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Development of Friction Factor Correlation for Single-phase Flow in Micro-fin Tube Using Logistic Dose Response Curve Fitting Method

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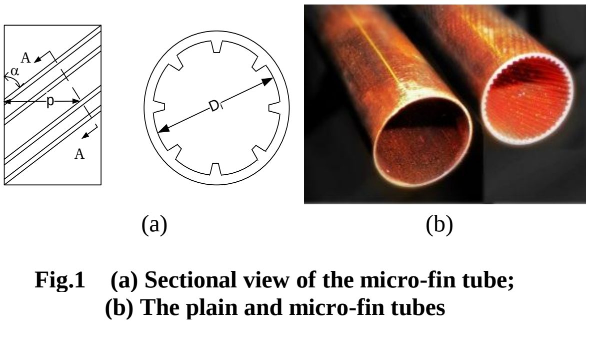
Abstract: Tam et al. [1] conducted simultaneous heat transfer and friction factor experiments for the plain and micro-fin tubes. The results showed that the heat transfer characteristics of the micro-fin tubes were the same as the plain tube. However, the friction factor characteristics of the micro-fin tubes in the transition region were different compared to the plain tube. This type of data cannot be easily correlated by the traditional regression method. Therefore, the logistic dose response curve fitting method is proposed in this study. This particular method has been used in correlating the friction factor data in plain tubes (Joseph and Yang [2]). In this study, one set of micro-fin tube friction factor data from Tam et al. [1] was correlated by the logistic dose response curve fitting method. All the fully-developed friction factor data for the entire flow regime can be predicted accurately by a composite logistic dose response function within ±10% deviation. The majority of the data (95%) was predicted with less than ±5% deviation.

Keywords: correlation, friction factor, logistic dose response curve, micro-fin tube.

1. INTRODUCTION

Single-phase liquid flow in internally micro-fin tubes (see Fig.1) is important in commercial HVAC applications. It has been observed that heat transfer can be increased when using the micro-fin tubes inside the flooded evaporators as well as shell-side condensers. This enables water chillers to reach high efficiency, which helps mitigate global warming concerns of HVAC systems. It is commonly understood that the micro-fin enhances heat transfer but at the same time increases the pressure drop as well. However, the understanding of the friction factor and heat transfer characteristics in the entire flow regime, especially, in the transition region, is insufficient. Furthermore, in the open literature, the friction factor and heat transfer correlations for the single-phase flow in micro-fin tubes are mainly focused on the turbulent region. The correlations for the transition region are not available in the open literature. Following the plain tube study, the micro-fin tube correlations can be developed based on the particular characteristics of friction factor and heat transfer.



(a) (b)

Fig.1 (a) Sectional view of the micro-fin tube; (b) The plain and micro-fin tubes

Referring to Tam et al. [1], it was observed that the heat transfer characteristics of the micro-fin tubes behaved the same way as the plain tube. However, as shown in Fig.2, the friction factor characteristics of the micro-fin tube in the transition region were different than the plain tube. Owing to the different characteristics, the plain tube correlations are not applicable to the micro-fin tube data. However, the correlating methods for the transition region data in plain tube may be applicable to the micro-fin tube data in the transition region.

For the plain tube transition region friction factors, several empirical correlations [3-6] have been developed. In the study of [3], those correlations were compared with the fully developed friction factor data with different inlet configurations. Based on the plain tube parabolic data trend in the transition region, Ghajar and Madon [3] and Hrycak and Andrushkiw [4] developed the transitional correlations with the second-order polynomial form. However, the second-order polynomial correlations are not suitable for the particular transition region data trend of the micro-fin tube. For the plain tube friction factor data, Bhatti and Shah [5] and Churchill [6] developed correlations for the entire flow regime (laminar-to-turbulent). Bhatti and Shah[5] developed a simple correlation form, which could be used for the entire flow regime. For different flow regimes, different sets of parameters were used in their correlation. Unfortunately, the form of their correlation was not sophisticated enough to provide accuracte predictions in the transition region. Compared to [5], the correlation of Churchill [6] was more complicated and more accurate. Their correlatimg method [6] was referred to as the asymptotic method. For the application of the asymtotic method to the plain tube data, the two asymptotic lines (laminar and turublent) were defined first and then the ”S” shaped curve between the asymptotic lines was adjusted by the correlation parameters based on the parabolic data trend in the transition region. However, the micro-fin tube data trend between the asymtotic lines (laminar and turbulent) is not smooth. Therefore, the asymptotic correlating method [6] is not suitable for the micro-fin tube transition region friction factor data.

In a recent study [2], an alternative correlating method called logistic dose response curve fitting method was used to correlate the plain tube friction factor data. Referring to [2], their curve fitting method separates the entire flow regime friction factor characteristics into several segments based on the critical points. After that, those segments are organized into a composite logistic dose response function. The advantage of the method is that the non-smooth and the sudden change data trend can be solved by this method. Therefore, it is possible to capture the particular micro-fin tube friction factor characteristics with this method. The objective of this study is to apply the logistic dose response curve fitting method to the micro-fin tube friction factor data of Tam et al. [1].

2. EXPERIMENTATL DATA

In Tam et al. [1], the friction factor and heat transfer experiments for one plain tube and three micro-fin tubes were conducted with two different inlets (square-edged and re-entrant) under the isothermal and uniform wall heat flux boundary conditions. In that study, the experiments covered a local bulk Reynolds number range of 1000 to 25,000, a local bulk Prandtl number range of 4.8 to 51.9, a local bulk Grashof number range of 1801 to 28619, a local bulk Nusselt number range of 12.4 to 326.9, and a friction factor range of 7.3×10^{-3} to 2.2×10^{-2}. The wall heat flux for the experiments ranged from 3.4 to 6.9kW/m^2. The maximum uncertainties for the friction factor and heat transfer were 2.1% and 12.6%, respectively.

In this study, the isothermal and squared-edged inlet fully developed friction factor data from one of the micro-fin tubes is adopted. Table 1 shows the specifications of the plain tube and the micro-fin tube used in this study. Fig.2 presents the fully developed friction factor characteristics for the plain tube and the micro-fin tube under isothermal boundary condition. As seen in the figure, the friction factor characteristics of the micro-fin tube in the transition region were different than the plain tube. The parallel shift from the classical laminar equation (C_f = 16/Re) is also observed for the micro-fin tube. The micro-fin tube has a slightly delayed transition when compared to the plain tube. In the transition region, the friction factor for the micro-fin tube goes through a steep increase followed by a relatively constant C_f section and then a parallel shift from the classical Blasius turbulent friction factor correlation (C_f = 0.0791 Re$^{-0.25}$).

Table 1. Specifications of the test tubes

Tube Type	Outer Dia., D_o (mm)	Inner Dia., D_i (mm)	Spiral angle, α	Fin height, e (mm)	Number of starts, N_s
Plain tube	15.9	14.9	–	–	–
Micro-fin tube	15.9	14.9	18 °	0.5	25

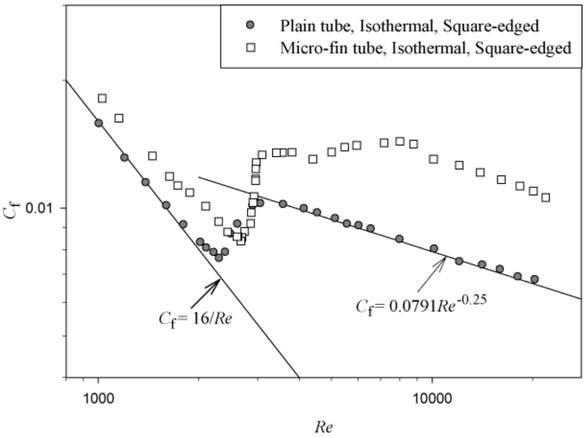


Fig.2 Friction factor characteristics for the plain and micro-fin tubes

The Reynolds number for the start and end of transition of friction factor data for the plain and micro-fin tubes are listed in Table 2. The transition range of the micro-fin tube is much wider than the plain tube transition range.

Table 2. Start and end of transition of friction factor data for plain and micro-fin tubes at x/D$_i$ of 200

Tube, Condition	Friction Factor			
	Re$_{start}$	C_f	**Re$_{end}$**	C_f
Plain, Isothermal (Square-edged)	2306	7.6e-3	3588	0.0102
Micro-fin, Isothermal (Square-edged)	2675	8.4e-3	8800	0.0144

3. APPLICATION OF THE LOGISTIC DOSE RESPONSE CURVE FITTING METHOD ON THE MICRO-FIN TUBE FRICTION FACTOR DATA

Refering to Joseph and Yang [2], a typical modified 5-parameter logistic dose response curve for the micro-fin tube friction factor data can be written as,

$$C_f = f(\mathrm{Re}) = f_L(\mathrm{Re}) + \frac{f_R(\mathrm{Re}) - f_L(\mathrm{Re})}{[1+(\frac{\mathrm{Re}}{\mathrm{Re}_c})^p]^q} \tag{1}$$

where f_L(Re) denotes the left side continuous function, f_R(Re) denotes the right side continuous function, p and q are the two parameters for the function f(Re) approaching to the two continuous functions [f_L(Re) *and* f_R(Re)], and Re$_c$ is the threshold value in the x-axis, which is the connecting point of the two continuous functions [f_L(Re) *and* f_R(Re)].

The procedure for applying the logistic dose response curve fitting method on the micro-fin tube friction factor data is summarized below:

(1) Separate the entire flow regime friction factor characteristics into several segments based on the critical (or turning) points;
(2) Determine the function for each segment;
(3) Find the threshold value or the intersection point of two continuous functions;
(4) Based on Eq.(1), establish a composite logistic dose response function (function F_1) for connecting the two functions;
(5) Repeat the steps (3) and (4). Establish another composite logistic dose response function (function F_2) for connecting another two functions;
(6) Finally, establish the single composite logistic dose response function for connecting the functions F_1 and F_2.

Referring to Fig.3, four line segments can be observed in the friction factor characteristic for the micro-fin tube. That is one segment in the laminar region; two segments in the transition region; and one segment in the turbulent region. Also, the friction factor data are in log-log coordinates in which power law behaves linearly. Therefore, four power-law functions (f_a, f_b, f_c, f_d) for the line segments are defined as:

$$f_a = 4.2\,\mathrm{Re}^{-0.79}, \quad f_b = 5.9\times10^{-16}\,\mathrm{Re}^{3.83},$$
$$f_c = 6.9\times10^{-3}\,\mathrm{Re}^{0.08} \text{ and } f_d = 1.9\times10^{-1}\,\mathrm{Re}^{-0.29}.$$

Based on the intersection point of the first two functions, f_a and f_b, the threshold value (Re_{c1} = 2,675) can be obtained.

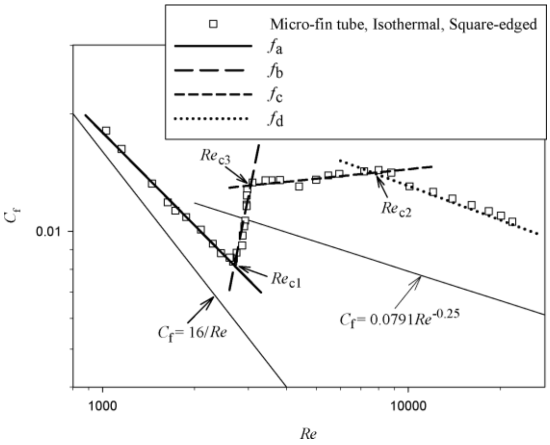


Fig.3 Power-law functions for the four line segments of the micro-fin tube friction factor characteristics

After determination of the four power-law functions and the threshold values, the logistic dose response curve Eq.(1) can be used for connecting the first two functions, f_a and f_b. In Eq.(1), the functions f_a and f_b can be treated as the continuous functions f_L and f_R. The parameters p and q can be determined by the non-linear regression method. Then, the composite logistic dose response function can be written as:

$$F_1 = f_a(\mathrm{Re}) + \frac{f_b(\mathrm{Re}) - f_a(\mathrm{Re})}{[1+(\frac{\mathrm{Re}}{\mathrm{Re}_{c1}})^p]^q} \tag{2}$$

where p, q and Re_{c1} are −134.2, 4,051 and 2,675, respectively.

Fig. 4 shows the composite logistic dose response function F_1 for connecting the functions f_a and f_b.

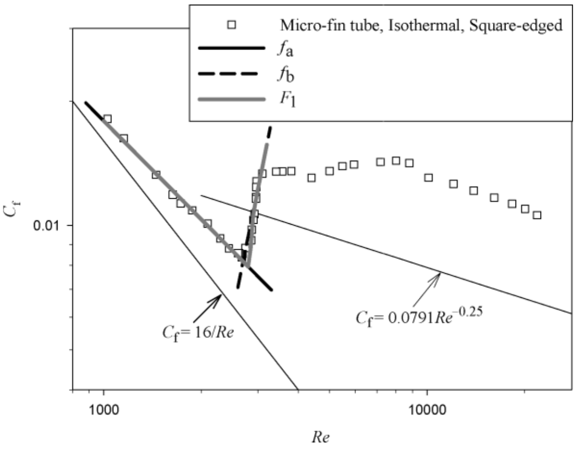


Fig.4 The composite logistic dose response function F_1 for connecting the functions f_a and f_b

Based on the intersection point of another two functions, f_c and f_d, the threshold value (Re_{c2} = 8,800) can be obtained. After that, Eq.(1) can be used for connecting the two functions, f_c and f_d. In Eq.(1), the functions f_c and f_d can be treated as the continuous functions f_L and f_R. The parameters p and q can be determined by the non-linear regression method. Then, the composite logistic dose response function can be written as:

$$F_2 = f_c(\mathrm{Re}) + \frac{f_d(\mathrm{Re}) - f_c(\mathrm{Re})}{[1+(\frac{\mathrm{Re}}{\mathrm{Re}_{c2}})^p]^q} \tag{3}$$

where p, q and Re_{c2} are -16.8, 0.9 and 8,800, respectively.

Fig.5 shows the composite logistic dose response function F_2 for connecting the functions f_c and f_d.

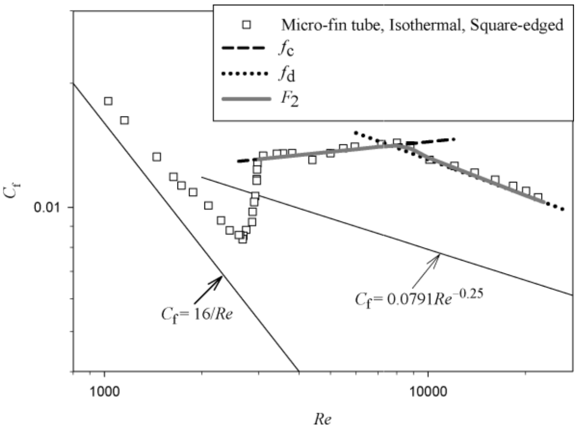


Fig.5 The composite logistic dose response function F_2 for connecting the functions f_c and f_d

Based on the intersection point of the two functions, F_1 and F_2, the threshold value (Re_{c3} = 2,973) can be obtained. After that, Eq.(1) can also be used for connecting the two functions, F_1 and F_2. In Eq.(1), the functions F_1 and F_2 can be treated as the continuous functions f_L and f_R. The parameters p and q can be determined by the non-linear regression method. Then, the composite logistic dose response function for connecting the two functions F_1 and F_2 can be written as:

$$C_{\mathrm{f}} = F_1(Re) + \frac{F_2(Re) - F_1(Re)}{[1 + (\frac{Re}{Re_{\mathrm{c3}}})^{p}]^{q}} \qquad (4)$$

where p, q and Re_{c3} are equal to -723.3, 0.4 and 2,973, respectively. For the two functions F_1 and F_2 refer to Eqs.(2) and (3).

Fig. 6 shows the single composite logistic dose response function C_f for connecting the two functions F_1 and F_2. In the figure, all the micro-fin tube friction factor data can be perfectly fitted with the final compsosite logistic dose response function, Eq.(4).

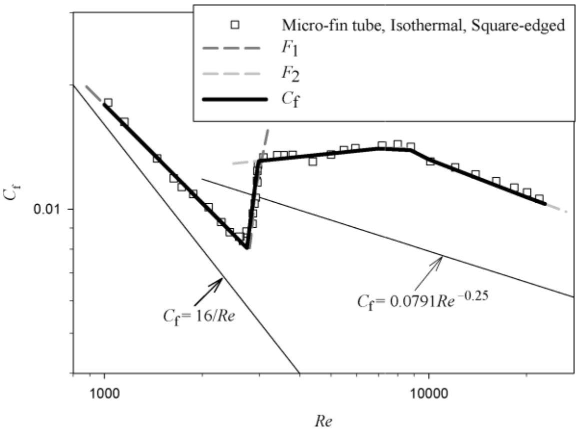


Fig.6 The final composite logistic dose response function C_f for connecting the functions F_1 and F_2

With using Eq.(4), all the experimental data (40 points) is accuractely predicted within -3.9% to +8.5%. The average absolute deviation of all the predictions is 1.9% and the majority, ninety five percent of all the data points (38 points), is predicted with less than ±5% deviation. Fig. 7 compares the predicted friction factors calculated from Eq.(4) with the measurements.

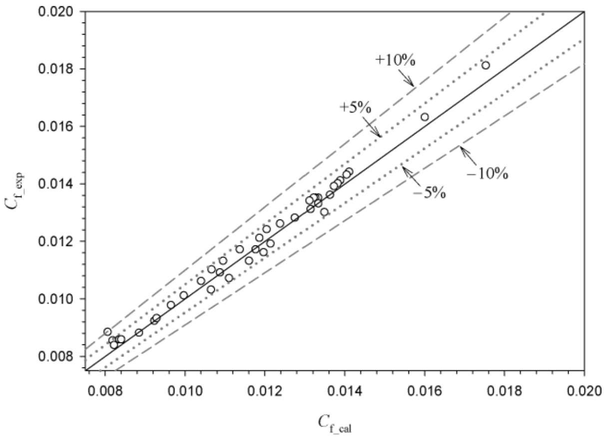


Fig.7 Comparison of the compositie logistic dose response function, Eq.(4), with the experimental friction factor data

4. CONCLUSION

In this study, the logistic dose response curve fitting method was succefully applied to the micro-fin tube friction factor data. The single composite logistic dose response function was developed for prediction of the micro-fin tube friction factor data with good accuracy. In the future, it is recommanded to apply the logistic dose response curve fitting method to more micro-fin tubes friction factor data with various fin geometries and various inlet configurations.

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References

[1] H. K. Tam, L. M. Tam, A. J. Ghajar, C. Sun, H. Y. Leung, Experimental investigation of the single-phase friction factor and heat transfer inside the horizontal internally micro-fin tubes in the transition region, *Proceedings of ASME-JSME-KSME Joint Fluids Engineering Conference*, 24-29 Jul 2011, Hamamatsu, Japan.

[2] D. D. Joseph, B. H. Yang, Friction factor correlations for laminar, transition and turbulent flow in smooth pipes, Physica D 239 (2010), 1318-1328.

[3] A. J. Ghajar, K. F. Madon, Pressure drop measurements in the transition region for a circular tube with three different inlet configurations, Experimental Thermal and Fluid Science 5 (1992), 129-135.

[4] P. Hrycak, R. Andruskhiw, Calculation of critical Reynolds number in round pipes and infinite channels and heat transfer in transition regions, Heat Transfer 1974 2 (1974), 183-187.

[5] M. S. Bhatti, R. K. Shah, *Turbulent and transition flow convective heat transfer in ducts*, *Hand book of single-phase convective heat transfer*, S. Kakac, R. K. Shah, and W. Aung, Eds., Chapter 4, p. 16, Wiley, New York, 1987.

[6] S. W. Churchill, Comprehensive correlating equations for heat, mass and momentum transfer in fully developed flow in smooth tubes, Ind. Eng. Chem., Fundam. 16 (1977), 109-116.

Nomenclature

C_f	fully developed friction factor coefficient (fanning friction factor), (=$\Delta P \cdot D_i/2 \cdot L \cdot \rho \cdot V^2$), dimensionless
D_i	inside diameter of the test section (tube), m
D_o	outside diameter of the test section (tube), m
e	internal fin height, mm
L	length of the test section (tube), m
N_s	number of starts/fins inside the cross-section area, dimensionless
p	axial fin pitch, [=$\pi \cdot D_i/(N_s \tan \alpha)$], m
Re	local bulk Reynolds number (=$\rho \cdot V \cdot D_i / \mu_b$), dimensionless
T_b	local bulk temperature of the test fluid, ℃
V	average velocity in the test section, m/s

Greek letters

α	spiral angle, degree
ΔP	pressure difference, Pa
μ_b	absolute viscosity of the test fluid evaluated at T_b, Pa s
ρ	density of the test fluid evaluated at T_b, kg/m^3

Subscripts

cal	referes to calculated value
exp	referes to experimental value