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Factor Analysis of Convective Heat Transfer for a Horizontal Tube in the Turbulent Flow Region Using Artificial Neural Network

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Abstract Artificial neural network (ANN) has shown its superior predictive power compared to the conventional approaches in many studies. However, it is always treated as a 'black box' because it provides little explanation to the relative influence of the independent variables in the prediction process. Ghajar et al. [1] used the ANN method to develop an empirical correlation for the heat transfer data in a horizontal tube with a reentrant inlet under uniform wall heat flux boundary condition in the transition region. In their work, the least and the most important variables were examined using the coefficient matrices based on a single training. However, the method was only applied to one set of experimental data. The applicability of their method to other data sets is not known. In this study, the method proposed in the previous study is modified and a new set of experimental data for different inlet configurations (square-edged and bell-mouth) from the work of Ghajar and Tam [2] in the turbulent region are used to further verify this method. An index of contribution is defined in this study. Furthermore, the gradient method used and the number of neurons and iterations for each training are carefully examined. Using the revised method and the index of contribution defined in this study, an ANN correlation is established and the Reynolds number (Re) and the Prandtl number (Pr) are observed as the most important parameters. The length-to-diameter ratio (x/D) and the viscosity ratio $(\mu_b/\mu_w)^{0.14}$ are found to be the least important parameters.

Key words: convective heat transfer, artificial neural network, turbulent flow, index of contribution
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INTRODUCTION

Heat transfer inside horizontal tubes in the transitional and turbulent flow regimes have been studied experimentally by various researchers in the past. Usually, the research results are presented in the form of heat transfer correlations. The form of the correlations is based either on different theoretical models or they are completely empirical. The coefficients of the correlations are usually determined by the conventional least squares method. Kakac et al. [3] documented some of the most well accepted correlations in the transition and turbulent flow regions. Recently, Ghajar et al. [1] proposed a new correlation in the transition region based on the method of artificial neural network (ANN) with excellent accuracy. In their paper, it was mentioned that ANN can also be used in the determination of the most and least important variables using the coefficient matrices obtained from the weight and bias matrices of the ANN correlation. However, there are some unanswered questions regarding this technique, such as (1) applicability of this technique to other data sets and (2) besides the most important variables, i.e., the normalized Reynolds and Grashof numbers, and least important variables, i.e., the normalized Sieder and Tate factor $(\mu_b/\mu_w)^{0.14}$, the importance of the normalized Prandtl number can not be seen. Therefore, this method is modified and then verified by using a different set of experimental data, the turbulent heat transfer data for uniformly heated horizontal tube fitted with different inlet configurations of Ghajar and Tam [2]. Furthermore, the gradient method selected, the number of neurons and iterations used for training will be analyzed and defined systematically in this study. Based on the defined network parameters, the index of contribution for each independent variable will be found from the ANN correlation by the revised method.

EXPERIMENTAL DATASET

The heat transfer experimental data used in this study, along with a detailed description of the experimental apparatus and procedures used, were reported by Ghajar and Tam [2]. A schematic of the overall experimental setup used for heat transfer measurements is shown in Figure 1. In this paper, only a brief description of the experimental setup and procedures will be provided. The local forced and mixed convective measurements were made in a horizontal, electrically heated, stainless steel circular straight tube with reentrant, square-edged, and bell-mouth inlets under a uniform wall heat flux condition. The pipe had an inside diameter of 1.58 cm and an outside diameter of 1.90 cm. The total length of the test section was 6.10 m, providing a maximum length-inside diameter ratio of 385. A uniform wall heat flux boundary condition was maintained by a dc arc welder. Thermocouples (T-type) were placed on the outer surface of the tube wall at close intervals near the entrance and at greater intervals further downstream. Twenty-six axial locations were designated, with four thermocouples placed at each location. The thermocouples were placed 90 degrees apart around the periphery. From the local peripheral wall temperature measurements at each axial location, the inside wall temperatures and the local heat transfer coefficients were calculated. In these calculations, the axial conduction was assumed negligible (RePr > 4,200 in all cases), but peripheral and radial conduction of heat in the tube wall were included. In addition, the bulk fluid temperature was assumed to increase linearly from the inlet to the outlet. As reported by Ghajar and Tam [2], the uncertainty analyses of the overall experimental procedures showed that there is a maximum of 9% uncertainty for the heat transfer coefficient calculations. Moreover, the heat balance error for each experimental run indicates that in general, the heat balance error is less than 5%. For Reynolds numbers lower than 2500 where the flow is strongly influenced by secondary flow, the heat balance error is slightly higher (5–8%) for that particular Reynolds number range. To ensure a uniform velocity distribution in the test fluid before it entered the test section, the flow passed through calming and inlet sections. The calming section had a total length of 61.6 cm and consisted of a 17.8 cm diameter acrylic cylinder with three perforated acrylic plates, followed by tightly packed soda straws sandwiched between galvanized steel mesh screens. Before entering the inlet section, the test fluid passed through a fine mesh screen and flowed undisturbed through 23.5 cm of a 6.5 cm-diameter acrylic tube before it entered the test section. The inlet section had the versatility of being modified to incorporate a reentrant or bell mouth inlet (see Figure 2). The reentrant inlet was simulated by sliding 1.93 cm of the tube entrance length into the inlet section, which was otherwise the square-edged (sudden contraction) inlet. For the bell-mouth inlet, a fiberglass nozzle with a contraction ratio of 10.7 and a total length of 23.5 cm was used in place of the inlet section. In the experiments, distilled water and mixtures of distilled water and ethylene glycol were used. The experiments covered the local bulk Reynolds number range of 280 to 49000, the local bulk Prandtl number range of 4 to 158, the local bulk Grashof number range of 1000 to 2.5×105, and the local bulk Nusselt number range of 13 to 258. The wall heat flux for the experiments ranged from 4 to 670 kW/m^2.

As mentioned in the previous section, only the turbulent heat transfer data is considered in this study. Because of that, Grashof number makes no contribution to Nusselt number. Therefore, the correlation only consists of five variables, which are Nu, Re, Pr, x/D, and μ_b/μ_w and the range of the variables is summarized as follows:

$57.4 \leqq \text{Nu} \leqq 235.3$, $3.21 \leqq x/D \leqq 173.08$, $9128 \leqq \text{Re} \leqq 46338$,

$4.35 \leqq \text{Pr} \leqq 31.04$, $1.1 \leqq \mu_b/\mu_w \leqq 1.54$

A total of 418 data points is used. Out of them, 173 data points are for the reentrant inlet, 143 data points are for the square-edged inlet, and 102 data points are for the bell-mouth inlet. Ghajar and Tam [2] correlated their data in the turbulent flow region in the following form:

$$\text{Nu}_t = 0.023\text{Re}^{0.8}\text{Pr}^{0.385}(x/D)^{-0.0054}(\mu_b/\mu_w)^{0.14} \tag{1}$$

The accuracy of the correlation is described in Table 1. Figure 3 shows that the majority of the turbulent data were predicted by this correlation within ±10%. The next sections will be devoted to how the variables in Equation (1) contribute to the Nusselt number.

Table 1: Prediction Results for Equation (1)

Inlet Configurations	Number of Data within ±10%	Number of Data within ±5%	Abs. Mean Dev. (%)	Range of Dev. Abs.(%)
Total data points (418 pts.)	389	291	4.02	-13.5% to12.5%
Reentrant (173 pts.)	147	100	4.99	-13.5% to12.5%
Square-edged (143 pts.)	141	113	3.26	-11.0% to 6.37%
Bell-mouth (102 pts.)	101	78	3.42	-4.09% to 10.28%

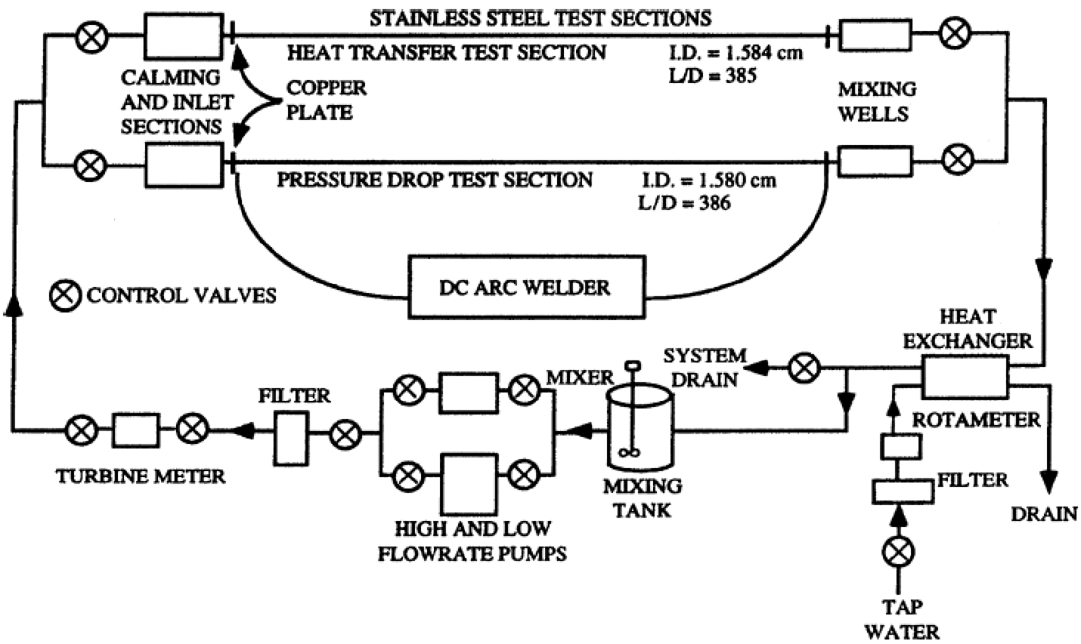

Figure 1: Schematic diagram of experimental setup

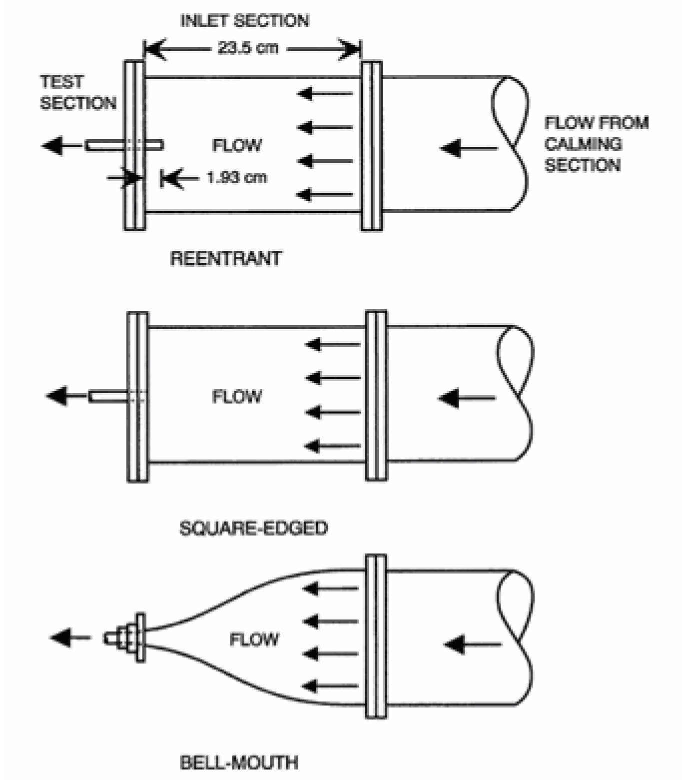

Figure 2: Schematic of the three different inlet configurations

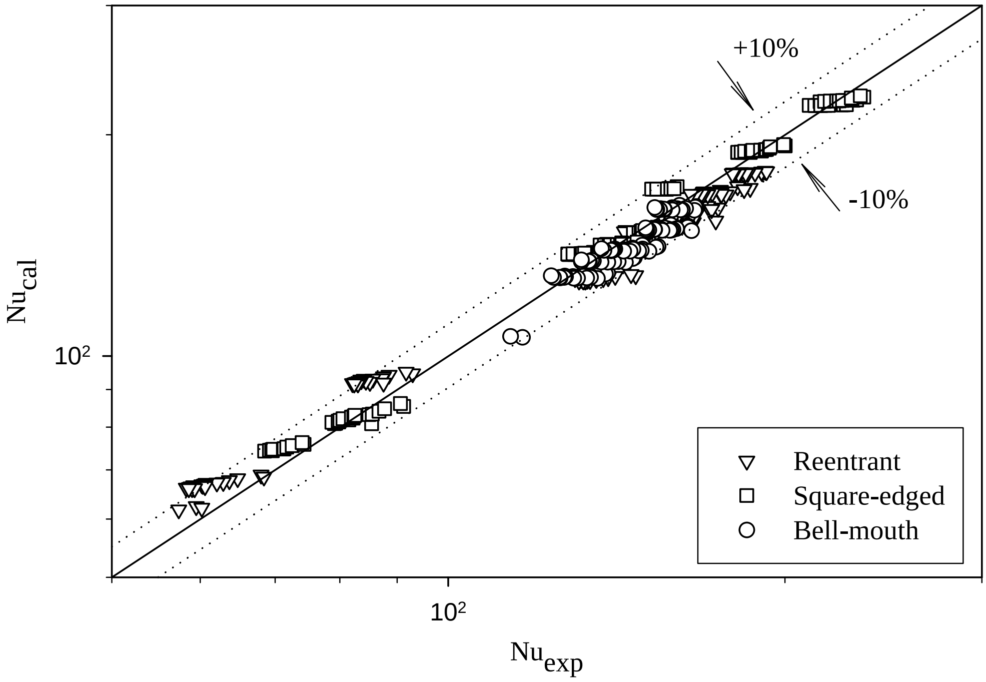

Figure 3: Comparison between experimental Nusselt numbers for three different inlet configurations and those predicted by the turbulent region heat transfer correlation, Equation (1)

ANN CORRELATION AND THE INDEX OF CONTRIBUTION ANALYSIS

To correlate the turbulent heat transfer data, the ANN with single hidden layer is employed. Figure 4 shows a typical example of ANN model of this kind. It has been shown that any continuous correlation can be modeled by the network [4]. The weight and the bias of the optimal ANN model are usually determined by the back propagation algorithms [5]. In order to determine the contribution of each independent variable to the correlation, the matrix form of the optimal ANN model has to be examined.

$$N(p_1,\cdots,p_R)=[w_1^2,\cdots,w_S^2]\begin{bmatrix} f(\sum_{j=1}^{R} w_{1j}^1 p_j + b_1^1) \\ f(\sum_{j=1}^{R} w_{2j}^1 p_j + b_2^1) \\ \vdots \\ f(\sum_{j=1}^{R} w_{Sj}^1 p_j + b_S^1) \end{bmatrix} + b_1^2 \qquad (2)$$

where ($p_1,\cdots,p_R$) are the R inputs, S is the number of hidden neurons, $f(t)=\dfrac{1}{1-e^{-t}}$ is the transfer function and w's and b's are the weights and biases of the ANN respectively. The contribution of the independent variables p_j to the output of the k^{th} neuron in the hidden layer $f(\sum_{j=1}^{R} w_{kj}^1 p_j + b_k^1)$ is simply $\left|w_{kj}^1\right|$ and the contribution of the k^{th} neuron output to the ANN model is

$$Ratio\_W_k^{2} = \frac{\left|w_k^2\right|}{\sum_{k=1}^{S}\left|w_k^2\right|} \qquad (3)$$

Therefore, the contribution of the independent variables p_j to the ANN model is

$$Product\_W_j^1 = \sum_{k=1}^{S} ratio\_W_k^{2} * \left|w_{kj}^1\right| \qquad (4)$$

Finally, in order to compare with the other independent variables, the index of contribution of p_j is defined to be

$$index(p_j) = \frac{Product\_W_j^1}{\sum_{j=1}^{R} Product\_W_j^1} \times 100\% \tag{5}$$

Hence, the most significant independent variable would have the largest index of contribution. On the other hand, the variable with small index appears to be less important.

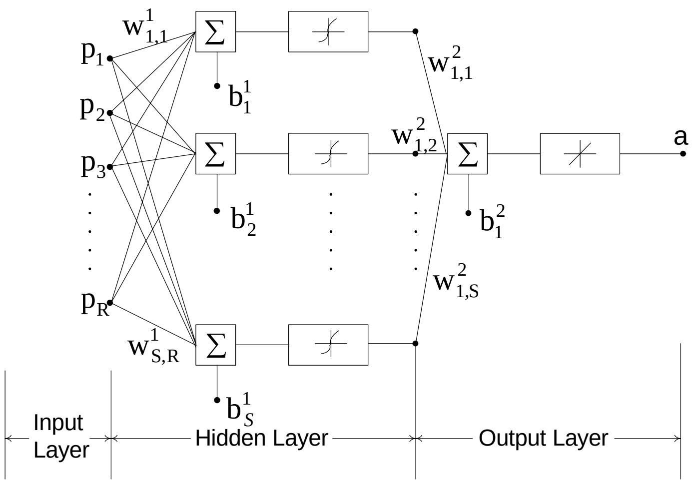

Figure 4: An ANN with S neurons in its hidden layer

RESULTS AND DISCUSSION

Before the usage of the index of contribution defined in Equation (5), it is worthwhile to evaluate the traditional least squares Equation (1). According to the form of Equation (1), it is obvious that Reynolds and Prandtl numbers are both important. However, it is not possible to tell which one is more important than the other by simply judging the exponents of them. It is also obvious that x/D, and μ_b/μ_w are the least important variables. According to the range of the data, the value of the terms $(x/D)^{-0.0054}$ and $(\mu_b/\mu_w)^{0.14}$ are forced to a value very close to one, hence they made less contribution to the Nusselt number. Theoretically, for turbulent flow, the thermal entry length is usually very short and the entrance effect is insignificant. Moreover, from Sieder and Tate [6], the data in the turbulent region show little variation between μ_b and μ_w , firstly, because fluids which give turbulent flow seldom have a large temperature coefficient of viscosity and secondly, because the heat transfer rates are high, preventing large temperature differences. Therefore, both the terms $(x/D)^{-0.0054}$ and $(\mu_b/\mu_w)^{0.14}$ act as correction factors to make the correlation more accurate.

Coming back to the ANN analysis, using the supervised three-layer feedforward neural network with fully weighted connection and the algorithm described in the previous section to determine the index of contribution for each independent variable, we are able to decide (a) the gradient method to be used, and (b) the reasonable number of iterations and neurons used for each network training.

For the gradient method to be used, according to Ghajar et al. [1,7], the Levenberg-Marquart algorithm (LM) was adopted as the gradient method in back propagation. According to Hagan and Menhaj [8], this method can speed up the network training. However, it is hard to tell whether this method works well with the index of contribution analysis. Therefore, in this study, the classical gradient method, which is the slower steep descent algorithm (SDA) is considered. The turbulent heat transfer data is correlated using the LM and SDA methods. The number of neurons is varied from 5 to 8 and the iteration for each training is fixed at 10,000. The index of contribution based on the algorithm described in the previous section using different number of neurons is calculated and shown in Figure 5. Each data point shown in the figure is an average of 10 trainings. From Figure 5, it is obvious that the SDA method gave more consistent information regarding the contribution from each independent variable irregardless of how many neurons were used. Moreover, the findings based on SDA agree with the findings according to the traditional least squares equation, where Re and Pr contribute the most and $(x/D)^{-0.0054}$ and $(\mu_b/\mu_w)^{0.14}$ contribute less. When the LM method is considered (see Figure 5b), the influence of $(\mu_b/\mu_w)^{0.14}$ is large and the same observations can not be seen. Since $(\mu_b/\mu_w)^{0.14}$ is proved to be a correction factor in the turbulent region, the LM method cannot clearly identify the contribution of each variable. Therefore, the SDA method should be selected as the gradient method in this study.

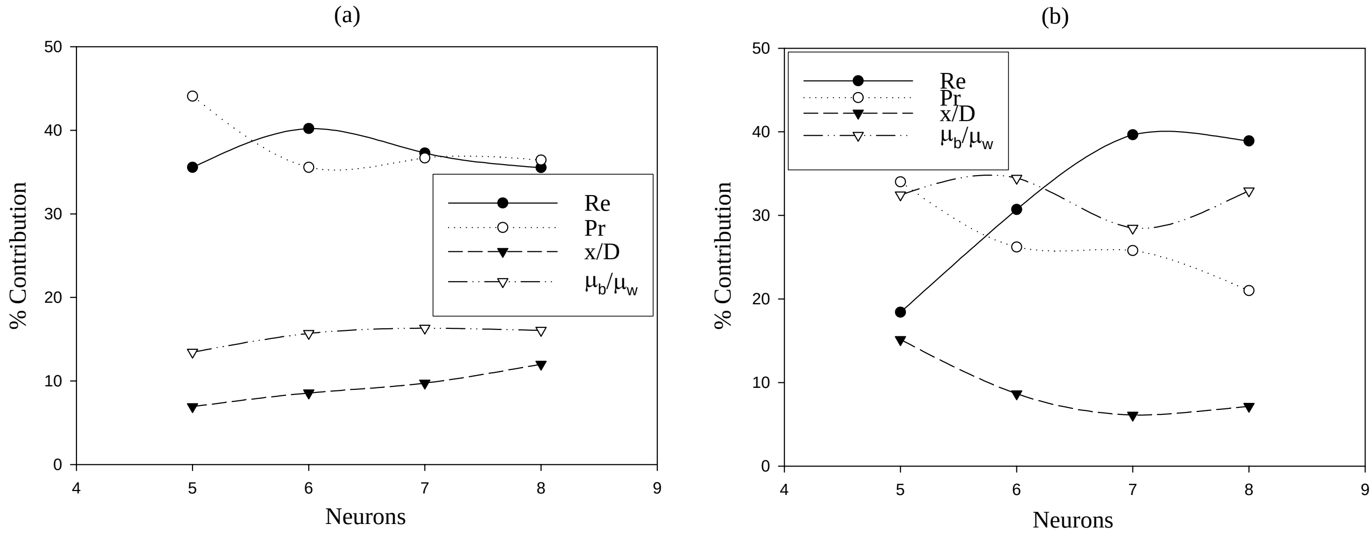

Figure 5: A comparison of the percent contribution by using two different gradient descent algorithms. (a) Slower algorithm – SDA method; (b) Faster algorithm – LM method. Each data point shown is the average value from 10 trainings.

For the reasonable number of iterations and neurons to be used for each network training, the following steps were taken. The ANN correlation and the index of contribution calculation is computed based on 5, 6, 7 and 8 neurons. The number of iterations for each training ranged from 1,000 to 100,000 times. The increment of the iterations is set as 1,000 when the number of iterations is from 1,000 to 10,000. The increment of the iterations is set as 10,000 when the number of iterations is from 10,000 to 100,000. The index of contribution for different number of neurons and iterations are shown in Figure 6. Again, each data point shown in these figures are the average of 10 trainings. According to these figures, it is important to find out that the contribution from each variable is not well established when the number of iterations is less than 10,000. When the number of iterations is more than 10,000, the trend for the contribution from each variable is highly consistent for different number of neurons. The contributions from Re and Pr are always close to 40% and the other variables contribute less than 20%. Therefore, the number of iterations based on Figure 6 is selected as 10,000. Since the trend for the contribution from each variable is relatively insensitive to the number of neurons used, the number of neurons is arbitrarily chosen as 6.

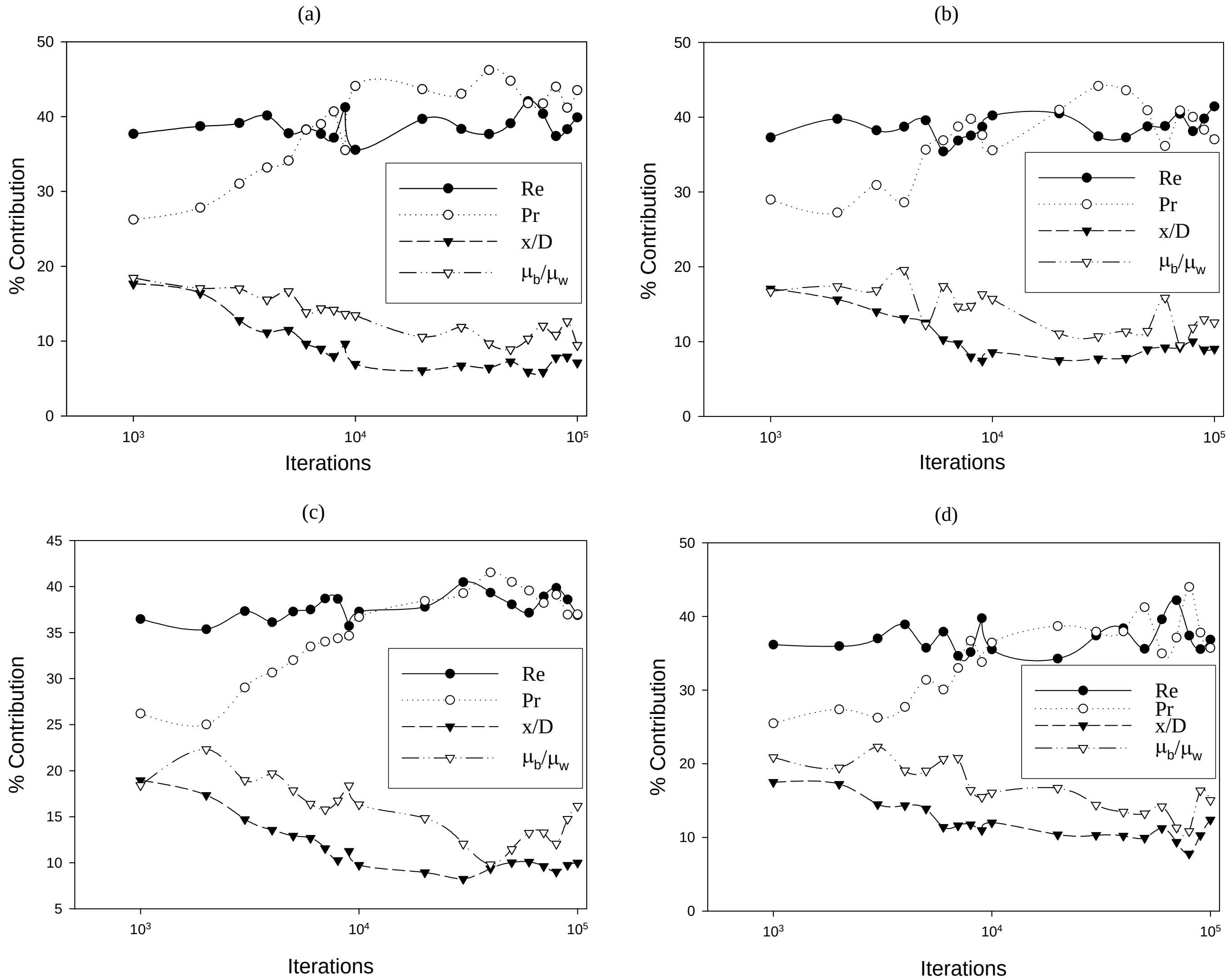

Figure 6: The percent contribution from ANN training for different dimensionless numbers to Nusselt number by adjusting the iterations and the number of neurons, (a) 5 neurons, (b) 6 neurons, (c) 7 neurons, (d) 8 neurons. Each data point is the average value from 10 trainings.

In summary, based on the observations made above, the gradient method is selected as the SDA, the number of iterations used is selected as 10,000 and the number of neurons used is selected as 6. Moreover, the form of the ANN correlation is given by Equation (2). With the establishment of the ANN correlation, the index of contribution according to the above mentioned criteria can now be computed. For reliability purposes, ninety percent of the total of 418 data points were used for training and the remaining data is for verification. The initial value of the free parameters (weights and biases) is randomly selected within ±1. For satisfying the log-sigmoid transfer function, the normalized input variables, Reynolds number, Prandtl number, length-to-diameter ratio, and viscosity ratio are arranged into the input vector, **p:**

$$\mathbf{p} = \begin{bmatrix} Re_{normal} \\ Pr_{normal} \\ \left(\frac{x}{D}\right)_{normal} \\ \left(\frac{\mu_b}{\mu_w}\right)_{normal} \end{bmatrix} = \begin{bmatrix} 2 \cdot \left[Re - Re_{min}\right] \;/\; \left[Re_{max} - Re_{min}\right] - 1 \\ 2 \cdot \left[Pr - Pr_{min}\right] \;/\; \left[Pr_{max} - Pr_{min}\right] - 1 \\ 2 \cdot \left[\left(\frac{x}{D}\right) - \left(\frac{x}{D}\right)_{min}\right] \;/\; \left[\left(\frac{x}{D}\right)_{max} - \left(\frac{x}{D}\right)_{min}\right] - 1 \\ 2 \cdot \left[\left(\frac{\mu_b}{\mu_w}\right) - \left(\frac{\mu_b}{\mu_w}\right)_{min}\right] \;/\; \left[\left(\frac{\mu_b}{\mu_w}\right)_{max} - \left(\frac{\mu_b}{\mu_w}\right)_{min}\right] - 1 \end{bmatrix}$$

In Equation (2), the dependent output is the Nusselt number for the turbulent heat transfer data. The $\mathbf{w^1}$, $\mathbf{w^2}$, $\mathbf{b^1}$, $\mathbf{b^2}$ terms used in Equation (2) are constant matrices or scalars. Their numerical values are shown in the following matrices:

$$\mathbf{w^1} = \begin{bmatrix} 0.11 & -0.09 & 0.77 & 0.21 \\ 0.11 & -0.76 & 0.01 & -0.75 \\ -0.34 & -2.09 & -0.06 & -0.25 \\ 0.22 & 1.21 & -0.04 & 1.00 \\ 1.11 & 0.25 & 0.25 & 0.69 \\ 1.50 & -0.07 & -0.04 & -0.11 \end{bmatrix}, \quad \mathbf{w^2} = \begin{bmatrix} -0.05 & -0.32 & -1.83 & -0.66 & 0.79 & 2.62 \end{bmatrix}, \quad \mathbf{b^1} = \begin{bmatrix} -0.62 \\ -0.69 \\ -1.76 \\ -0.49 \\ 0.63 \\ 1.03 \end{bmatrix}, \quad \mathbf{b^2} = -0.81$$

As shown in Table 2, all the experimental data are predicted within -11% to 11.38%. The absolute deviation of all the predictions is 3.21%. About 98% of the data (412 data points) are predicted within ±10% deviation. About 77% of all the data (323 data points) are predicted within ±5% deviation. The prediction of bell-mouth data points is most accurate, i.e. all the data points are predicted within ±10% deviation. As compared to the previous correlation, significant improvement is observed.

Table 2: Prediction Results for the Improved Correlation by Using ANN

Inlet Configurations	**Number of Data within ±10%**	**Number of Data within ±5%**	**Abs. Mean Dev. (%)**	**Range of Dev. Abs.(%)**
Total data points (418 pts.)	412	323	3.21	-11.00% to11.38%
Reentrant (173 pts.)	170	122	3.57	-10.47% to11.38%
Square-edged (143 pts.)	140	116	3.05	-11.00% to 11.22%
Bell-mouth (102 pts.)	102	143	2.85	-9.69% to 7.67%

For the calculation of the index of contribution for each variable, Equations (3) to (5) are employed according to the weight matrices, $\mathbf{w^1}$ and $\mathbf{w^2}$ shown above. The computation process is as follows:

i. For each hidden neuron k, the absolute value of the hidden-output layer connection weight is divided by the summation of the hidden-output layer connection weight.

	Hidden Neuron 1	Hidden Neuron 2	Hidden Neuron 3	Hidden Neuron 4	Hidden Neuron 5	Hidden Neuron 6
$Ratio\_W_k^2$	0.01	0.05	0.29	0.11	0.13	0.42

ii. For each hidden neuron k, multiply the $Ratio\_W_k^2$ by the absolute value of the hidden-input layer connection weight. Then, sum up the values to obtain the $Product\_W_j^1$ for each input variable p_j .

	Re	Pr	x/D	μ_b/μ_w
$Product\_W_j^1$	0.89	0.84	0.08	0.35

iii. Compute the index of contribution in percentage by dividing $Product\_W_j^1$ by the sum of the $Product\_W_j^1$ corresponding to each input variable. Finally, the index of contribution for each variable is established as:

	Re	Pr	x/D	μ_b/μ_w
$index(p_j)$	41.4 %	38.8 %	3.6 %	16.2 %

Through the computation, the contribution of each variable can be obviously seen. The result demonstrates that the Prandtl number and Reynolds number are the most important variables since they contribute more to the Nusselt number. On the other hand, the x/D and viscosity ratio are the least important variables since they contribute less to the Nusselt number. That is absolutely identical to the observations made from the conventional heat transfer correlations.

CONCLUSIONS AND RECOMMENDATIONS

Development of correlations using ANN is considered as a black box approach. However, in this study, besides the establishment of the ANN heat transfer correlation, the contribution from each variable is evaluated by using the index of contribution defined in this study. Moreover, the gradient method used, the number of neurons employed and the number of iterations adopted in this study were carefully examined. Based on the numerical work done in this study, it can be concluded that not only the ANN method can be used in accurately correlating the data, but also can be used to determine the contribution from each variable. Unlike the analysis for the most and least important variables based on the traditional least squares correlation, which can only determine what is important, the proposed method in this study can clearly quantify the contribution for different variables. Therefore, this method not only can evaluate the contributions qualitatively, but also quantitatively. However, it is still recommended that more experimental data sets should be examined to verify this interesting method.

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