Applied Energy **23** (1986) 205–224

Computer Simulation of Stratified Heat Storage

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SUMMARY

A one-dimensional, explicit, finite difference model of a single tank for stratified thermal storage has been developed and tested on published experimental data. The model covers through-flow conditions for charging or discharging the thermal storage tank and conduction and turbulent mixing within the water. It was demonstrated that careful design of the algorithm and choice of the time-step are necessary to avoid computational errors, which create apparent mixing in the through-flow calculation.

In order to handle variable flow rates, a 'conceptual buffer-tank' algorithm was developed which makes the procedure flexible and useful. Turbulent mixing across the thermocline is simulated through thermal eddy conductivity factors which have been determined from published experimental data. The complete computational model is simple and efficient enough to interface with a larger program simulating an entire air-conditioning system, which includes chillers, building loads, weather, etc. At the same time, the model reproduces hydraulic model test data better than a recent one-dimensional model found in the literature.

INTRODUCTION

There are substantial benefits to the use of thermal storage in both heating and cooling:

(1) Storing heat can make solar energy available at night for heating.

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Applied Energy 0306-2619/86/$03·50

(2) Storing chilled water can utilize air-conditioning equipment at night, during hours of maximum coefficient of performance and low off-peak electricity rates.

The design of the device used to store heat is important: the energy introduced into this device should be extractable when needed. The simplest design of such a device is the single well-mixed storage tank. This tank does not separate the hot and cold fluids in the tank; therefore, this design recovers only a small portion of this energy.

Another approach is the use of multiple storage tanks. This system can improve on the recoverable energy extracted from the system, but it requires the cost and complexity of several tanks and many connections.

The design discussed in this paper consists of using only one tank but preserving the thermocline within the tank, thus taking advantage of the density difference of the hot and cold fluids. The storage medium is placed in the tank with minimum mixing with the fluid already in the tank. The success of this scheme depends upon the design of the inlet; one design of inlet has been demonstrated by Loehrke *et al.*,[1] achieving the performance of multiple tanks but using a single tank. In order to assess the true benefits of a stratified thermal storage system, there is a definite need for a simulation program.

To simulate a stratified thermal storage tank, a number of basic topics must be investigated:

(1) The significant physical parameters of the water tank and its inlet configuration.
(2) The governing equations for change in storage status with the passage of time due to various through-flows and inlet temperatures.
(3) The thermal eddy conductivity in these governing equations, as obtainable from physical reasoning or from experimental data in the literature.
(4) The uncertainties due to the various assumptions, as well as the identification of proposed experiments to reduce these uncertainties.

With the investigation of these basic topics, a computer program can be developed which will predict the temperature profile in the storage tank as a function of time, if the history of inlet flows and temperatures is provided. These profiles will predict the water temperature at the tank outlet. Such a program is not only a testable end product and an enhancement of our stratification simulation capability, but is also a useful input into a total system simulation project.

PHYSICAL MODEL

The storage geometry modeled is a vertical cylindrical tank. The assumptions on which the model is based are as follows:

(1) One-dimensional fluid flow and heat conduction, which means that the thermocline is axisymmetric and independent of the radial distance. Corroboration of this assumption through experiments is acknowledged by Loehrke *et al.*[1]

(2) Small losses due to conduction through the wall of the tank, achieved by insulating the tank. This assumption predicts that the changes of the thermocline are dominated by conduction and convection in the fluid instead of conduction through the walls. This is in agreement with the findings of Abdoly[2] if the tank is insulated to obtain maximum efficiency.

(3) The walls of the tank are not overly massive, so reducing the tendency of the tank to retain heat within the walls and minimizing conduction of heat down the walls of the tank. This is in agreement with Abdoly.[2]

(4) The inlet temperature of the flow is beyond the extremes of the temperatures within the tank. That is, the temperature of fluid flowing into the top of the tank must be at least as hot as the temperature of fluid at the top of the tank, and the converse must be true for the relatively low temperature at the bottom inlet. This condition is met in modern chilled water systems.

The equation governing the stratified thermal model for conduction and convection is the energy equation:

$$\frac{DT}{Dt} = \frac{k}{\rho C} \nabla^2 T \tag{1}$$

Now, applying this to one-dimensional flow in the x direction, which is assumed to be up the tank, the above equation takes the form:

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{2}$$

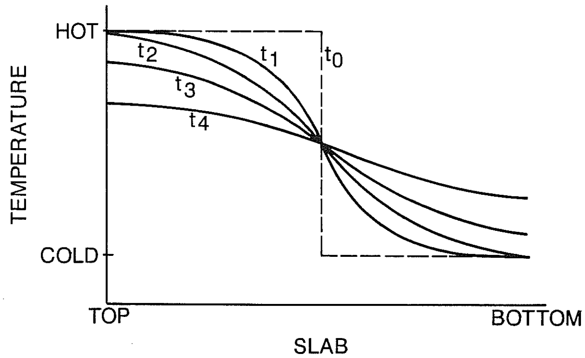

Fig. 1. Temperature profile in the tank for the theoretical conduction-only case as time increases.

where $\alpha = k/\rho C$ (which is defined as the thermal diffusivity of the fluid), k is the thermal conductivity, ρ is the density, C is the specific heat, t is the time, T is the temperature and D/Dt is the substantial derivative. Equation (2) can be split into two special cases; namely, the conduction case (involving only mixing with no flow) and the convection case (involving only flow with no mixing). Numerical procedures will be applied to eqn (2) for these two special cases in order to verify the simulated results, because the theoretical results for both cases are known, as shown in Figs 1 and 2 for the conduction and convection cases, respectively.

Conduction-only model

For this special case, depleting the thermocline occurs when the velocity terms in the governing equation are zero. Thus, eqn (2) reduces to the following form:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{3}$$

The numerical approximation of the derivatives in eqn (3) is obtained via the finite difference method.[3] We define $\text{Fo} = \alpha \Delta t/\Delta x^2$ (the 'finite difference' Fourier number) and $AMIX = (1 + \varepsilon)\text{Fo}$ (the non-dimensional mixing parameter), where ε depends on the mixing and is similar to an eddy conductivity. For laminar mixing, $\varepsilon = 0$ and

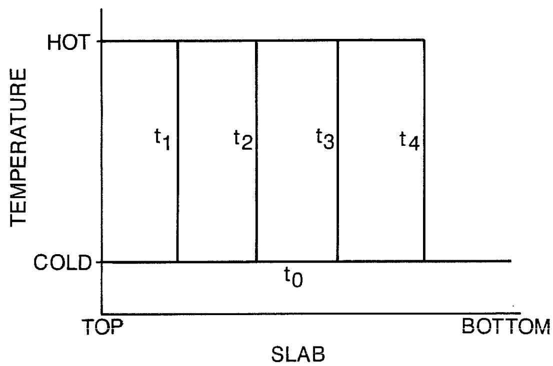

Fig. 2. Temperature profile in the tank for the theoretical convection-only case as time increases.

$AMIX = \text{Fo}$. We solve eqn (3) in the explicit form to obtain the numerical update equation for conduction:

$$T_{t+1,x} = AMIX(T_{t,x+1} + T_{t,x-1}) + (1 - 2AMIX)T_{t,x} \quad (4)$$

To obtain stability and ensure convergence of eqn (4), $AMIX$ must be less than, or equal, 1/2. Therefore, Δt and Δx must be properly chosen for a given α.

Convection-only model

The convection model, also known as the flow-only model, involves water flowing through the tank with no mixing between the water initially in the tank and the incoming water. Thus, we obtain perfect stratification in the tank and recover 100 per cent of the energy put into the tank. The simplified equation for this situation for the one-dimensional case with the conduction term equal to zero is as follows:

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x} = 0 \quad (5)$$

To obtain the numerical equation, the upwind differencing technique was used in order to compensate for the directional change of water when flowing into either the top or bottom of the tank.[3] Figure 3 depicts the notation used for the tank. Solving for the temperature at the new time level and defining $FLOW = V\Delta t/\Delta x$ (also known as the Courant number[3]) where V is the velocity, we obtain eqn (6) for water flowing into

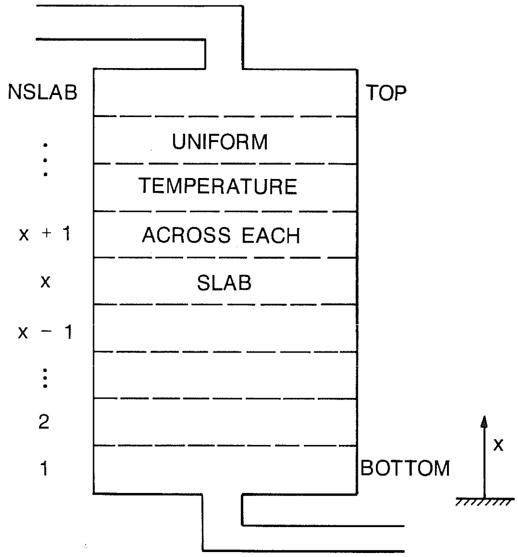

Fig. 3. One-dimensional tank pictorial.

the top of the tank and eqn (7) for water flowing into the bottom of the tank:

$$T_{t+1,x} = (FLOW)T_{t,x+1} + (1 - FLOW)T_{t,x} \tag{6}$$

$$T_{t+1,x} = (FLOW)T_{t,x-1} + (1 - FLOW)T_{t,x} \tag{7}$$

To ensure stability of eqns (6) and (7), the *FLOW* parameter must not be greater than unity. Since the stratified case or convection-only case contains no mixing, the temperature profile should resemble the plot

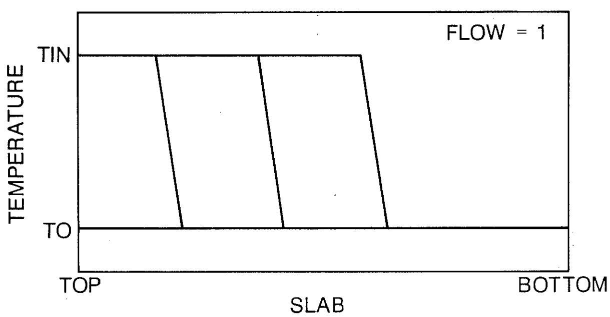

Fig. 4. Temperature profile in the tank for the numerical convection-only ($FLOW = 1$) case as time increases.

shown in Fig. 2. Notice that the temperature of the incoming flow replaces the previous temperature of the slab and continues to march down the exit of the tank as the time elapses. The equation that would produce the temperature profile for the top flow case, as shown in Fig. 2, is given by:

$$T_{t+1,x} = T_{t,x+1} \tag{8}$$

When trying to simulate the temperature profile in Fig. 2 by eqn (6), we see that the *FLOW* parameter must be equal to unity. The simulated results with $FLOW = 1$ are shown in Fig. 4. If *FLOW* is less than unity, our algorithm produces a temperature profile as shown in Fig. 5, which is

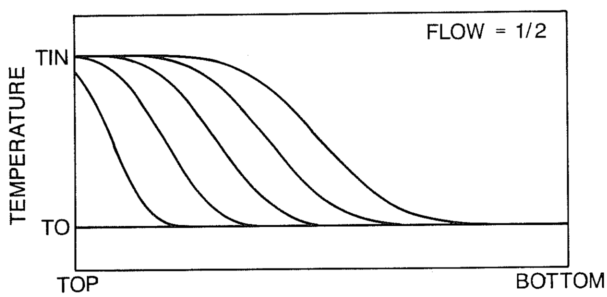

Fig. 5. Temperature profile in the tank for the numerical convection-only ($FLOW = 1/2$) case as time increases.

physically incorrect. The situation of $FLOW = 1$ implies that the incoming flow of water must fill up one slab volume in the tank during the time interval of calculation, Δt. Therefore, if Δt and Δx are fixed, the velocity of the incoming flow is restricted to $V = \Delta x/\Delta t$ for $FLOW = 1$. Thus, the flow rate must remain constant. If $FLOW < 1$, then we obtain pseudo-mixing (also known as numerical diffusion or artificial viscosity).[3]

To overcome the problem of not being able to vary the flow rate of the incoming flow in the algorithm, two fictitious buffer tanks are placed at the ends of the main tank, as shown in Fig. 6, each capable of holding up to two slab volumes. The purpose of using the buffer tanks is to allow for a variable flow rate and eliminate the pseudo-mixing in the algorithm when $FLOW < 1$. The inlet buffer tank stores the incoming flow of water and is examined at regular time intervals; it continues to accumulate the incoming flow of water until the amount of water in the buffer tank equals at least one slab volume in the tank. Then one slab volume of water in the buffer tank is pulsed into the main tank. Numerically this means that the

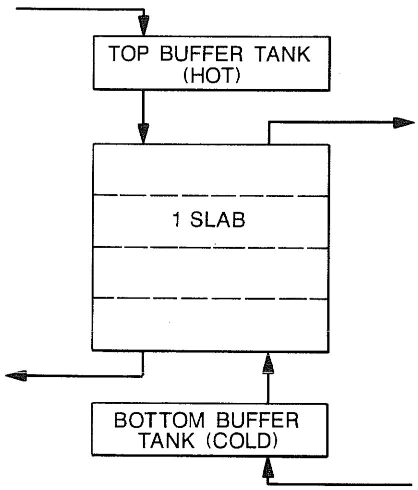

Fig. 6. Buffer tank concept.

equations derived for stratified flow are used with $FLOW = 1$ leading to the following equations.

For water flowing into the top of the tank:

$$T_{t+1,x} = T_{t,x+1} \tag{9}$$

For water flowing into the bottom of the tank:

$$T_{t+1,x} = T_{t,x-1} \tag{10}$$

Figure 7 shows the correct temperature profile of the convection-only case when using the buffer-tank concept.

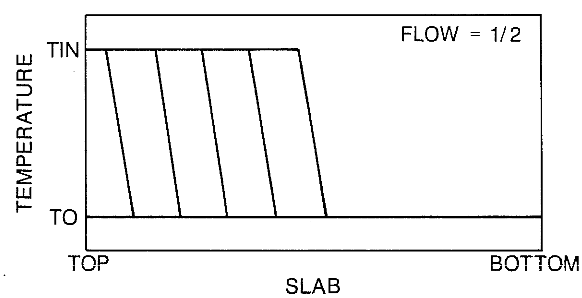

Fig. 7. Temperature profile in the tank for the buffer tank concept ($FLOW = 1/2$) as time increases.

Combination of flow and conduction

To obtain the combined effect, the methods described above can be added together. The conduction routine will be applied at each time interval whereas the through-flow routine will be applied only if there is enough backlog that we can set $FLOW = 1$. The through-flow routine might be invoked only occasionally; for example, every third or fourth time, depending on the flow rate. Thus, we can now simulate a combined condition without introducing pseudo-mixing through numerical procedures, by executing the flow calculations at variable time intervals which are integral multiples of the minimal times.

COMPUTER IMPLEMENTATION

A partial listing of the variables used in the computer program and discussed in this paper is shown in Table 1. The user of the program must input several variables to the program such as: *A*, *H*, *RMAX*, *TO*, *TIN*. One of the two variables, *DELT* or *NSLAB*, must be input to the program based upon the choice of the user, while the other variable will be calculated from the stability criteria. The values of the remaining variables listed in Table 1 will be either calculated or chosen by the program.

This program will choose the eddy conductivity factor, ε, called *EDDY* in the program. Since *EDDY* can vary for each slab in the tank, some flexibility is introduced into the one-dimensional flow model. By selecting certain values of *EDDY* for different slabs, some of the two-dimensional flow properties can be absorbed into this weighting factor *EDDY* for our one-dimensional flow model.

As mentioned above, *DELT* or *NSLAB* will be calculated from the stability criteria, where $NSLAB = H/DELX$. Recall the two stability requirements for the flow and conduction routines:

$$AMIX = (1 + EDDY) * [ALPHA * DELT/(DELX)^2] \leq 0{\cdot}5 \qquad (11)$$

$$FLOW = VEL * DELT/DELX \leq 1{\cdot}0 \qquad (12)$$

To obtain stability in this simple explicit updating algorithm, eqns (11) and (12) must be satisfied. Notice that the maximum values of *EDDY* and *VEL*; that is, *EMAX* and *VMAX*, respectively, must be known. *VMAX*

TABLE 1
Computer Nomenclature

A	Area of tank.
ALPHA	Thermal diffusivity.
AMIX	Non-dimensional mixing constant.
D	Tank inside diameter.
DELT	Time interval in program.
DELX	Distance between slabs in the tank.
EDDY	Thermal eddy conductivity factor.
EMAX	Maximum eddy conductivity factor.
FLOW	Non-dimensional flow constant.
H	Height in tank.
NSLAB	Number of slabs in the tank.
RMAX	Maximum flow rate of incoming flow.
T	Temperature.
TIN	Temperature of incoming flow.
TO	Initial temperature in the tank.
VEL	Velocity of water through the tank.
VMAX	Maximum velocity of water through the tank.

can be calculated from the input value *RMAX*. *EMAX* will be determined from simulation of various experimental data. The following equations can calculate either *DELT* or *NSLAB*:

$$DELT \leq H/(NSLAB * VMAX)$$

and (13)

$$DELT \leq (H/NSLAB)^2/[(2 * ALPHA) * (1 + EMAX)]$$

$$NSLAB \leq H/(VMAX * DELT)$$

and (14)

$$NSLAB \leq [H^2/(2 * ALPHA * DELT)]^{0.5}$$

Appropriate integer values satisfying eqns (13) and (14) for *DELT* (calculated from the user supplied *NSLAB*) or *NSLAB* (calculated from the user supplied *DELT*) will be used in the program.

Evaluating boundary conditions

Since the assumption of a well insulated tank is used in this work, the temperature gradient across the boundaries; that is, the top and bottom of the tank, is assumed to be zero. Thus, a fictitious slab is introduced

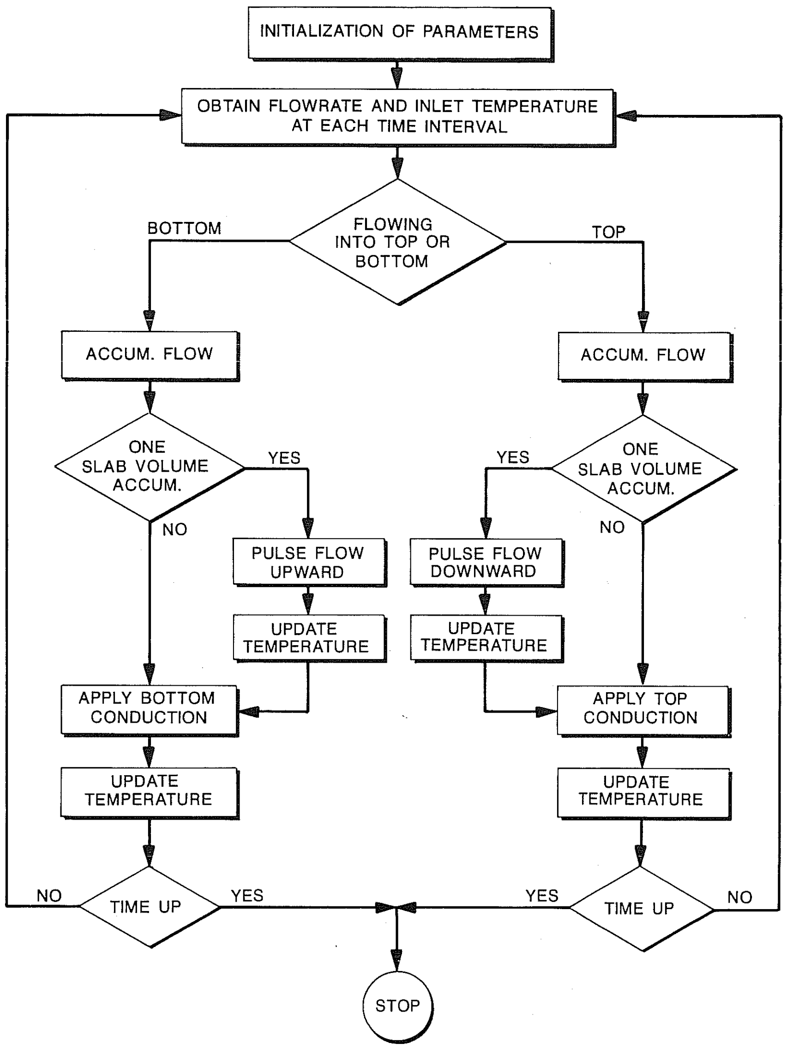

Fig. 8. Flowchart.

outside the end walls of the tank. These fictitious slabs have the same properties as their corresponding interior end slabs as if a mirror image occurred. Thus, the equations for the boundary conditions are as follows:

$$T_{t+1,1} = (1 - AMIX)T_{t,1} + (AMIX)T_{t,2} \tag{15}$$

$$T_{t+1,NSLAB} = (1 - AMIX)T_{t,NSLAB} + (AMIX)T_{t,NSLAB-1} \tag{16}$$

With these boundary conditions, the order of sequentially updating the temperature profile should start where the flow enters and end at the exit. Therefore, the direction of the incoming flow is important in the conduction routine in order to produce the correct order of updating the temperature.

The buffer tank concept resolves the difficulty of handling the boundary conditions in the flow routine by using a top and bottom buffer tank. Conservation of mass is satisfied by this method also. For example, when the inlet flow changes from the top to the bottom, the amount of water left in the top buffer is retained until water flows into the top buffer again; likewise, for the bottom buffer tank. Also note that conservation of energy within the buffer tanks was not considered because we assume that the inlet temperature flowing into the buffer tanks remains constant, where the top buffer tank remains hot and the bottom buffer tank remains cold. Thus, the mixing effects in the buffer tanks will be insignificant due to the constant inlet temperature. Even if the inlet temperature does vary somewhat, the mixing effect occurring inside the buffer tank will be insignificant because the volume of the buffer tanks is very small compared with the volume of the main tank.

With the development of the numerical equations and the employment of the above boundary conditions, an overall program can be produced. Figure 8 shows the flowchart logic of this program.

SIMULATION RESULTS

Having developed the computer program in the previous sections, the next step is to determine the eddy conductivity factor, ε, in *AMIX* needed to reproduce the experimental data. The best value was determined through numerous simulation runs.

Several models of stratified thermal storage have been developed in the literature, including a fully stratified model[4] and a viscous entrainment model.[5] Figure 9 shows a comparison of predictions from these one-dimensional models with experimental data.[6] Note that the temperature at the top of the tank, predicted by both of the above one-dimensional models, is consistently lower than the experimental values; that is, the models underestimated the amount of energy extracted from the tank during the experiment. The reported experimental data[6] did not contain enough information to input into our model to simulate these tests and compare our simulated results with the data.[4,5] A three-dimensional

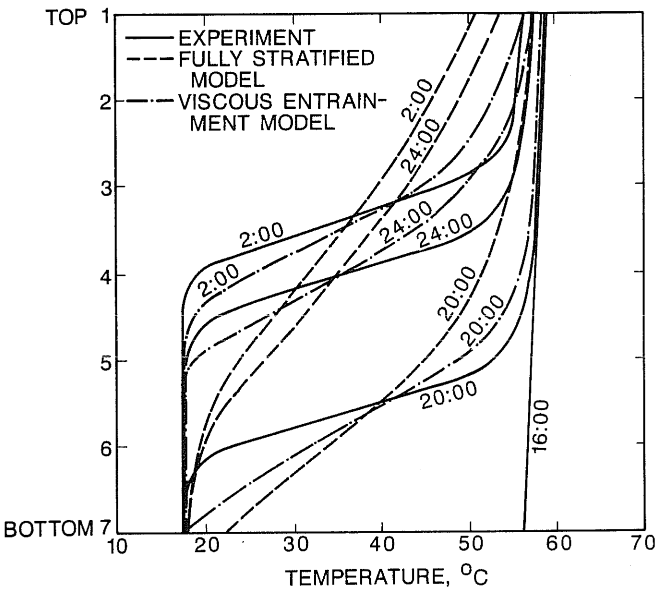

Fig. 9. Comparison of the storage tank temperature profiles during the load time, taken from reference 6.

model was developed by Sha and Lin;[7] their model was not compared against any experimental data in their paper. Any three-dimensional model would increase the degree of difficulty and cost of a computer program. A model based entirely on conduction was developed by Abdoly.[2] His model was not accurate for dynamic cases which involved turbulent mixing created in through-flow situations. Loehrke *et al.*[1] developed a one-dimensional explicit finite difference model. Figure 10 shows a comparison of their model predictions with their experimental data. The reported data[1] do contain sufficient information for input to our computer model.

The type of experiments conducted in Fig. 10 consisted of charging the initially cold tank with hot water through the inlet at the top of the tank. Notice that the inlet temperature decreased during the operation of the experiment. We will examine each curve individually, assuming that the inlet temperature for that curve remained constant. An appropriate eddy conductivity factor will be chosen for each curve. Thus, the variation of eddy conductivity factors throughout the tank may be determined. Only two curves will be simulated (namely, for 0·5 h and 1 h) because the last curve (for 1·5 h) does not contain the full thermocline.

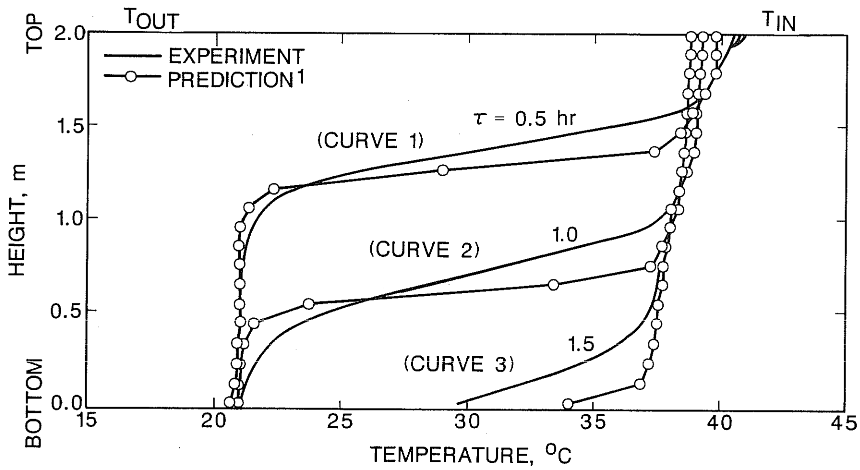

Fig. 10. Prototype manifold, charging $\dot{m} = 1364$ kg/h (6 gpm), taken from reference 1.

The data needed and used for input to our model are listed in Table 2. The thermal properties used were evaluated at the average temperature, between *TO* and *TIN*. The parameter *NSLAB* was chosen as 20 because there were 20 thermocouple locations within the tank. With the parameter *NSLAB* fixed, *DELT* must be calculated from the stability criteria. *DELT* must be less than 4·28 min (*FLOW* criterion) and also less than 527 min (*AMIX* criterion) as shown in Table 2. The *FLOW* criterion limits the choice of *DELT* for most cases. Both curves were simulated for three different cases. The first case contained only laminar conduction with $\varepsilon = 0$. The second case used a constant turbulent eddy conductivity factor

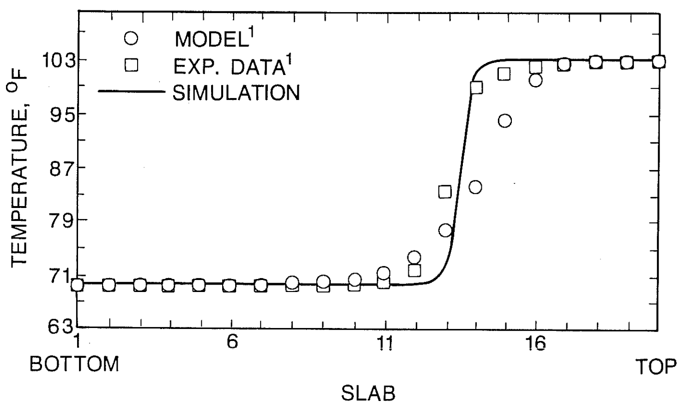

Fig. 11. Comparison of storage tank temperature profiles for the laminar case of curve 1 in Fig. 10.

TABLE 2
Computer Input for Simulation[1]

Tank specifications	*Fluid properties*
$H = 6{\cdot}34$ ft	$TIN = 102\,°F$
$D = 3{\cdot}8$ ft	$TO = 69\,°F$
$A = 11{\cdot}34$ ft^2	$\rho = 62{\cdot}4$ lbm/ft^3
$NSLAB = 20$	$k = 0{\cdot}355$ Btu/h °F ft
$DELX = 0{\cdot}317$ ft	$C_p = 0{\cdot}998$ Btu/lbm °F
Insulated tank	$\alpha = 0{\cdot}0057$ ft^2/h
Inlet manifold designed by Ref. 1	$RMAX = 52$ lbm/min
	$VMAX = 7{\cdot}35 \times 10^{-2}$ ft/min
Stability criteria	*Non-dimensional parameters based on DELX, VMAX*
$FLOW$: $DELT < 4{\cdot}28$ min	Pick $DELT = 3{\cdot}0$ min
$AMIX$: $DELT < 527$ min	$FLOW = 0{\cdot}70$
	Fourier number $= 2{\cdot}846 \times 10^{-3}$

throughout the tank. The third case consisted of varying the eddy conductivity factor within four equal regions of the tank.

Simulating the first curve (for 0·5 h) will help predict the degree of mixing occurring near the inlet. The results of the laminar case shown in Fig. 11 dictate that turbulent mixing does occur, as was expected. Figure 12 shows the uniform turbulent case with an eddy conductivity factor of 20. This simulated profile matches the experimental profile better than the

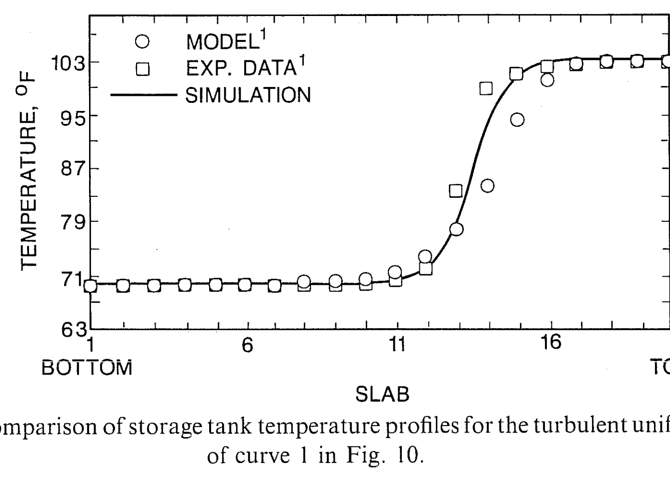

Fig. 12. Comparison of storage tank temperature profiles for the turbulent uniform case of curve 1 in Fig. 10.

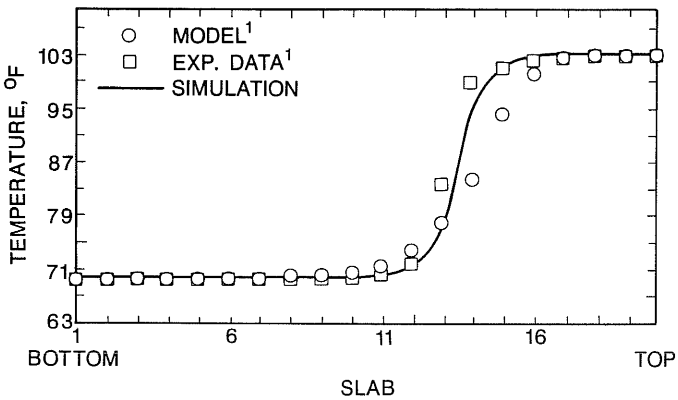

Fig. 13. Comparison of storage tank temperature profiles for the turbulent varying case of curve 1 in Fig. 10.

model developed by Loehrke *et al.*[1] For the varying case, the eddy conductivity factor was decreased in the regions closer to the outlet of the tank. Figure 13 shows the results of the varying eddy conductivity factor. Notice that the varying profile is about the same as for the uniform case in Fig. 12. This may be attributed to the fact that the eddy conductivity value near the inlet of the tank for the varying case was nearly equal to the uniform eddy conductivity value. Thus, the eddy conductivity values at the outlet of the tank essentially have no effect because the temperatures have not changed yet.

Simulating the second curve (for 1·0 h), which occurs later in time and

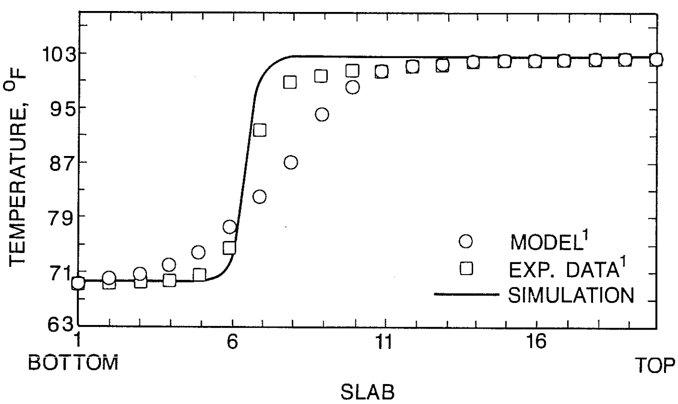

Fig. 14. Comparison of storage tank temperature profiles for the laminar case of curve 2 in Fig. 10.

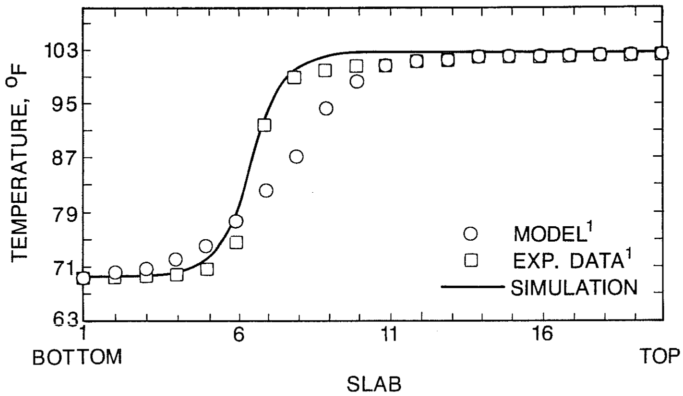

Fig. 15. Comparison of storage tank temperature profiles for the turbulent uniform case of curve 2 in Fig. 10.

further down the tank toward the outlet, will determine how the eddy conductivity changes with time and distance into the tank. The results of the laminar case for the second curve shown in Fig. 14 reveal that a smaller amount of turbulence is occurring as the thermocline advances toward the exit. In Fig. 15, a uniform eddy conductivity factor of 10 produced a simulated profile similar to the experimental profile. Therefore, the turbulent eddy conductivity factor has decreased during the movement of the thermocline toward the outlet. Notice that the experimental temperature above the thermocline has decreased due to the variation of the inlet temperature.[1] If the inlet temperature had remained

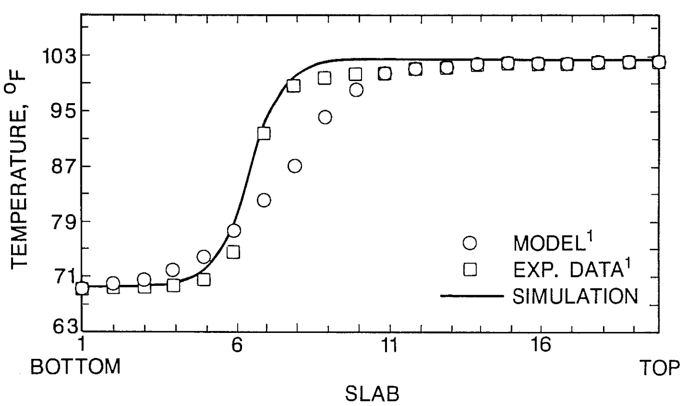

Fig. 16. Comparison of storage tank temperature profiles for the turbulent varying case of curve 2 in Fig. 10.

TABLE 3
Simulation Results of Experimental Data[1]

Figure	*Curve*	*Case*	*Eddy conductivity factor used*	*Allowable range of eddy conductivity*	*Our model accumulated error*	*Reference model accumulated error*
11	1	Laminar uniform	1	—	22·8	34·7
12	1	Turbulent uniform	20	15–25	12·0	34·7
13	1	Turbulent varying	1. 25[a] 2. 15 3. 5 4. 1	—	12·3	34·7
14	2	Laminar uniform	1	—	27·2	40·1
15	2	Turbulent uniform	10	5–15	17·3	40·1
16	2	Turbulent varying	1. 25[a] 2. 15 3. 5 4. 1	—	15·4	40·1

[a] Eddy conductivity factors used in the four regions of the tank (for the turbulent varying case) where 1 refers to the region next to the inlet and 4 refers to the region near the exit of the tank.

constant during the experiment, this decrease would not have been as noticeable. Figure 16 shows the results of the varying case. The simulated profile matches the experimental profile except at the location above the thermocline. Note that the eddy conductivity values in the four regions of the tank for the varying case of the second curve were not changed from the values of the first curve.

Table 3 summarizes the results of both curves. The Table contains the eddy conductivity values used in the simulation, the range of allowable eddy conductivity values and the total accumulated error between the simulated and experimental values for both our model and the model of reference 1. Observing this Table, along with the temperature plots, it can be concluded that the eddy diffusivity factor does vary somewhat inside the tank as the thermocline moves from its development at the inlet to its depletion at the exit. In comparing our one-dimensional model with the one-dimensional model of reference 1, note that our model simulated results that consistently lay on, or near, the thermocline, whereas their one-dimensional model produced a thermocline wider and flatter than the experimental thermocline. This suggests that their model did not consider enough mixing in the portion of the tank near the inlet.

CONCLUSIONS

Our results show that a very simple numerical model can accurately simulate the behavior of a stratified storage tank. The only precaution which proved to be necessary was to separate the conduction and flow algorithms, in order to eliminate the gradual 'smearing' of the temperature profiles in the flow case. The conduction algorithm was applied at each time step and the flow algorithm was applied whenever the buffer tank contained at least one slab volume. A variable integer relationship between time steps was achieved by means of conceptual buffer tanks.

Mixing was combined with conduction by choosing an eddy conductivity factor of value ten or twenty for the set of experimental data used; the exact value did not prove to be critical to produce good results. We believe that the precise value should be a function of Reynolds number, Richardson number and inlet configuration. In order to obtain the dependence of ε upon these parameters, numerous simulations of various experimental data must be examined. We are currently trying to determine this relationship. This paper covers only one specific case, yet encouraging results are produced.

This one-dimensional model is efficient enough to be incorporated in simulations of complex chilled-water systems such as our campus-wide air-conditioning at Oklahoma State University. At the same time, it proved to be more accurate than the one-dimensional model reported in the literature, because of the care taken with the points mentioned above.

Our continuing efforts include conducting flow-visualizations and measurements using dye and salt solutions or heated water to study inflows and mixing rates in stratified tanks for several styles of inlet configurations and various density ranges and inflow conditions (Richardson and Reynolds numbers), for the purpose of obtaining general results for the value of the thermal eddy conductivity factor.

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