

HT2007-32408

## SINGLE-PHASE HEAT TRANSFER IN MICRO-TUBES: A CRITICAL REVIEW

Chandramoulee Krishnamoorthy, School of  
Mechanical and Aerospace Engineering,  
Oklahoma State University, Stillwater,  
Oklahoma, 74078, USA  
[chandra.krishnamoorthy@okstate.edu](mailto:chandra.krishnamoorthy@okstate.edu)

Rahul P. Rao, School of Mechanical and  
Aerospace Engineering, Oklahoma State  
University, Stillwater, Oklahoma, 74078, USA  
[rahulpr@okstate.edu](mailto:rahulpr@okstate.edu)

Afshin J. Ghajar, School of  
Mechanical and Aerospace  
Engineering, Oklahoma State  
University, Stillwater, Oklahoma,  
74078, USA  
[ghajar@ceat.okstate.edu](mailto:ghajar@ceat.okstate.edu)

### ABSTRACT

This review paper specifically concentrates on heat transfer in micro-tubes and eleven experiments (on liquid flow) and two experiments (on gaseous flow) from 1991 to 2007 are reviewed critically with respect to measurement techniques, instrumentation; and factors like surface roughness and diameter that may play an important role at these small scales. Moreover, a comprehensive list of numerical and analytical results (for both liquid and gaseous flows) is presented in this paper. Interestingly, the effect of surface roughness on heat transfer does not seem to have been investigated thoroughly, as it has been observed to play a key role in influencing heat transfer at small diameters. The state-of-art review thus provides the contemporary experimenters in the field of mini-micro channel heat transfer, this tabulated data that can be used to understand how the different parameters affect the heat transfer in these small scales and a data-bank to validate future numerical and experimental work. The present study identifies the various factors that have contributed in the disparity of results found in the literature and finds that there is a need to investigate certain issues like the effects of roughness, diameter, and secondary flow due to buoyancy on heat transfer and transition. Moreover, it was observed that the start and end of the transition region at these small diameters are not validated by the any of the existing macro-scale correlations.

### INTRODUCTION

Advances in micro-fabrication techniques have resulted in miniaturization of technology in almost all fields of engineering

and bio-mechanics. Devices like micro heat-exchangers show enhanced heat transfer owing to the increase in surface-to-volume ratio at small scales. Electronics cooling, micro-refrigeration, cooling of reactor cores and blankets are some of the key areas where micro heat-exchangers are employed. The development of Micro-Electro Mechanical Systems (MEMS) has found major applications in sensor technology, micro-fluidic actuators and a myriad of new applications in bio-medical engineering. However, previous experiments in single-phase heat transfer in micro-channels have shown a lot of disagreement with classical behavior. No concrete conclusion regarding the adherence to macro-scale behavior has been confirmed so far. Prior reviews (Obot, 2000; Palm, 2001; Papautsky et al., 2001; Sobhan and Garimella, 2001; Rostami et al., 2002) along with recent reviews (Celata, 2004; Morini, 2004; Hestroni, 2005) present a comprehensive information on heat transfer and fluid flow in micro-channels and micro-tubes. Micro-tubes are chosen in this study over other non-circular micro-channels to negate the aspect ratio effects on the flow that might affect the heat transfer at such small scales. Measurement and instrumentation techniques are reviewed for all the experiments to gain insight so as to identify the factors responsible for the observed deviation from macro-scale correlations. The study has been necessitated due to the large scattering of results amongst the existing experimental studies. The following are the objectives of this paper:

- Provide a tabulated data-bank with information on temperature measurements, instrumentation,

uncertainties and observations for future researchers in the field of single-phase heat transfer in micro-tubes.

- Identify the various causes for discrepancies or contradictions observed in the existing literature.
- Investigate and tabulate the effect of roughness and micro-tube diameters on the transition Reynolds number

## NOMENCLATURE

a = constant in Eq. (6)  
b = constant in Eq. (6)  
Br = Brinkmann number  
c = constant in Eq. (6)  
C<sub>p</sub> = specific heat of fluid, J/(kg-K)  
D = diameter of micro-tube, m  
f = Darcy friction factor  
h = heat transfer coefficient, W/m<sup>2</sup>-K  
H = height of substrate, m  
j<sub>H</sub> = Colburn j-factor  
k = thermal conductivity, W/(m-K)  
Kn = Knudsen number  
L = length of micro-tube, m  
n = power-law exponent  
N = inverse power-law exponent (= 1/n)  
Nu = Nusselt number  
Pr = Prandtl number  
Re = Reynolds number  
u = flow velocity, m/s  
z = axial coordinate, m

## Greek Symbols

Δ = aspect ratio (=D/H)  
ε = roughness height, m  
η = kinematic viscosity, m<sup>2</sup>/s  
λ = thermal conductivity ratio (=k<sub>s</sub>/k<sub>f</sub>)  
ξ = dimensionless axial coordinate (=z/L)  
ρ = density of fluid, kg/m<sup>3</sup>  
μ = viscosity of fluid, Pa-s

## Subscripts

f = fluid  
l = laminar  
w = wall  
s = substrate  
t = turbulent  
tr = transition

## MACRO-SCALE HEAT TRANSFER CORRELATIONS

Various macro-scale correlations are used by researchers to compare and validate their experimental results. These correlations have performed with exceptional accuracy for heat transfer data in macro-tubes. However, at the micro-scales, there is considerable doubt regarding their validity. They are:

### Laminar regime:

- Sieder-Tate correlation (Sieder and Tate, 1936)

For  $Re_f Pr_f > 10$

$$Nu_f = 1.86 \left( Re_f Pr_f \frac{D}{L} \right)^{1/3} \left( \frac{\eta_f}{\eta_w} \right)^{0.14} \quad (1)$$

- Hausen correlation (Hausen, 1959)

$$Nu_f = 3.66 + \frac{0.19 \left( Re_f Pr_f \frac{D}{L} \right)^{0.8}}{1 + 0.117 \left( Re_f Pr_f \frac{D}{L} \right)^{0.467}} \quad (2)$$

Range of validity:

$$Re_f < 2200, Pr_f = 0.5 - 17000, Re_f Pr_f < 10$$

$$\eta_f / \eta_w = 0.044 - 9.8$$

- Shah correlation (Shah, 1975)

$$Nu_f = \left( 4.364 + 0.0722 Re_f Pr_f \frac{D}{L} \right) \left( \frac{\mu_f}{\mu_w} \right)^{0.14} \quad (3)$$

$$\text{For } Re_f \leq 2200; Re_f Pr_f \frac{D}{L} \leq 33.3$$

If  $Re_f Pr_f \frac{D}{L} \geq 33.3$ , then Shah Correlation becomes

$$Nu_f = 1.953 \left( Re_f Pr_f \frac{D}{L} \right)^{1/3} \left( \frac{\mu_f}{\mu_w} \right)^{0.14} \quad (4)$$

### Transitional regime:

- Hausen correlation (Hausen, 1959)

$$Nu = 0.116 (Re^{2/3} - 125) Pr^{1/3} \left[ 1 + \left( \frac{D}{L} \right)^{2/3} \right] \left( \frac{\mu_f}{\mu_w} \right) \quad (5)$$

- Tam - Ghajar correlation (Tam and Ghajar, 2006)

$$Nu = Nu_l + \{ \exp[(a - Re)/b] + Nu_t^c \}^c \quad (6)$$

Nu<sub>l</sub> = laminar flow Nusselt Number

Nu<sub>t</sub> = turbulent flow Nusselt Number

a, b, c = Constants depending on inlet geometry  
(Please refer to Tam and Ghajar (2006) for further details on experiments, expressions for Nu<sub>l</sub> and Nu<sub>t</sub> and values of constants.)

- Gnielinski correlation (Gnielinski, 1976):

$$Nu_f = \frac{(f/2)(Re_f - 1000) Pr_f}{1 + 12.7(f/2)^{1/2} (Pr^{2/3} - 1)} \quad (7)$$

$$f = \frac{1}{(3.64 \log(Re_f) - 3.28)^2} \quad (8)$$

Range of Validity:  $2300 < Re_f < 5 \times 10^6$

**Turbulent regime:**

- Dittus-Boelter correlation (Dittus and Boelter, 1930):  

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \quad (9)$$

- Colburn analogy (Colburn, 1933):  

$$\frac{8j_H}{f} = 1 \quad (10)$$

$$j_H = \frac{Nu}{Re Pr^{1/3}} = \frac{h Pr^{2/3}}{u \rho C_p} \quad (11)$$

- Petukhov analogy (Petukhov, 1970):  

$$\frac{8j_H}{f} = \frac{Pr^{2/3}}{K + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \quad (12)$$

$$K = 1.07 + (900/Re) - [0.63/(1 + 10Pr)] \quad (13)$$

Range of Validity:  $10^4 < Re < 10^6$  and  $0.5 < Pr < 2000$

**LITERATURE REVIEW**

The review treats the experimental and numerical studies on liquids and gases separately. Experiments on heat transfer in micro-tubes exhibit a mixed bag of results and some of the numerical studies on liquid flow confirm some contradictions with the experiments. A summary of experiments is shown in Table 1 while the details of the measurements and uncertainties are shown in Table 2. Owing to the difficulty in conducting experiments on gases, it is observed that most of the gas-flow studies are either numerical or analytical. For gas flows, the effect of slip-flow and compressibility on Nusselt number and heat transfer is discussed in brief.

Experimental studies on liquid flow:

Yu et al. (1995) performed heat transfer experiments in fused-silica tubes using distilled water as test fluid. Their experiments found that at low Reynolds number, the micro-tube heat transfer results agree with theory. However, for turbulent flows higher Nu values are observed and the divergence was found to increase with Re. The authors observed that the index of Prandtl number is reduced and Colburn analogy, Equation (10), was found to be invalid as  $j_H$  changed with Reynolds number instead of being a constant value for macro-scale flow.

Adams et al. (1998) investigated the heat transfer characteristics in micro-channels with diameters of 760  $\mu\text{m}$  and 1090  $\mu\text{m}$  for turbulent, single-phase forced convection of water. They found the Nusselt numbers to be higher than those predicted by classical correlations. Moreover, the authors found the deviation in the results increased with decrease in the tube diameter and increase in Reynolds numbers. Thus, their results were similar to those of Yu et al. (1995). A modified version of the Gnielinski correlation, Equations (7) and (8), was obtained

to accommodate the smaller diameters, see Equations (14) and (15). The difference between Nusselt numbers for theoretical values and experimental values was observed in the range of  $\pm 18.6\%$ .

$$Nu = Nu_{Gn}(1 + F) \quad (14)$$

Where F is given by:

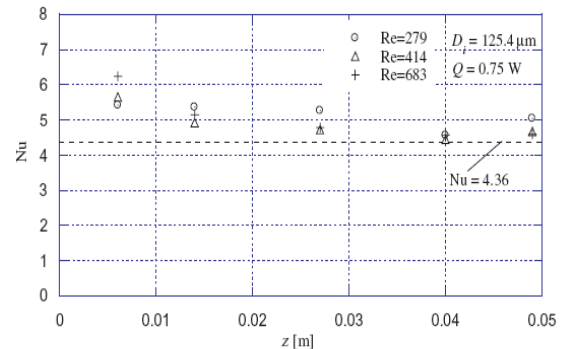
$$F = C Re \left[ 1 - \left( \frac{D}{D_o} \right)^2 \right] \quad (15)$$

$C = 7.6 \times 10^{-5}$  and  $D_o = 1.164\text{mm}$ .

Kandlikar et al. (2001) investigated the effect of roughness on heat transfer in micro-tubes. The roughness was changed by etching the tubes with different acids. They observed that for the larger tube (1067  $\mu\text{m}$ ), the effect of roughness is negligible. For smaller tube (620  $\mu\text{m}$ ) an increase in roughness resulted in enhanced heat transfer.

Celata et al. (2002) performed experiments on the thermal characteristics of capillary tubes of diameter 130  $\mu\text{m}$ . They concluded that the conventional correlations were not valid to calculate the heat transfer coefficient. The range of transition regime for micro-scale and macro-scale were found to be in good agreement. They concluded that more investigations are necessary to uncover the reason for the variations.

Bucci et al. (2003) examined the flow characteristics and forced convective heat transfer of water flowing in micro-tubes. The traditional correlations were not sufficiently adequate for the calculation of heat transfer coefficients. Heat transfer coefficient was found to be higher in the laminar regime while in the turbulent regime the variation was small. The smaller micro-tube showed a higher deviation compared to the others, but was in better agreement with the Adams et al. (1998) correlation; see Equations (14) and (15).

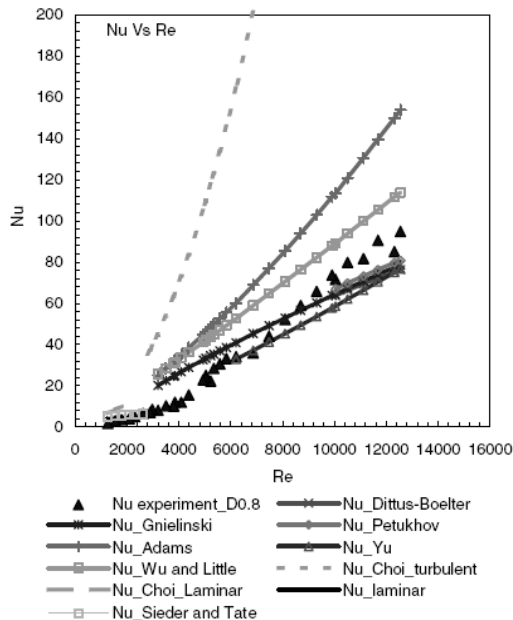


**Figure 1: Results of Nusselt number for tube diameter of 125.4 $\mu\text{m}$  with a heat input of 0.75 W (From Lelea et al., 2004)**

Lelea et al. (2004) performed experiments on micro-tube heat transfer using water as the working fluid with a Reynolds number range of up to 800. In reference to their experimental set-up, they found the Nusselt numbers to be in good agreement to the conventional theories (see Figure 1). It was found that the

Nusselt number was higher than theory at very low Reynolds numbers but reached a constant value as the Reynolds number increased. The heat input was found to be in the order of 0.5, 0.75, 1, and 2W.

Owhaib and Palm (2004) conducted heat transfer experiments for single-phase forced convection flow of R134a through micro-tubes. They found that the data obtained agreed well with the classical macro-scale correlations but were not in agreement with the micro-scale correlations for the experimental data (see Figure 2). The variation of heat transfer coefficient is higher for smaller diameter tubes for turbulent flows.



**Figure 2: Comparison of experimental Nusselt number with classical and micro-scale correlations. (From Owhaib et al., 2004)**

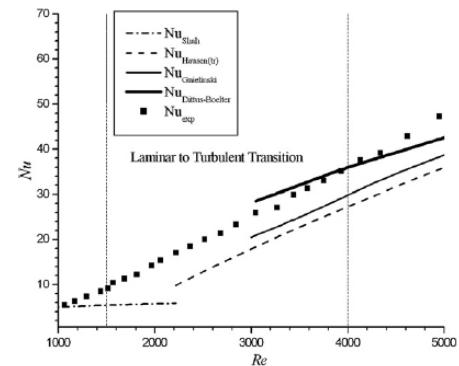
Grohmann (2005) conducted experiments on heat transfer measurements of micro-tubes using argon as the test fluid. The data obtained were in agreement with conventional correlations of macro-scale heat transfer coefficients. They explained the enhanced heat transfer coefficients to the increased influence of roughness which could be the decisive factor. They found there was no diameter dependence on the enhancement. They added a new roughness parameter to accommodate the enhancement in the correlation and the maximum error was less than 2.6%.

Zhao and Liu (2006) studied the heat transfer characteristics and flow visualization of three working fluids using two different materials of tubes. The transition from laminar to turbulent was found to be much earlier than those predicted by the macro-scale theory. They found that the experimental Nusselt numbers were in agreement with the classical correlations only for low Reynolds number. For turbulent flow higher Nu values were observed.

Yang and Lin (2006) conducted an experimental investigation on convective heat transfer of water flowing

through micro-tubes of six different diameters. Their experimental data were in agreement with the conventional correlations for both laminar and turbulent regimes. For  $Re > 1000$ , heat transfer coefficients were found to increase with increasing Reynolds number. They attribute this to the fact that the flow is in the developing condition and the tube is not long enough to achieve fully developed flow.

Celata et al. (2006) investigated experimentally the behavior of single-phase flow in micro-tubes with water as the working fluid. Thermally developing flow effects were observed in larger diameter while smaller diameter exhibited low Nu values at small Reynolds numbers. They concluded that the latter phenomena might be the effect of an unaccounted dissipation term which is negating the convective heat absorption, thereby giving a decreased value. Investigations on a tube of diameter  $50 \mu\text{m}$  proved to be unrealistic due to the difficulty in the experimental set-up and the sensitivity of measurement errors resulting in large variation of heat transfer coefficient.



**Figure 3: Comparison of Nusselt number with Reynolds number for constant heat flux. (From Liu et al., 2007)**

Liu et al. (2007) performed experiments on the forced convective heat transfer of de-ionized water flowing through quartz micro-tubes. They conducted experiments with both the iso-flux and iso-thermal boundary conditions and found the difference considering both methods is small for laminar and transitional regimes. The authors found that Nusselt numbers were in agreement with the classical correlations in the laminar regime. For very low Reynolds numbers, the authors attribute the deviation to conjugate heat transfer effects. For the turbulent regime, they observed higher experimental Nusselt numbers than those predicted by conventional correlations. Moreover, the authors found that the effect of type of heating method diminishes at higher Reynolds numbers.

#### Numerical studies on liquid flow:

Maynes and Webb (2004) investigated electro-osmotic flow through micro-tubes and found that viscous heating has no effect on Nusselt number at micro-scales. The authors note that reduction in Nusselt number for the iso-thermal boundary

**Table 1: Summary of Experiments**

Author (Year)	Test Fluid	Tube Material	Micro tube Diameters (μm)	Re Range	Correlation compared
Choi et al. (1991)	Nitrogen	Fused Silica	3,7,10,53,81	20 - 20000	Colburn , Petukhov , Dittus-Boelter
Yu et al. (1995)	Nitrogen, Distilled water	SS 304	19,52,102	250 - 20000	Sieder-Tate (Laminar ) Dittus-Boelter (Turbulent)
Adams et al. (1998)	Distilled water	Copper block with passage created by EDM	760,1090	2600 - 26000	Petukhov, Gnielinski
Kandlikar et al. (2001)	Distilled water	SS	620,1032	500 - 3000	-
Celata et al. (2002)	R114	AISI SS 316	130	100 - 8000	Hausen (Laminar) Dittus-Boelter , Gnielinski, Modified Gnielinski
Bucci et al. (2003)	De-mineralized and degassed water	SS 316L	172,190,520	200 - 6000	Hausen (Laminar) Gnielinski, Modified Gnielinski
Lelea et al. (2004)	Water	SUS 304	100,300,500	Up to 800	-
Owhaib and Palm (2004)	R134a	SS	1700,1200,800	Less than 1000 up to 17000	Sieder-Tate (Laminar) Petukhov, Gnielinski, Dittus-Boelter
Grohmann (2005)	Argon	Copper	250,500	Around 2000 - 12000	Gnielinski
Celata et al. (2006)	Degassed and De-mineralized water	Glass	528,325,259,120,50	≥100	Gnielinski
Zhao and Liu (2006)	Water, Ethanol, Tetra-chloromethane	Quartz Glass and Rough SS	168,242,315,399,520,799	-	Hausen, Sieder-Tate
Yang and Lin (2006)	Water	SS	123,220,308,416,764,962	Around 100 - 13000	Dittus-Boelter, Petukhov Gnielinski, Shah & Bhatti
Liu et al. (2007)	De-ionized water	Quartz	242, 315, 520	100 - 7000	Shah Hausen, Sieder-Tate (Laminar) Hausen, Gnielinski (Transition) Gnielinski, Dittus-Boelter (Turbulent)

condition and an increase in Nu for the iso-thermal condition. Moreover, it is interesting to note that electro-osmotic phenomena takes place at extremely small Reynolds number, that are non-existent for high heat flux electronics-cooling applications. Liechty et al. (2005) extended the work of Maynes and Webb (2004) and studied the effect of arbitrary wall zeta potential on electro-osmotic heat transfer in micro-tubes. Their numerical results indicate that the predicted Nusselt numbers are lower than those calculated by assuming low wall zeta potential.

Gari and Rahman (2005) numerically analyzed the effect of magnetic field on water flow through gadolinium micro-tube. Periodic application and removal of the magnetic field resulted in similar periodic variation in Nusselt number. The authors found that Nusselt number increased with higher magnetic field strength and Reynolds number. Moreover, they observed that changes in diameter had the highest effect on Nusselt number.

Croce and D'Agaro (2004) investigated the effect of roughness on heat transfer in micro-tubes by employing a finite-element code. They concluded that the effect of roughness on heat transfer is quite small and within the Experimental uncertainties. However, they note that the geometry of the duct and shape of roughness may significantly alter Nusselt numbers. Bahrami et al. (2005) in their numerical

study observed that higher surface roughness results in an increase in surface area that ultimately causes an enhancement in heat transfer. However, owing to the wide-scale disparity in experimental studies, the authors did not validate their model. Koo and Kleinstreuer (2005) modeled the surface roughness by a porous medium layer and found that roughness doesn't significantly affect the heat transfer. Moreover, the authors suggest roughness does not play any role in Reynolds number dependence of Nusselt number. They found that thermal conductivity ratio between porous medium layer and bulk fluid is the most significant parameter affecting the heat transfer phenomenon.

Sharath et al. (2004) numerically simulated conjugate heat transfer in micro-tubes with rectangular substrates. They found that heat transfer coefficients and Nusselt number depend on substrate and coolant properties. Their results compare with the experimental results of Owhaib and Palm (2004) as seen from Figure 2. However, the results do not agree with laminar macro-scale correlations of Hagen-Poiseuille and Sieder-Tate, Equation (1). Sharath et al. (2006) developed a correlation to predict conjugate heat transfer in a micro-tube, given by Equation (16).

$$Nu = (Re)^{0.225}(Pr)^{0.465}(\lambda)^{0.015}(\xi)^{-0.675}(\Delta)^{0.585} \quad (16)$$

**Table 2: Measurements and Uncertainties**

Author (Year)	Boundary condition	Temperature measurement	Uncertainty		
			T	h	Nu
Choi et al. (1991) (COMPRESSIBLE)	Iso-flux	T-type thermocouple (Copper-Constantan)	12.7%	23.8%	24%
Yu et al. (1998)	Iso-thermal	Thermocouple	-	Value higher than convention	23%
Adams et al. (1998)	Iso-thermal	T-type thermocouple (Copper-Constantan)	±0.3°C	Value higher than convention	High Nu obtained after using Adams et al. (1998) correlation with a difference of 18.6%
Kandlikar et al. (2001)	Iso-flux	K-type thermocouple	-	Value higher	Nu - 5% deviation at Re - 2000 At entry less than theoretical
Celata et al. (2002)	Iso-thermal	No thermocouple. Vapor temperature measured.	-	h value higher than convention	20% for Re<1000 5% for Re<2500 0.5% for Re >2500
Bucci et al. (2003)	Iso-thermal	No thermocouple. Condensed vapor temperature measured.	Generally less in Laminar region	±22.24%	±22.25%
Lelea et al. (2004)	Iso-flux	K-type thermocouple	≤7.9%	≤8.9%	Nu higher at low Re
Owhaib and Palm (2004)	Iso-flux	T-type thermocouple	±0.2 °C	-	-
Grohmann (2005)	Iso-flux	PT 100 sensors	-	h – up to 2.6% for laminar Up to 3.8% for turbulent	Nu – up to 2%
Celata et al. (2006)	Iso-flux	K-type thermocouple	0.1K	-	A decrease in Nu up to transition has been found
Zhao and Liu (2006)	Iso-flux	Infrared Camera K-type thermocouple	0.3 °C	-	±8.5%
Yang and Lin (2006)	Iso-flux	Liquid Crystal Thermography	0.4 °C	-	For laminar flow Nu is in agreement 2 – 12%
Liu et al. (2007)	Iso-flux & Iso-thermal	K-type thermocouple	0.1°C	Difference of around 15% in Nu values at low Re for the two heating methods	For laminar flow Nu is in agreement

Experimental studies on gas flow:

Choi et al. (1991) were the first researchers to investigate the heat transfer coefficients for flow of nitrogen gas in micro-tubes. Their data indicated significant deviations from the conventional correlations of macro-scale tubes. The Nusselt numbers on the turbulent flow for the heat transfer coefficients were found to be much larger than those predicted by the Colburn analogy. None of the correlations were in agreement with the experimental data obtained. They suggested that suppression of the turbulent eddy motion in the radial direction because of the small diameter of the tube to be one of the reasons for this deviation. Their correlation for laminar and turbulent regimes is shown in Equations (17) and (18), respectively.

For Re < 2000,

$$Nu = 0.000972 Re^{1.17} Pr^{1/3} \quad (17)$$

For 2500 < Re < 20,000

$$Nu = 3.82(10^{-6}) 0.000972 Re^{1.96} Pr^{1/3} \quad (18)$$

Numerical and analytical studies on gas flow:

Guo and Wu (1997) were one of the first to numerically investigate the effect of compressibility on heat transfer in micro-tubes. They used the forward difference forward marching procedure to solve the governing equations of momentum and energy. The authors note that a fully developed condition can never occur for a compressible gas flow for both the velocity and temperature profiles. They observed that Nu increased along the flow direction with an increase in compressibility.

Ameel et al. (1997) presented an analytical solution of laminar flow for gases in very small micro-tubes. The authors studied the effect of Knudsen number on heat transfer characteristics of the fluid. Their results showed that for the constant heat flux (iso-flux) boundary condition, the Nusselt number decreased with an increase in Knudsen number (see Figure 4). Also shown in Figure

4 are the results for the iso-thermal from Barron et al. (1997).

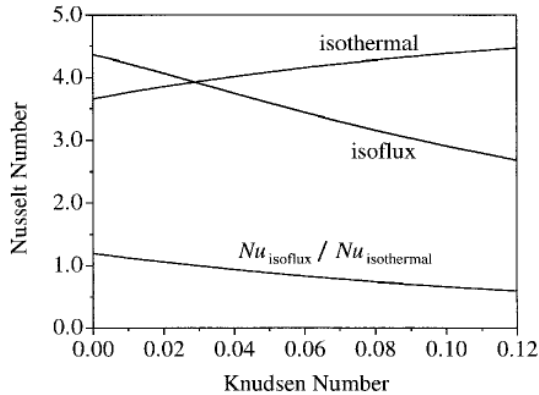


Figure 4: Effect of Kn on Nu for fully developed flow (From Ameer et al., 1997)

Wang et al. (1998) extended the work done by Ameer et al. (1997) and Barron et al. (1997) to non-Newtonian (power-law) fluids. For the iso-flux boundary condition, their results indicated that the Nusselt number increased linearly with inverse power-law exponent (N) but decreased with Knudsen number.

Tunc and Bayazitoglu (2001) analyzed the effects of slip flow and viscous heating on heat transfer of gases in micro-tubes. The effect of slip flow is characterized by Knudsen number (Kn) while viscous heating can be represented by changes in Brinkmann number (Br). The authors observed that when the temperature-jump is considered, Nu decreases with Kn and opposite effects are observed when they are not included in the analyses. Similar to previous results, the authors report decrease in Nu with Kn. Moreover, the effect of viscous dissipation increased Nu for iso-thermal case and decreased Nu for the iso-flux case.

Chen and Kuo (2003) investigated low Reynolds number gaseous flow analytically and numerically. They observed good correlation between analytical and numerical solutions at very low Reynolds number ( $0.0025 < Re < 0.02$ ). Moreover, the authors note that as the Reynolds number increases, the iso-thermal assumption does not hold and hence analytical solutions are incapable of predicting correct values of Nu at higher Reynolds number. The authors validated their iso-thermal case numerical results with the experimental results of Choi et al. (1991) and found that the numerical predictions hold well only till  $Re \approx 100$  to 200. The authors attribute the deviation to the expansion process occurring at the micro-tube outlet and temperature measurement chamber. This causes a reduction on outlet temperature and consequently an increase in Nusselt number. Moreover, they noted that at high Reynolds number this effect may be more pronounced resulting in further deviation.

Chen and Kuo (2004) extended their previous study to include turbulent flow in a  $50 \mu\text{m}$  tube. For modeling turbulent flow, the authors used the Baldwin and Lomax two-equation algebraic model. Their observation is tabulated as shown in Table 3.

Table 3: Summary of observation by Chen and Kuo (2004)

Boundary Condition	Flow Regime	Observation (Nu vs. Re)	Agreement with theory for micro-tubes
Iso-flux	Laminar	Nu decreases	No
	Turbulent	Nu decreases	No
Iso-thermal	Laminar	Nu increases	-
	Turbulent	Nu increases	-

### DIAMETER AND ROUGHNESS MEASUREMENT

The most significant factor that affects the uncertainty is the diameter measurement. Precise measurement of micro-tube diameter is hence an inherent part of micro-tube studies. Scanning Electron Microscope (SEM) is widely used for accurate diameter measurement. Figure 5 shows a SEM image from Celata et al. (2002).

In early studies Choi et al. (1991) and Yu et al. (1995) measured the roughness using computer enhanced laser interferometric technique (WYCO Corp.). They used a computer-software to determine the roughness by scanning images of the surface profiles. The authors reported a measurement uncertainty of 0.2 nm in this process. Kandlikar et al. (2001) created different roughness values by etching the micro-tubes with two different acids. The roughness was measured by a profilometer (Alpha-Step 200). The effect and significance of roughness in micro-tube heat transfer studies is explained in detail later on in this paper.

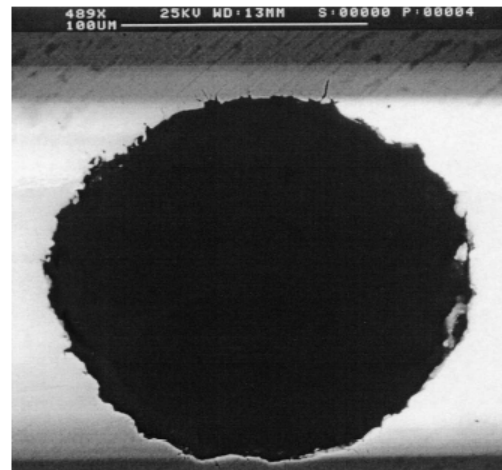


Figure 5: SEM image of  $130 \mu\text{m}$  stainless steel tube from Celata et al. (2002)

## TEMPERATURE MEASUREMENT

It is generally observed that pressure drop measurement and flow rate measurement do not significantly contribute to the experimental uncertainties. However, surface temperature measurement and bulk temperature measurement of a fluid flowing in a micro-tube may prove to be challenging and a leading source of error in heat transfer experiments. The various temperature measurement techniques are discussed in this section. They are:

- Thermocouples (K-type or T-type)
- Platinum resistance thermometers (PRT)
- Infrared (IR) camera
- Liquid Crystal Thermograph (LCT)
- Measuring vapor pressure of condensed water

Table 2 shows the method used by different experimenters along with the uncertainty associated with it. It is observed from the literatures that thermocouples and PRTs are adequate and perform well for bulk fluid temperature measurements at inlet and outlet manifolds. At these sections the velocity of the fluid is extremely small compared to those inside the micro-tube and hence static conditions prevail. However, surface temperature measurements (especially local temperature distributions) may prove to be quite challenging. The thermocouples and its adhesive may be a source of additional thermal resistance and the cause of losses through conduction.

Recent non-contact methods (IR camera and LCT) have the advantage of faster time response and hence can detect surface temperature fluctuations quite effectively. Yang and Lin (2006) make use of LCT while Zhao and Liu (2006) use the IR camera for surface temperature measurements.

An indirect method in iso-thermal experiments is the evaluation of vapor pressure to determine the condensing vapor temperature (Celata et al., 2002; Bucci et al., 2003 and Liu et al., 2007). So, at atmospheric pressure, the saturation temperature of steam is 100 °C. These high temperatures may cause variation in fluid properties like viscosities, resulting in change in Reynolds numbers.

## OBSERVATIONS

Various factors contributing to the disparity of results can be identified from the literature. They are:

- Measurement of local heat flux at the inner wall of the micro-tubes: Analytical solutions and numerical modeling is often sought to obtain local properties at the inner wall. Moreover, finding local temperature distribution is often quite challenging.
- Measurement of average wall-temperature: Different data reduction techniques are used depending on the definition of heat transfer coefficient.
- Conjugate heat transfer may be one of the causes of disparity at very low Reynolds number; hence the Nu values are less than theory and Reynolds number

dependence is observed. This dependence was found to reduce as Reynolds number increased (Liu et al., 2007).

- Entrance length effects may be predominant in larger diameter micro-tubes. (Liu et al., 2007; Yang and Lin, 2006; Celata et al., 2006). Yang and Lin (2006) found that the largest diameter followed correlations of Shah and Bhatti (1987) in thermal developing length region. Other smaller diameter were found to agree only later downstream of the channel.
- High Surface Roughness: Roughness may be one of the causes influencing enhanced heat transfer. Kandlikar et al. (2001) is the only study that experimentally investigated the effect of roughness on heat transfer. Moreover, results of the numerical studies on roughness (Croce and D'Agaro, 2004; Bahrami et al., 2005; Koo and Kleinstreuer, 2005) contradict the experimental observation of Kandlikar et al. (2001).
- Heat loss can occur by conduction through the connecting wires, leads, tubing etc. This is predominant only in cases of extremely low fluid flow and high heat input (Celata et al., 2006).
- Viscous heating might be one of the causes for enhancement in heat transfer at these small scales (Celata et al., 2006).
- For iso-thermal boundary condition, at pressures of 1 atm, condensation of steam occurs at a high temp of 100 °C. Change in properties like viscosity with temperature can cause errors in measurement. Hence it is advisable to limit the temperature drop across the micro-tube to be around 20 °C (Yu et al., 1995).
- The present study observes that the effect of secondary flow due to buoyancy has not been studied in detail in the literature and this effect might also play a key role in understanding the heat transfer phenomena in micro-tubes.

Effect of roughness and diameter on heat transfer (see Table 4):

- For laminar flows from column 5 of Table 4, one can observe that Yang and Lin (2006) agree with theory. Most of the studies agree till a particular Reynolds number after which higher values of Nu are observed. This Reynolds number seems to be a function of diameter for low roughness tubes (Liu et al., 2007) and a function of roughness (Zhao and Liu, 2006) for high roughness values. Apart from these studies, results of Lelea et al. (2004) and Owhaib and Palm (2004) also tend to be in agreement with theory. It is seen that lowest diameter used by Owhaib and Palm (2004) is 800  $\mu\text{m}$ , and no



**Table 4: Effect of Roughness and Diameter on Heat Transfer in Micro-tubes**

Authors	Diameter (μm)	Roughness (μm)	Relative Roughness ε/D	Nu vs. Re (Laminar)	Nu vs. Re (Turbulent)	Thermal Transition
Choi et al. (1991)	3 6.9 9.7 53 81.2	0.175 0.0797 0.0104 0.0144 0.014	0.0058 0.0116 0.0011 0.00027 0.00017	Varies with Re	High	Re <sub>tr</sub> ≈ 2000 – 3000 (Figure 5, Choi et al., 1991)
Kandlikar et al. (2001)	1067	2.4 1.9 3.0	0.00225 0.00178 0.00281	Effect of roughness negligible (falls within error band)	Effect of roughness negligible (falls within error band)	Re <sub>tr</sub> ≈ 2300 – 3000 (Figures 10 – 16, Kandlikar et al., 2001)
	620	2.2 1.8 1.0	0.00355 0.00290 0.00161		High Nu for higher roughness	
Celata et al. (2002)	130	3.42	0.0265	Lower than Hausen (1959)	Higher than Gnielinski (1976) OK with Adams et al. (1998) for high inlet temperature	Re <sub>tr</sub> ≈ 2500 - 3500 No correlation compared
Bucci et al. (2003)	520 290 170	1.609 2.166 1.488	0.00309 0.0075 0.00871	OK till Re ≈ 900-1000; then increases (Figures 9,10,11 Bucci et al., 2003)	Lower than Gnielinski (1976) and Adams et al. (1998). (Figures 9,10,11, Bucci et al., 2003)	Re <sub>tr</sub> ≈ 2500 Re <sub>tr</sub> ≈ 3000 Re <sub>tr</sub> ≈ 3500 No correlation compared
Zhao and Liu (2006)	168 399 799	1.68 4.00 7.8	0.08 to 0.1 0.03 to 0.04 0.01	OK till Re ≈ 650 OK till Re ≈ 1200 OK till Re ≈ 1450	High	Re <sub>tr</sub> ≈ 750 Re <sub>tr</sub> ≈ 1550 Re <sub>tr</sub> ≈ 1600 Early transition observed
Yang and Lin (2006)	123 220.4 308.3 416.1 763.5 962.0	1.4 1.48 1.34 1.46 1.16 1.4	0.0114 0.00672 0.00435 0.00351 0.00152 0.00146	OK (Nu ≈ 4.36)	OK (agrees with Gnielinski, 1976)	From Re = 2500 to 3500 Slope different than Gnielinski (1976)
Liu et al. (2007)	242 315 520	1.21 1.58 2.60	0.005 0.005 0.005	OK till Re ≈ 1500 OK till Re ≈ 1600 OK till Re ≈ 1900	High High High	Re = 1500 – 4000 Re = 1600 – 4500 Re = 1900 – 5500 Slope different than Hausen (1959)

roughness values are mentioned. Lelea et al. (2004) use extremely low heat fluxes of 0.5, 0.75, 1 and 2 W along with small flow rates of 0.04 kg/s. Even though the results are in agreement with theory such low heat fluxes are never encountered in most of the micro-tube applications.

- It can be observed for Turbulent flows (see column 6 of Table 4) that other than Bucci et al. (2003) and Yang and Lin (2006) all other experiments exhibit higher Nusselt numbers than classical correlations. In most experiments, the deviation decreases as with an increase in diameter or decrease in Reynolds number.
- Transition in micro-tubes doesn't seem to compare with any of the classical correlations (see

column 7 of Table 4). While some experimenters find an increase in transition Reynolds number with increasing micro-tube diameters (Zhao and Liu, 2006; Liu et al., 2007) others observe an exact opposite result (Bucci et al., 2003). Tam and Ghajar (2006) obtained correlations for transitional regime in macro-tubes, Equation (6). They also investigated in detail the effect of mixed convection and inlet configurations on the start and end of the transition region. Their correlation performed remarkably well when compared to the experimental data and is recommended for validation purposes in the transition region. Further experiments are thus needed to confirm accurately, the behavior of start and end of the transition region.

## CONCLUSIONS

- Accurate temperature and heat-flux measurements play a key role in heat transfer experiments. The inherent difficulty in obtaining precise inner surface temperature profile may be the primary cause of errors in data reduction.
- Surface roughness is one of the major factors that influence heat transfer at small diameters. More experiments are required to understand its effects. The present numerical studies do not agree with the experimental results of Kandlikar et al. (2001) – which shows enhanced heat transfer for rough tubes in small diameter tubes.
- Recent experiments indicate that laminar and turbulent regimes in micro-tubes compare well with classical correlations. However, the effect of roughness, diameter and heat-flux magnitude cannot be overlooked. Hence, this study is unable to find a concrete conclusion that can encompass all the experimental, numerical and analytical studies.
- Transition in micro-tubes does not compare well with any of the available macro-scale correlations. Moreover, the precise start and end of the transition region and the different factors affecting it have not been observed and discussed in the literature. Controlled experiments are required in the near future to comprehend the exact behavior of transition region at these small-scales.

## ACKNOWLEDGMENTS

This work was partially funded by the Sandia National Laboratories, Albuquerque, New Mexico.

## REFERENCES

Adams, T. M., Abdel-Khalik, S. I., Jeter, S. M., and Qureshi, Z. H., 1998, "Experimental Investigation of Single Phase Forced Convection in Microchannels," *International Journal of Heat and Mass Transfer*, Vol. 41, No. 6-7, pp. 851-857.

Ameel, T. A., Wang, X., Barron, R. F., and Warrington, R. O., 1997, "Laminar Forced Convection in Circular Tubes with Constant Heat Flux and Flip Flow," *Microscale Thermophysical Engineering*, Vol. 1, pp. 303-320.

Bahrami, M., Yovanovich, M. M., and Culham J. R., 2005, "Convective Heat Transfer in Laminar, Single Phase Flow in Randomly Rough Microtubes," *Proceedings of International Mechanical Engineering Congress and Exposition*, pp. 449-457.

Barron, R. F., Wang, X. M., Ameel, T. A., and Warrington, R. O., 1997, "The Graetz Problem Extended to Slip Flow," *International Journal of Heat Mass Transfer*, Vol. 40, No. 8, pp. 1817 – 1823.

Bucci, A., Celata, G. P., Cumo, M., Serra, E., and Zummo, G., 2003, "Water Single Phase Fluid Flow and Heat Transfer in Capillary Tubes," *First International Conference on Microchannels and Minichannels*, pp. 319-326, Rochester, New York, USA, April 24 – 25, 2003.

Celata, G. P., 2004, "Single-Phase Heat Transfer and Fluid Flow in Micropipes," *Heat Transfer Engineering*, Vol. 25, pp. 13-22.

Celata, G. P., Cumo, M., Guglielmi, M., and Zummo, G., 2002, "Experimental Investigation of Hydraulic and Single Phase Heat Transfer in 0.130 mm Capillary Tube," *Microscale Thermophysical Engineering*, Vol. 6, pp. 85-97.

Celata, G. P., Cumo, M., Marconi, V., McPhail, S. J., and Zummo G., 2006, "Microtube Liquid Single-Phase Heat Transfer in Laminar Flow," *International Journal of Heat and Mass Transfer*, Vol. 49, pp. 3538-3546.

Chen, C. S. and Kuo, W. J., 2003, "Heat Transfer and Flow Friction for Gaseous Flow in Microtubes," *Transactions of Aeronautical and Astronautical Society of the Republic of China*, Vol. 35, No. 3, pp. 257-266.

Chen, C. S. and Kuo, W. J., 2004, "Heat Transfer Characteristics of Gaseous Flow in Long Mini- and Microtubes," *Numerical Heat Transfer*, Vol. Part A, 46, pp. 497-514.

Choi, S. B., Barron, R. F., and Warrington, R. O., 1991, "Fluid Flow and Heat Transfer in Microtubes," *Micromechanical Sensors Actuators and Systems*, Vol. DSC-32, pp. 123-134.

Colburn, A. P., 1933, "A Method of Correlating Forced Convection Heat Transfer Data and a Comparison with Fluid Friction," *Trans. Am. Inst. Chem. Eng.*, Vol. 29, pp. 174 – 209.

Croce, D. and D'Agaro, P., 2004, "Numerical Analysis of Roughness Effect on Microtube Heat Transfer," *Superlattices and Microstructures*, Vol. 35, pp. 601-616.

Dittus, F. W. L. and Boelter, M. k., 1930, "Heat Transfer in Automobile Radiators of the Tubular Type," *Univ. Calif. Publ. Eng.*, Vol. 2, No. 13, pp. 443 – 446.

Gari, A. A. and Rahman, M. R., 2005, "Conjugate Heat Transfer Analysis of Circular Microtube Under Time Varying Time Source," Vol. 376, No. 2, pp.663 – 668.

Gnielinski, V., 1976, "New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow," *Int. Chem. Engrg.*, Vol. 16, pp. 359 – 368.

Grohmann, H., 2005, "Measurement and Modeling of Single-Phase and Flow-Boiling Heat Transfer in Microtubes," *International Journal of Heat and Mass Transfer*, Vol. 48, pp. 4073-4089.

Guo, Z. Y. and Wu, X. B., 1997, "Compressibility Effect on Gas Flow and Heat Transfer in a Microtube," *International Journal of Heat and Mass Transfer*, Vol. 40, No. 13, pp. 3251-3254.

Hausen, H., 1959, "Neue fur die Wärmeübertragung bei frir und Erzwungener Stromung," *Allg. Warmetchnik*, Vol. 9, pp. 75-79.

Hestroni, G., 2005, "Heat Transfer in Microchannels: Comparisons of Experiments with Theory and Numerical

Results,” *International Journal of Heat and Mass Transfer*, Vol.48, No. 25 – 26, pp. 5580 – 5601.

Kandlikar, S. G., Joshi, S., and Tian, S., 2001, “Effect of Channel Roughness on Heat Transfer and Fluid Flow Characteristics at Low Reynolds Numbers in Small Diameter Tubes,” 35th National Heat Transfer Conference, Anaheim, California, USA, pp. 1-10, June 10 – 12, 2001.

Koo, J. and Kleinstreuer, C., 2005, “Analysis of Surface Roughness Effects on Heat Transfer in Micro-Conduits,” *International Journal of Heat and Mass Transfer*, Vol. 48, pp. 2625-2634.

Lelea, D., Nishio, S., and Takano, K., 2004, “Experimental Research on Microtube Heat Transfer and Fluid Flow of Distilled Water,” *International Journal of Heat and Mass Transfer*, Vol. 47, pp. 2817-2830.

Liechty, B. C., Webb, B. W., and Maynes, R. D., 2005, “Convective Heat Transfer Characteristics of Electro-Osmotically Generated Flow in Microtubes at High Wall Potential,” *International Journal of Heat and Mass Transfer*, Vol. 48, pp. 2360 – 2371.

Liu, Z. G., Liang, S. Q., and Takei, M., 2007, “Experimental Study on Forced Convective Heat Transfer Characteristics in Quartz Micro-Tube,” *International Journal of Thermal Sciences*, Vol. 46, pp. 139-148.

Maynes, D. and Webb, B. W., 2004, “The Effect of Viscous Dissipation in Thermally Fully Developed Electro-Osmotic Heat Transfer in Micro-Channels,” *International Journal of Heat and Mass Transfer*, Vol. 47, pp. 987-999.

Morini, G. L., 2004, “Single-Phase Convective Heat Transfer in Microchannels: A Review of Experimental Results,” *International Journal of Thermal Sciences*, Vol. 43, pp. 631-651.

Obot, N. T., 2000, “Towards a Better Understanding of Friction and Heat/Mass Transfer in Microchannels – A Literature Review,” *Proceedings of the International Conference on Heat Transfer and Transport Phenomena in Microscale*, Banff, Canada, October 15 – 20.

Owhaib, W. and Palm, B., 2004, “Experimental Investigation of Single Phase Convective Heat Transfer in Microchannels,” *Experimental Thermal and Fluid Science*, Vol. 28, pp. 105-110.

Palm, B., 2001, “Heat Transfer in Microchannels,” *Microscale Thermophysical Engineering*, Vol. 5, pp. 155-175.

Palm, B. and Peng, X. F., 2004, “Single Phase Convective Heat Transfer, in Celata, G. P., (Ed), *Heat Transfer and Fluid Flow in Microchannels*,” Begell House, New York.

Papautsky, I., Ameel, T., and Frazier, A. B., 2001, “A Review of Laminar Single Phase Flow in Microchannels,” *Proceedings of ASME International Mechanical Engineering Congress and Exposition*, pp. 3067-3075, Vol. 2, New York, NY, November 11 – 16, 2001.

Petukhov, B. S., 1970, “Heat Transfer and Friction in Turbulent Pipe Flow with Variable Physical Properties,” *Advances in Heat Transfer*, Vol. 6, Academic Press, Inc., New York, pp. 504 – 564.

Rostami, A.A., Majumdar, A.S., and Saniei, N., 2002, “Flow and Heat Transfer for Gas Flowing in Microchannels: A review,” *Heat and Mass Transfer*, Vol. 38, pp. 359-367.

Shah, R. K. and Bhatti, M. S., 1987, “Laminar Convective Heat Transfer in Ducts”, in S. Kakac, R. K. Shah and W. Aung eds., *Handbook of Single Phase Convective Heat Transfer*, Wiley, New York.

Shah, R. K., 1975, “Thermal Entry Length Solutions for Circular Tube and Parallel Plates,” *Proc. 3<sup>rd</sup> National Heat Mass Transfer Conference*, Indian Inst. Technology, Bombay, Vol. 1, HTM-11-75.

Sharath, P., Rao, C., and Muhammad, M. R., 2004, “Analysis of Steady State Conjugate Heat Transfer in a Circular Micro-Tube Inside a Substrate,” *Proceedings of International Mechanical Engineering Congress and Exposition*, Anaheim, California, USA, pp. 743-751, November 13 – 20, 2004.

Sharath, P., Rao, C., Muhammad, M. R., and Hassan, M. S., 2006, “Numerical Simulation of Steady State Conjugate Heat Transfer in a Circular Microtube Inside a Rectangular Substrate,” *Numerical Heat Transfer*, Vol. 49, pp. 635-654.

Sieder, E. N. and Tate, G. E., 1936, “Heat Transfer and Pressure Drop of Liquids in Tubes,” *Industrial and Engineering Chemistry*, Vol. 28, No. 12, pp. 1429 – 1435.

Sobhan, C. B. and Garimella, S.V., 2001, “A Comparative Analysis of Studies on Heat Transfer and Fluid Flow in Microchannels,” *Microscale Thermophysical Engineering*, Vol. 5, pp. 293-311.

Tam, L. M. and Ghajar, L. M., 2006, “Transitional Heat Transfer in Plain Horizontal Tubes,” *Heat Transfer Engineering*, Vol. 27, No. 5, pp. 23-38.

Tunc, G. and Bayazitoglu, Y., 2001, “Heat Transfer in Microtubes with Viscous Dissipation,” *International Journal of Heat and Mass Transfer*, Vol. 44, pp. 2395-2403.

Wang, M., Bruno, F. A., Ameel, T., and Warrington, R., 1998, “Micro-Tube Convection Heat Transfer for a Power-Law Fluid in Laminar Slip Flow with an Isoflux Boundary Condition,” *Proceedings of the ASME Heat Transfer Division*, pp. 157-164.

Yang, C. Y. and Lin, T. Y., 2006, “Heat Transfer Performance of Water Flow in Microtubes,” *13th International Heat Transfer Conference*, Sydney, Australia, MIC-14, August 13-18, 2006.

Yu, D., Warrington, R., Barron, R., and Ameel, T., 1995, “An Experimental and Theoretical Investigation of Fluid Flow and Heat Transfer in Microtubes,” *ASME/JSME Thermal Engineering Conference*, Vol. 1, pp. 523-530.

Zhao, Y. and Liu, Z., 2006, “Experimental Studies on Flow Visualization and Heat Transfer Characteristics in Microtubes,” *13th International Heat Transfer Conference*, Sydney, Australia, MIC-12, August 13-18, 2006.