COMPRESSIBLE FLOW SOFTWARE FOR PROPERTIES CALCULATION AND AIRFOIL ANALYSIS

L. M. Tam [1,*], A. J. Ghajar [2,#], and C. W. Pau [1]

[1]Electromechanical Engineering Program, University of Macau, Macau
[2]School of Mechanical and Aerospace Engineering, Oklahoma State University, USA

ABSTRACT

For engineering applications involving compressible flow analysis, it is inevitable that tedious tables and charts have to be used in order to calculate the compressible flow properties. However, there are restrictions when using those tables and charts, for example, the specific heat ratio which indicates the type of fluid used is one of them. Most of the tables and charts are constructed using a particular value for the specific heat ratio. For other values, tables may not be available and the original equations for a particular flow may need to be solved numerically in order to obtain the desired properties. Moreover, when a situation involving shock waves is encountered, such as an oblique shock or a conical shock, it is very difficult for the user to accurately read the properties from the charts if the desired Mach number is not shown and visual interpolation has to be used. In these situations it is very easy to make calculation mistakes. The software developed in this study resolves the above-mentioned problem. The computer languages used are Fortran and Delphi. Fortran is used to build a dynamic link library, which handles all the numerical calculations. Delphi is used to construct an interactive user-friendly graphical interface for the user to have a convenient way to access the library. There are six modules in this computer program. The first five modules calculate the properties for: **Isentropic Flow**, **Normal Shock Wave**, **Oblique Shock Wave**, **Fanno Flow**, and **Rayleigh Flow**. The last module is for **Supersonic Airfoil Analysis**. For the first five modules, the user can input data and obtain the output through a dialog box or from a graph, which is generated using the flow equations. For supersonic airfoil analysis, a CAD environment is developed for the user to define the dimensions and shape of an airfoil. The software can then calculate the lift force, the drag force, and the pressure distribution of the airfoil according to the flow Mach number and the airfoil angle of attack. The software can be downloaded free of charge in order to support engineering education.

[*] Address correspondence to Dr. L. M. Tam, Electromechanical Engineering Program, Faculty of Science and Technology, P.O. Box 3001, University of Macau, Macau. Tel: (853)3974457, E-mail: fstlmt@umac.mo

[#] Regents Professor and Director of Graduate Studies, School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, Oklahoma, 74078, USA.

1. INTRODUCTION

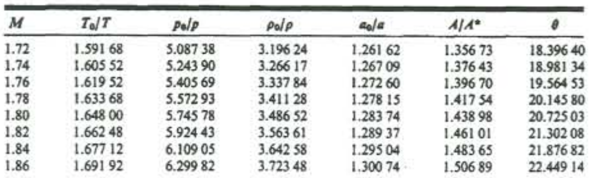
Compressible fluid mechanics is a study of flow in which significant density variations occur throughout the fluid. Traditional practices are the utilization of tables or graphs which give the variations of such quantities as p_0/p, and T_o/T with M in isentropic flow for a fixed value of the specific heat ratio γ. Such tables and graphs are available for a limited range of the specific heat ratio but care must be taken to ensure that a table for the correct value of γ is used. It is even more troublesome with shock wave problems such as an oblique shock or a conical shock since not only the graph with the correct γ value needs to be used but also visual interpolation within the figures is almost always necessary. Most of the mistakes or inaccurate readings are from the above-mentioned processes. This paper discusses the modules adopted in the software, the integrated environment for flow-properties calculations, and the numerical methods used in the program.

2. MODULES

The software consists of six modules, the isentropic flow, normal shock wave, oblique shock wave, Fanno flow, Rayleigh flow, and supersonic airfoil analysis. The software provides user-friendly interfaces for each module (as seen in later sections) to communicate with a dynamic link library to solve equation listed in Table I. The specific heat ratio (γ) value used in the calculations can be easily defined in the main screen of the software.

2.1 *Isentropic Flow*

An isentropic flow is an adiabatic flow (a flow in which there is no heat exchange) in which viscous losses are negligible, i.e. it is an adiabatic frictionless flow. Variations in properties are brought about by area change. The terms that are denoted by the subscript 0 in isentropic flow equations represent stagnation conditions. Traditionally, calculation of isentropic compressible flow properties is done with the aid of tables. Figure 1 is a representative table taken from Oosthuizen (1997) for one dimensional isentropic flow with γ =1.4. User-friendly is an important characteristic of a good software. Figure 2 shows the input dialog box for the isentropic flow module. The user only needs to enter the value of any one parameter shown in the input dialog box, and the other parameters will be calculated and displayed. Another useful tool of this software is the "Pick from Graph" function. The user can trace the input and output values directly from the graph as shown in Figure 2. This function assists students or engineers to have a visual physical picture of how the various properties change with the Mach number. Numerical methods need to be used when the input parameter is not the Mach number.



M	T_0/T	p_0/p	ρ_0/ρ	a_0/a	A/A^*	θ
1.72	1.591 68	5.087 38	3.196 24	1.261 62	1.356 73	18.396 40
1.74	1.605 52	5.243 90	3.266 17	1.267 09	1.376 43	18.981 34
1.76	1.619 52	5.405 69	3.337 84	1.272 60	1.396 70	19.564 53
1.78	1.633 68	5.572 93	3.411 28	1.278 15	1.417 54	20.145 80
1.80	1.648 00	5.745 78	3.486 52	1.283 74	1.438 98	20.725 03
1.82	1.662 48	5.924 43	3.563 61	1.289 37	1.461 01	21.302 08
1.84	1.677 12	6.109 05	3.642 58	1.295 04	1.483 65	21.876 82
1.86	1.691 92	6.299 82	3.723 48	1.300 74	1.506 89	22.449 14

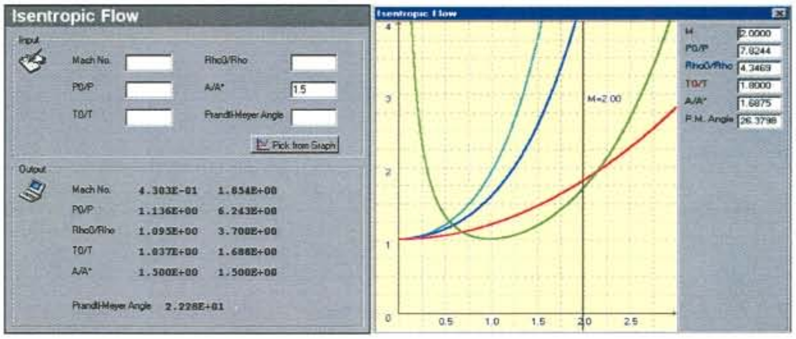
Figure 1. Typical isentropic flow table for $\gamma = 1.4$

Table 1 List of Equations

Isentropic Flow		**Adiabatic Fanno Flow**	
$\frac{T_0}{T} = (1+\frac{\gamma-1}{2}M^2)$	(1)	$\frac{4f}{D}l^* = \left(\frac{1-M^2}{\gamma M^2}\right)+\frac{\gamma+1}{2\gamma}\ln\frac{(\gamma+1)M^2}{2\left(1+\frac{1}{2}(\gamma-1)M^2\right)}$	(13)
$\frac{p_0}{p} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$	(2)	$\frac{p}{p^*} = \frac{1}{M}\left[\frac{(\gamma+1)/2}{1+(\gamma+1)M^2/2}\right]^{1/2}$	(14)
$\frac{\rho_0}{\rho} = \left(1+\frac{\gamma-1}{2}M^2\right)^{1/(\gamma-1)}$	(3)	$\frac{T}{T^*} = \frac{(\gamma+1)/2}{1+(\gamma-1)M^2/2}$	(15)
$\frac{A}{A^*} = \frac{\sqrt{\gamma}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(2-2\gamma)}}{M\sqrt{\gamma}\left(1+\frac{\gamma-1}{2}M^2\right)^{(\gamma+1)/(2-2\gamma)}}$	(4)	$\frac{p_0}{p_0^*} = \frac{1}{M}\left[\frac{1+(\gamma-1)M^2/2}{(\gamma+1)/2}\right]^{\gamma+1/2(\gamma+1)}$	(16)
Normal Shock Wave		**Isothermal Fanno Flow**	
$\frac{p_2}{p_1} = \frac{2\gamma M_1^2-(\gamma-1)}{(\gamma+1)}$	(5)	$\frac{4fl^*}{D} = \left[\frac{1-\gamma M^2}{\gamma M^2}\right]+\ln[\gamma M^2]$	(17)
$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$	(6)	$\frac{\rho}{\rho^*} = \frac{1}{\sqrt{\gamma}M}$	(18)
$\frac{T_2}{T_1} = \left\{\frac{[2\gamma M_1^2-(\gamma-1)]\cdot[2+(\gamma-1)M_1^2]}{(\gamma+1)^2M_1^2}\right\}$	(7)	$\frac{T_0}{T_0^*} = \frac{2\gamma}{3\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)$	(19)
$\frac{p_{02}}{p_{01}} = \left\{\frac{(\gamma-1)}{2}\cdot\frac{M_1^2}{\left(1+\frac{(\gamma-1)}{2}M_1^2\right)}\right\}^{\gamma/(\gamma-1)} \cdot\left\{\left(\frac{2\gamma}{\gamma+1}\right)M_1^2-\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^{-1/(\gamma-1)}$	(8)	**Rayleigh Flow**	
		$\frac{T_0}{T_0^*} = \frac{2(\gamma+1)M^2[1+(\gamma+1)M^2/2]}{(1+\gamma M^2)^2}$	(20)
$M_2^2 = \frac{(\gamma-1)M_1^2+2}{2\gamma M_1^2-(\gamma-1)}$	(9)	$\frac{T}{T^*} = \frac{(1+\gamma)^2M^2}{(1+\gamma M^2)^2}$	(21)
$\frac{p_{02}}{p_1} = \frac{[(\gamma+1)M_1^2/2]^{\gamma/(\gamma-1)}}{\left[\left(\frac{2\gamma M_1^2}{\gamma+1}\right)-\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{1/(\gamma-1)}}$	(10)	$\frac{p}{p^*} = \frac{(1+\gamma)}{(1+\gamma M^2)}$	(22)
Oblique Shock Wave		$\frac{\rho}{\rho^*} = \frac{(1+\gamma M^2)}{(1+\gamma)M^2}$	(23)
$\tan\delta = \frac{2\cot\beta(M_1^2\sin^2\beta-1)}{2+M_1^2(\gamma+\cos 2\beta)}$	(11)	$\frac{p_0}{p_0^*} = \left(\frac{1+\gamma}{1+\gamma M^2}\right)\left\{\left(\frac{2}{\gamma+1}\right)\left[1+\frac{(\gamma-1)}{2}M^2\right]\right\}^{\gamma/\gamma-1}$	(24)
$\sin^2\beta_{max} = \frac{\gamma+1}{4\gamma}-\frac{1}{\gamma M_1^2}\left[1-\sqrt{(\gamma+1)\left(1+\frac{\gamma-1}{2}\right)M_1^2+\frac{\gamma+1}{16}M_1^4}\right]$	(12)		

2.2 *Normal Shock Wave*

Under some circumstances, it is possible for an almost spontaneous change to occur in a flow, the velocity decreasing and the pressure increasing through this region of sharp change. Such regions of sharp change can only occur if the initial flow is supersonic. In the case of a normal shock wave, the velocities both ahead (i.e. upstream) of the shock and after (i.e. downstream) of the shock are at right angle to the shock wave. Figure 3 shows a sample input/output and the graph generated by the software for a normal shock wave.



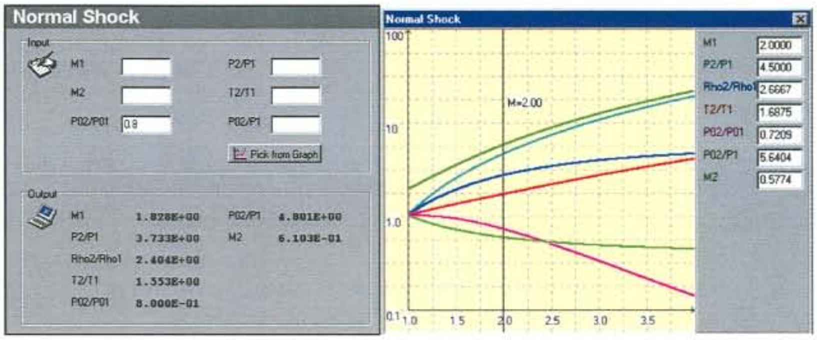
A/A* 1.5

```
Mach No.    4.303E-01   1.854E+00
P0/P        1.136E+00   6.243E+00
Rho0/Rho    1.095E+00   3.700E+00
TO/T        1.037E+00   1.688E+00
A/A*        1.500E+00   1.500E+00

Prandtl-Meyer Angle   2.228E+01
```

M 2.0000
P0/P 7.8244
Rho0/Rho 4.3469
TO/T 1.8000
A/A* 1.6875
P.M. Angle 26.3798

Figure 2. Input dialog box and the pick from graph function for isentropic flow module



P02/P01 0.8

```
M1          1.828E+00   P02/P1   4.801E+00
P2/P1       3.733E+00   M2       6.103E-01
Rho2/Rho1   2.404E+00
T2/T1       1.553E+00
P02/P01     8.000E-01
```

M1 2.0000
P2/P1 4.5000
Rho2/Rho1 2.6667
T2/T1 1.6875
P02/P01 0.7209
P02/P1 5.6404
M2 0.5774

Figure 3. Input dialog box and the pick from graph function for the normal shock module

2.3 *Oblique Shock Wave*

In case of an oblique shock wave, there is a change in flow direction across the shock (see Figure 4). The oblique shock relations can be deduced from the normal shock relations by noting that the oblique shock can produce no momentum change parallel to the plane in which it lies. Hence, if in the normal shock relations M_1 is replaced by $M_1 \sin\beta$ and M_2 by $M_2 \sin(\beta-\delta)$, relations for oblique shocks can be developed by using normal shock equations. For example, an oblique shock can occur when there is supersonic flow over a concave corner.

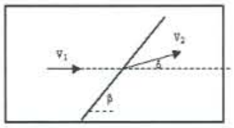
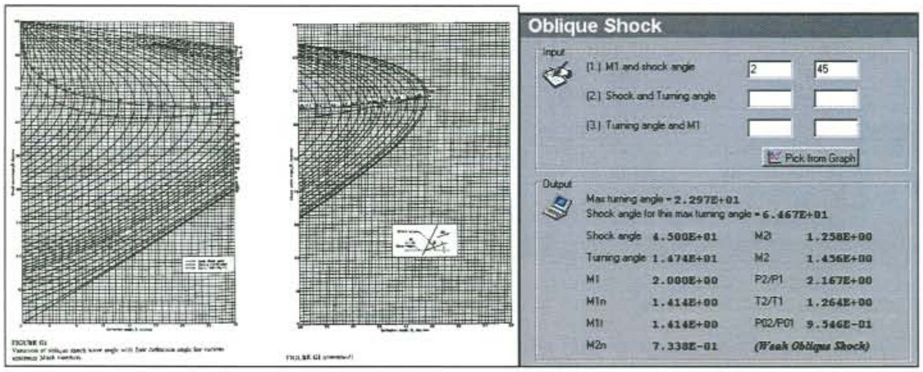


Figure 4. Velocity changes across oblique shock wave

For proper calculations of oblique shock properties, certain constraints must be satisfied. After the inputs have been entered in the dialog box, the following constrained are automatically checked by the software: $M_1 \sin\beta \geq 1$, $\sin^{-1}\left(\frac{1}{M_1}\right) \leq \beta \leq 90^\circ$, $\delta \leq \delta_{\max}$, $M_2 \sin(\beta-\delta) \leq 1$.

Traditionally, the calculation of oblique shock properties requires the utilization of some very complicated charts (refer to the left hand side of Figure 5). The user always needs to locate the intersection of three parameters; shock angle, turning angle, and Mach number in the chart to obtain the desired results. Such a procedure is not an easy task, usually only an approximation can be obtained. Figure 6 also shows the output sample generated by the software (refer to the right hand side of the figure) for a shock with an upstream Mach number of 2 and a shock angle of 45°.



(1.) M1 and shock angle: 2, 45

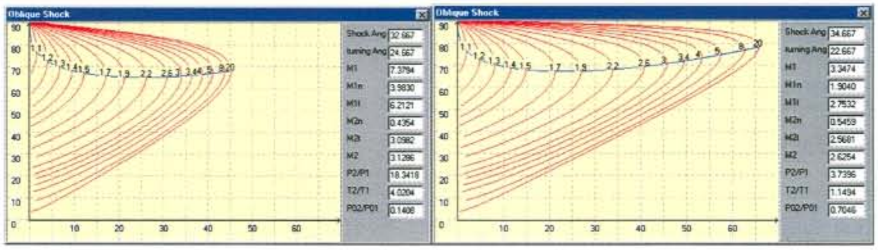
(2.) Shock and Turning angle

(3.) Turning angle and M1

```
Max turning angle = 2.297E+01
Shock angle for this max turning angle = 6.467E+01
Shock angle   4.500E+01      M2t       1.258E+00
Turning angle 1.474E+01      M2        1.456E+00
M1            2.000E+00      P2/P1     2.167E+00
M1n           1.414E+00      T2/T1     1.264E+00
M1t           1.414E+00      P02/P01   9.546E-01
M2n           7.338E-01      (Weak Oblique Shock)
```

Figure 5. Comparison between traditional oblique shock wave graph and the simple dialog box for the oblique shock module

Although advance users may feel comfortable with entering the required inputs in the dialog box and get the results, some novice users may prefer to use the graphs. Therefore, the software also provides this option and the number of graphs that is provided in the software, is even more than what can be found in any compressible flow textbook. Figure 6 shows two graphs generated by the software, one for Air and one for Butane.

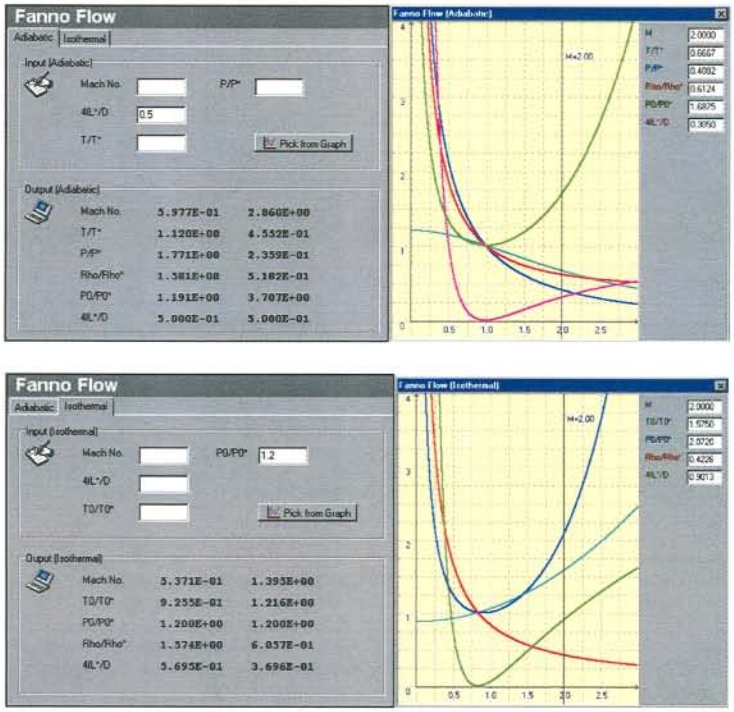


	Air	Butane
Shock Ang	32.667	34.667
turning Ang	24.667	22.667
M1	7.3794	3.3474
M1n	3.9830	1.9040
M1t	6.2121	2.7532
M2n	0.4354	0.5459
M2t	3.0982	2.5681
M2	3.1286	2.6254
P2/P1	18.3418	3.7396
T2/T1	4.0204	1.1494
P02/P01	0.1408	0.7046

Figure 6. Variation of oblique shock wave angle with flow deflection angle and various upstream Mach numbers for Air (γ = 1.400) and Butane (γ = 1.091)

2.3 Fanno Flow

The function of this module is to calculate compressible flow characteristics in a constant area duct with frictional effects. The software can handle two different cases. One deals with the effect of friction in constant area adiabatic (insulated) ducts and the other deals with flow with friction in constant area ducts, in which the fluid temperature is assumed constant (isothermal). Figures 7 shows a sample input/output and the graph generated by the software for adiabatic and isothermal Fanno flow.



Input (Adiabatic)

Mach No.
P/P*
4L*/D 0.5
T/T*

Output (Adiabatic)

Mach No.	5.977E-01	2.860E+00
T/T*	1.120E+00	4.552E-01
P/P*	1.771E+00	2.359E-01
Rho/Rho*	1.381E+00	5.182E-01
P0/P0*	1.191E+00	3.707E+00
4L*/D	5.000E-01	5.000E-01

M 2.0000
T/T* 0.6667
P/P* 0.4082
Rho/Rho* 0.6124
P0/P0* 1.6875
4L*/D 0.3050

Input (Isothermal)

Mach No.
P0/P0* 1.2
4L*/D
T0/T0*

Ouput (Isothermal)

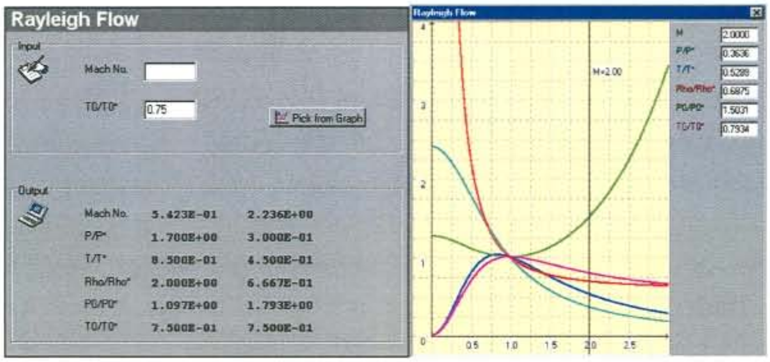
Mach No.	5.371E-01	1.395E+00
T0/T0*	9.255E-01	1.216E+00
P0/P0*	1.200E+00	1.200E+00
Rho/Rho*	1.574E+00	6.057E-01
4L*/D	5.695E-01	3.696E-01

M 2.0000
T0/T0* 1.5750
P0/P0* 2.0720
Rho/Rho* 0.4226
4L*/D 0.9013

Figure 7. Input dialog box and the pick from graph function for adiabatic (above) and isothermal (below) Fanno flow module

2.4 Rayleigh Flow

Rayleigh flow is a compressible flow in a constant area duct with heat addition and negligible friction. Figure 8 shows a sample input/output and the graph generated by the software for Rayleigh flow.



Mach No.
TO/TO* 0.75

```
Mach No.    5.423E-01   2.236E+00
P/P*        1.700E+00   3.000E-01
T/T*        8.500E-01   4.500E-01
Rho/Rho*    2.000E+00   6.667E-01
PO/PO*      1.097E+00   1.793E+00
TO/TO*      7.500E-01   7.500E-01
```

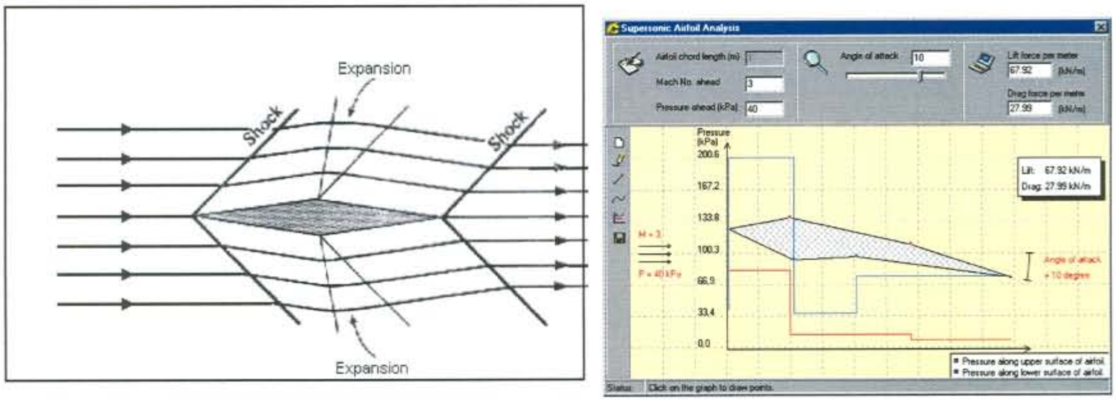
M 2.0000
P/P* 0.3636
T/T* 0.5289
Rho/Rho* 0.6875
PO/PO* 1.5031
TO/TO* 0.7934

Figure 8. Input dialog box and the pick from graph function for Rayleigh flow module

2.5 Supersonic Airfoil Analysis

An airfoil can be assumed to consist of a series of flat surfaces. For example, a diamond airfoil is composed of four flat surfaces. During analysis, the turning angle for each surface should be examined in order to find out whether there is an expansion wave or a shock wave in the flow, and then apply corresponding theories to find out the pressure on each surface. Lift and drag forces are determined by the summation of these pressure forces. For a diamond airfoil, as seen on the left-hand side of Figure 9, there are leading-edge oblique shocks on both surfaces, then there are expansion fans on each shoulder of the diamond. Finally, the trailing-edge pattern can be ignored in the calculations because it has no effect on the pressure of the surface.

The analysis of a supersonic airfoil flow is rather time-consuming especially when dealing with an airfoil composed of many surfaces. A specially designed CAD environment is developed for specifying the shape and dimension of the airfoil. Figure 9 is the sample output from the supersonic airfoil module with some typical inputs such as Mach number ahead of the airfoil, the chord length of the airfoil, the pressure ahead of the airfoil, and the airfoil's angle of attack. From this figure, it can be seen that the airfoil being analyzed consists of three flat surfaces on the top and three flat surfaces on the bottom. The pressure distributions, which are the main objective of the supersonic airfoil analysis, are also shown in this figure. The supersonic airfoil environment would enable the user to observe how the shape of an airfoil or the airfoil's angle of attack will influence the pressure distribution along the surfaces of an airfoil and consequently the airfoil lift. Figure 10 demonstrates the effect of airfoil's angle of attack on the pressure distribution along the upper and lower surfaces of an airfoil.



Airfoil chord length (m): 1
Mach No. ahead: 3
Pressure ahead (kPa): 40
Angle of attack: 10
Lift force per meter: 67.92 (kN/m)
Drag force per meter: 27.99 (kN/m)
Status: Click on the graph to draw points.

Figure 9. Typical diamond air foil and Sample output screen for the supersonic airfoil analysis

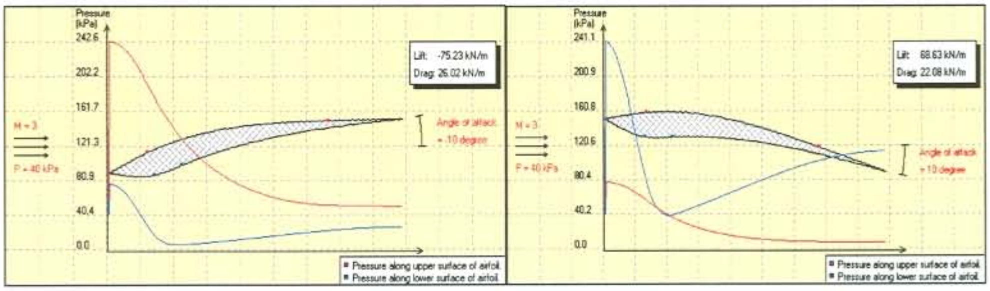


Figure 10. Effect of angle of attack (ranged from –10 to 10 degree) on pressure distribution

3. COMPUTER LANGUAGES AND NUMERICAL METHODS USED

The software is programmed using mixed languages. Microsoft Fortran Power Station 4.0 is used to build a dynamic link library (.DLL file) for calculations and Borland Delphi 4.0 is used to set up a windows-based interface for user to have a convenient way to access the library. When Mach number is an unknown, numerical methods such as Newton-Raphson methods as demonstrate in Mathews (1992) have to be used.

4. CONCLUSION

The development of an interactive software for compressible flow property calculations has been described. Six compressible flow modules are programmed into the software. Traditionally, when engineers or students analyze compressible fluid flow problems, it is inevitable that tables and graphs have to be used for property predictions. The software described provides an integrated environment for flow-properties calculation and can be utilized as an effective design or instructional tool. The software is available free of charge to the engineering educators and can be downloaded from the following Web page: http//umac.8k.com.

5. NONMENCLATURE

A	area
A^*	area at section where M=1
D	hydraulic diameter of a duct
f	friction coefficient
l^*	length at section where M=1
M	Mach number
M_n	component of Mach number normal to wave
M_t	component of Mach number tangent to wave
p	pressure
p^*	pressure at section where M=1
p_0	stagnation pressure
T	temperature
T_0	stagnation temperature
T^*	temperature at section where M=1
V	velocity
α	Mach angle
β	shock angle
γ	specific heat ratio
δ	turning angle
δ_{max}	maximum oblique shock wave turning angle
θ	Prandtl-Meyer angle
ρ	density (Rho)
ρ^*	density (Rho) at section where M=1
ρ_0	stagnation density

6. REFERENCES

1. Oosthuizen, P. H. and Carscallen, W. E., Compressible Fluid Flow, McGraw-Hill, New York, 1997.
2. Mathews, J. H., Numerical Methods for Mathematics, Science, and Engineering. Imprint Englewood Cliffs, New Jersey, Prentice Hall, 2nd edition, 1992