-
Bucci et al. (2003)	0.39%	-	-	1.8%	3.6%
Brutin and Tadrist (2003)	(0.57Pa/μV) Accuracy =20Pa	Accuracy 76.29μV= 10mg	1.17%(530μm) 3.2%(100μm)	-	1.7% to 4.56%
Yang et al. (2003)	0.1%	2%, 0.6% 0.015 g	-	-	4.1-9%(w), 4.1-9%- R134a, 3.1-4.8%(air)
Sharp and Adrian (2004)	-	-	3%	-	2.5% r.m.s. random error
Lelea et al. (2004)	-	-	-	5.3%, 0.98% 0.4%	18.7% (125.4), 3.9%(300) 1.5%(500)
Cui and Silber-Li (2004)	0.3%	-	0.1 μm for SEM, 3.3%, 2%, 1%	-	-
Asako et al. (2006)	-	1% (0.1 mg)	0.16 % (2μm)	-	-
Hwang and Kim (2005)	-	-	-	-	8.9%
Rands et al. (2006)	410 kPa or 60 psi	-	1 μm	-	-
Silber-Li et al. (2006)	-	4.8 , 8.6 , 13.6 (10, 5 and 3)	0.1 μm for SEM, 1 - 3.3% for 3- 10 μm tubes	-	-
Yang and Lin (2006)	-	-	-	-	-
Zhao and Liu (2006)	0.2%	2.1%	5 %	5.4%	6.5%
Celata et al. (2006a)	-	-	-	-	13 % and 34 %
Celata et al. (2006b)	-	-	-	-	-

PRESSURE DROP MEASUREMENT AND DATA REDUCTION TECHNIQUES

Friction factor in a macro-scale tube is related to the pressure drop across the tube, as stated by Hagen-Poiseuille law:

$$f = \frac{D}{l} \frac{2}{\rho u^2} \Delta p \tag{1}$$

From Equation (1), it can be observed that the uncertainty in pressure drop measurement is directly proportional to the uncertainty in estimating the friction factor.

In the estimation of pressure drop across a micro-tube, pressure sensors (transducers) and pressure gauges are generally used by various researchers in their experiments.

Column 3 of Table 3 presents a summary of instrumentation used for pressure drop measurement. The accuracies of these instruments are tabulated in column 2 of Table 4. It is observed that the uncertainty in pressure drop measurement varies from 0.1 % to 5 %. However, for pressure drop measurements in a micro-tube in comparison to similar measurements in a conventional macro-scale tube, the location of pressure transducers and various minor losses may play an important role due to the comparatively smaller scales involved.

Pressure drop, Δp , is determined from the following equation:

$$\Delta p = (p_1 - p_2) - \sum L \tag{2}$$

where p_1 and p_2 are inlet and outlet pressures and $\sum L$ is the sum of minor losses. The location of measurement of p_1 and p_2 is shown in Figure 1. The uncertainties in the measurement of Δp originate from (a) the accuracy of instrumentation used and (b) the technique used to determine the minor loss term. Moreover, the following data reduction techniques to estimate the pressure drop across a micro-tube can be found in the literature:

- Neglecting the minor loss term
- Considering the minor loss term
- Using different lengths of the same micro-tube
- Considering minor loss term and using different lengths of micro-tube (to compare the results)
- Other methods

Neglecting the Minor Loss Term

Equation (2) can be approximated to a simple algebraic difference in inlet and outlet pressure. This is done by neglecting the minor loss term (Choi et al., 1991; Yu et al., 1995; Adams et al., 1998; Hwang & Kim, 2005). The ℓ / D ratio of the tubes used in their experiments ranged from about 500 for Hwang & Kim (2005) and from 640 to 8100 for Choi et al. (1991). It can be observed from Table 2 that a consideration of the loss term would have resulted in a further deviation of experimental friction factor, when compared to the theory (Choi et al., 1991; Yu et al., 1995).

Considering the Minor Loss Term

A more accurate version of pressure drop measurement is the consideration of minor loss term. Table 5 summarizes the values of loss coefficients used by different investigators. From Table 5, it can be observed that the sum of loss coefficients considered by Judy et al. (2000, 2002) and Rands et al. (2006) is 3.1, which is almost 2.5 times the value considered by Li et al. (2003). Thus, the coefficients are different for every experiment and the minor-loss term may play an important role in certain cases.

Using Different Lengths of Micro-tubes

To eliminate the effects of inlet and outlet pressure losses various researchers have performed pressure drop experiments using two different lengths of the tube for the same micro-tube (see Figure 2). Since the minor losses are proportional to velocity and not a function of pressure drop, it is required that the flow-rate (i.e. velocity) in the two tubes must be maintained constant. An algebraic difference between the two readings results in a pressure drop value that can be used directly in Equation (1) to determine the friction factor, thereby eliminating the calculation of loss coefficients. The obvious advantage of this method is the elimination of the fudge factor involved in determining the coefficients which have different values in literature. The disadvantage is that precise machining of the micro-tubes is required to obtain burr free identical ends.

Table 5: Minor Loss Coefficients from Literature

Authors	Year	Coefficients	Value
Li et al.	2003	Contraction	0.5
		Vortex, acceleration, striking and veering	0.25
		Variation of momentum	0.33
		Variation of velocity profile	0.1
Judy et al.	2000	Exit	1.0
Judy et al.	2002	Entrance	0.8
Rands et al.	2006	Developing Length	1.3
Sharp and Adrian	2004	Sum of Minor Losses	0.6 %
		Entrance Length (worst case)	3 %
Lelea et al.	2004	Contraction and Expansion coefficients depend on cross-section ratios	

Considering Minor Loss Term and using Different Lengths of Micro-tube (to compare the results)

One way to check the effects of minor losses on the pressure drop calculation is to perform data reduction by methods explained in previous sections. In the experiment of Asako et al. (2005), the minor losses were considered in an identical manner as Lelea et al. (2004) and results obtained from both methods are plotted in Figures 3 and 4. Good agreement is observed between both the methods, which led Asako et al. (2005) to conclude that minor loss coefficients for macro-scale flow are valid for micro-tubes.

Other Methods

Kandlikar et al. (2001) used EDM (Electrical Discharge Machining) process to make slits on the surfaces of micro-tubes. Pressure tapping (see Figure 5) was done from these slits and the authors claim that the geometry was free from sudden changes in cross-section due to which the authors neglect the effect of minor losses.

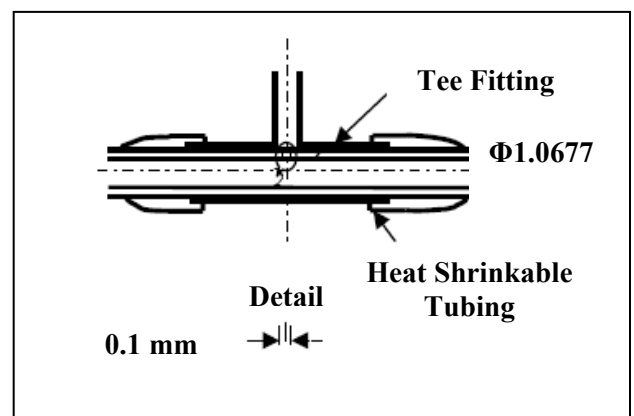


Figure 5: Pressure tapping on the micro-tube from Kandlikar et al. (2001)

Celata et al. (2002) used headers that are much larger than the micro-tube diameters. The velocity of the test-fluid in the headers can be made negligibly small. This way the authors assume static pressure conditions at the micro-tube inlet and exit. The schematic of their experimental set-up is shown in Figure 6.

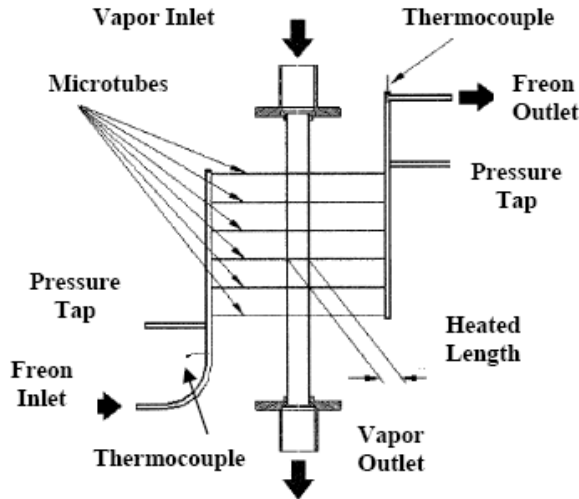


Figure 6: Pressure taps on inlet and outlet headers from Celata et al. (2002)

Analyzing all the above mentioned methods of pressure measurement and data reduction it can be concluded that there is not an appreciable difference between the different methods and loss coefficients can be safely assumed from the macro-scale theory for water flow up to diameters of 800 μm . Future experimenters using tube-cutting method for pressure drop measurement should compare their data with results obtained using minor loss coefficients. This will enable us to find the diameters up to which the minor loss coefficients can be confidently used.

FLOW RATE MEASUREMENT

The various flow measurement techniques used in literature are tabulated in column 4 of Table 3. They are:

- Coriolis flow meters (Bucci et al., 2003 ; Celata et al., 2002)
- Rotameters (Adams et al., 1998; Choi et al., 1991; Kandlikar et al., 2001 ; Mala & Li, 1999 ; Yang et al., 2003)
- Flow sensor or transducer (Asako et al., 2005; Mala & Li, 1999)
- Measuring mass and time at exit using graduated cylinder and stop-watch (Brutin & Tadrist, 2003; Hwang & Kim, 2005; Mala & Li, 1999 ; Sharp & Adrian, 2004 ; Yang & Lin, 2006 ; Zhao & Liu, 2006)

The uncertainties of the different flow measurement techniques are tabulated in column 3 of Table 4. In most cases, the uncertainty does not seem to exceed 2%. However, a few researchers report uncertainties as high as 7 % (Celata et al., 2002) and 13 % (Silber-Li et al., 2006: The method by which flow is measured is not mentioned).

Mala & Li (1999) performed the flow rate measurement by three different ways: (a) reading directly from the pump, (b) using a flow sensor and (c) by collecting the liquid at a known time interval. They found that the variation in all the three readings were less than 1%. Thus they showed that flow rate measurement may not be the most important factor in uncertainty analysis.

DIAMETER AND ROUGHNESS MEASUREMENT

The most significant factor that affects the uncertainty is the diameter measurement. Precise measurement of micro-tube diameter is hence an inherent part of micro-tube studies. As observed from Table 3, almost 15 out of 23 researchers have used Scanning Electron Microscope (SEM) for accurate diameter measurement. Figure 7 shows a SEM image from Celata et al. (2002). The uncertainties reported in literature vary from 0.16 % to 5 %. Judy et al. (2002) reported that the uncertainty involved is at least 2.5 % for fused-silica tubes and 5 % for stainless steel tubes.

In early studies Choi et al. (1991) and Yu et al. (1995) measured the roughness using computer enhanced laser interferometric technique (WYCO Corp.). They used a computer-software to determine the roughness by scanning images of the surface profiles. The authors reported a measurement uncertainty of 0.2 nm in this process. Brutin and Tadrist (2003) used an Atomic Force Microscope (AFM) to measure the roughness of a (5 $\mu\text{m} \times 5 \mu\text{m}$) sample surface. They observed an average surface roughness of two orders: (a) a low basic roughness below 2 nm (about AFM resolution) and (b) a higher roughness of order 10 nm which was calculated from a surface profile.

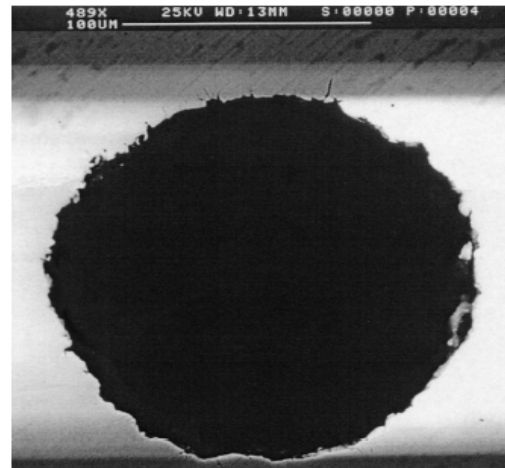


Figure 7: SEM image of 130 μm stainless steel tube from Celata et al. (2002)

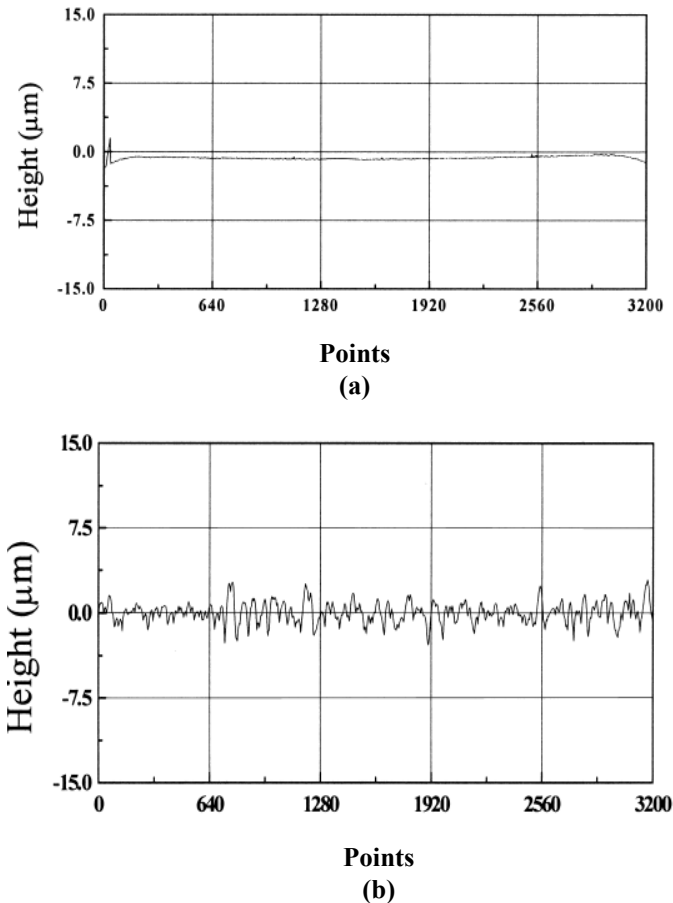


Figure 8: Roughness profile of (a) fused silica and (b) stainless steel tube from Li et al. (2003)

Kandlikar et al. (2001) created different roughness values by etching the micro-tubes with two different acids. The roughness was measured by a profilometer (Alpha-Step 200). Li et al. (2003) used the Talysurf-120 tester to measure the peak-to-valley roughness. Their analyses showed their fused-silica tubes to be much smoother than the stainless steel tubes, as shown in Figure 8. The effect and significance of roughness in micro-tube pressure drop studies is explained in detail in the next section.

OBSERVATIONS

- While some experimenters find an increase in transition Reynolds number with increasing micro-tube diameters (Li et al., 2003; Zhao & Liu, 2006) others observe an exact opposite result (Bucci et al., 2003; Yang et al., 2003; Rands et al., 2006). Further experiments are thus needed to confirm the behavior of transition Reynolds numbers with change of scales. Moreover, the exact start and end of transition region is something that has still not been confirmed by the experimenters.

- Roughness does seem to play a major role in the dynamics of the fluid. As seen from Table 6 for fused-silica tubes, the only case where (a) $f-Re$ product varies with Reynolds number and (b) there is evidence of early transition; is the experiment by Mala and Li (1999).

The following are observations from Table 6:

- Mala and Li's (1999) experiment is the only one in the table with large relative roughness (0.017 and 0.035) compared to other studies – Almost 10 times more than the others.
- Even though, the other experiments exhibit inconsistent results (which may be attributed to experimental uncertainty or inaccuracies in instrumentation), it is interesting to note that none of them talk about (a) variation of $f-Re$ product with Re and (b) early transition.
- For stainless steel tubes roughness and diameter do seem to play a major role.

The following are observations from Table 7:

- Yang and Lin (2006): All the diameters have low roughness, so the tubes behave just like macro-scale tubes. This behavior is similar to fused-silica tubes.
- Kandlikar et al. (2001):
 - At 1067 μm , relative roughness varying from 0.0018 to 0.0028 does not affect the flow. The 962 μm (from Yang and Lin, 2006) can also be included in this group due to similar diameter and roughness.
 - But for 620 μm tube, relative roughness of 0.00355 results in transition at $Re < 2000$ and higher friction factors are observed; Also, the 520 μm tube (Bucci et al., 2003) exhibits similar trends.
- Bucci et al. (2003): Their roughness is one order of magnitude more than the tubes used by Yang and Lin (2006) for the similar diameters. Their behavior is akin to tubes of Kandlikar et al. (2001) and Yang and Lin (2006) (for similar tube sizes).
- Mala and Li (1999): The tubes have high relative roughness (0.028), thereby resulting in higher friction factors.
- Li et al. (2003): These tubes also have high relative roughness (0.031 – 0.043); hence exhibit higher friction factors.
- Anomalies: Bucci et al. (2003): Transition Reynolds number increases with decrease in micro-tube diameter.

Table 6: Effect of diameter or roughness on fused silica and glass micro-tubes

Authors	Diameter (μm)	Roughness (μm)	Relative Roughness ϵ/D	f·Re	Transition
Choi et al. (1991)	3	0.175	0.0058	f·Re = 53 (Figure 4, Choi et al., 1991)	Between Re=2000 to 3000 (Figure 3, Choi et al., 1991)
	6.9	0.0797	0.0116		
	9.7	0.0104	0.0011		
	53	0.0144	0.00027		
	81.2	0.014	0.00017		
Yu et al. (1995)	52.1	-	0.0003	f·Re= 50.13 (Figure 3, Yu et al., 1995)	Between Re=2000 to 3000 (Figure 2, Yu et al., 1995)
Mala and Li (1999)	50	1.75	0.035	f·Re varies with Re (Figure 5, Mala and Li, 1999)	Early transition (Figure 4, Mala and Li, 1999)
	101		0.017		
Brutin and Tadrst (2003)	52.81 107.45 152.28 262 320.7 539.69	0.010	Less than 0.0002	81.5 79.6 77.7 71.2 68 66.7 (Figure 4, Brutin and Tadrst, 2003)	Laminar flow regime (no transition observed)
Li et al. (2003) (GLASS)	79.9 108.3 137.5 155.5 166.3	0.05 (Peak-to-Valley)	0.001	60 67 60 58 61 (Figure 7, Li et al., 2003)	Between Re=2000 to 2500 (Figure 6, Li et al., 2003)

Table 7: Effect of diameter or roughness on stainless steel micro-tubes

Authors	Diameter (μm)	Roughness (μm)	Relative Roughness ϵ/D	f·Re	Transition
Mala and Li (1999)	63.5	1.75	0.02775	f·Re varies with Re (Figure 5, Mala and Li, 1999)	Early transition (Figure 4, Mala and Li, 1999)
Kandlikar et al. (2001)	1067	2.4	0.00225	Effect of roughness negligible (falls within error band)	Between Re=2000 to 3000 Early transition for $\epsilon/D = 0.00355$ (Figures 17,19, Kandlikar et al., 2001)
		1.9	0.00178		
620	3.0	0.00281			
	2.2	0.00355			
Celata et al. (2002)	130	3.42	0.0265	OK till Re=580; then f·Re increases (Figure 7, Celata et al., 2002)	Between Re=1880 to 2480 (Figure 6, Celata et al., 2002)
Li et al. (2003)	128.76 136.5 179.9	5.5	0.04271	15% higher	Between Re=1750 to 2500 (Figure 8, Li et al., 2003)
			0.04029	37% higher	
			0.0309		
Bucci et al. (2003)	520 290 170	1.609	0.00309	OK till Re=800; then f·Re increases (Figure 8, Bucci et al., 2003)	Between Re=1800 to 3000 (Figures 5,6,7, Bucci et al., 2003)
		2.166	0.0075		
		1.488	0.00871		
Yang and Lin (2006)	123	1.4	0.0114	No significant effect of roughness and diameter	Between Re=2000 to 3000 (Figure 4, Yang and Lin, 2006)
	220.4	1.48	0.00672		
	308.3	1.34	0.00435		
	416.1	1.46	0.00351		
	763.5	1.16	0.00152		
962.0	1.4	0.00146			

CONCLUSIONS

- There does not seem to be a significant effect of pressure and flow measurement on the experimental uncertainty as compared to uncertainties in diameter measurement.
- Observations from Table 6 indicate in the assumption that all other studies that have not provided any roughness data for their fused silica tubes:
 - Those which report (a) experimental friction factor data and (b) transition similar to macro-scale theory → “*must have used smooth tubes*”.
 - Those which report (a) higher experimental friction factor data and (b) hints of early transition → “*must have used rough tubes*”.
- Observations from Table 7 indicate that:
 - For smooth tubes, transition follows macro-scale theory and does not depend on diameter.
 - For rough tubes, a slight dependence of transition on the diameter is observed. However this variation of transition cannot be explained by the present experiments. Further controlled research is recommended to understand this behavior of transition.

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