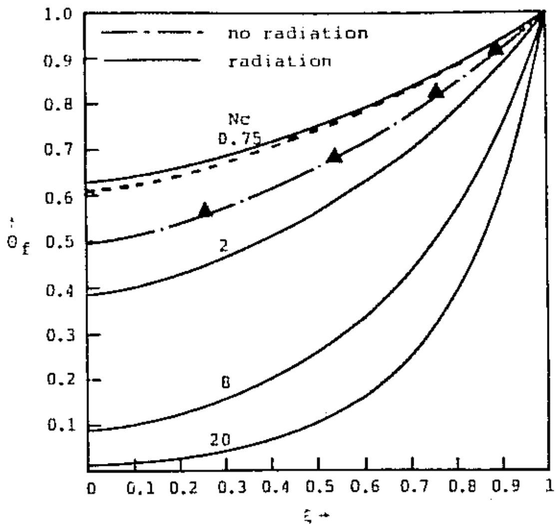

Fig. 3 The temperature distribution along the plate fin for $N=1$, $C_T=0.5$, and various values of Nc. —, simple model; ▲, Sparrow.[4]

$$\frac{Q}{k(T_0-T_\infty)Re_L^{1/2}}=2\int_0^1\left(-\frac{\partial\theta}{\partial\eta}\right)\times\left[1+\frac{4(\theta+C_T)^3}{3N}\right]\Big/\xi^{1/2}\mathrm{d}\xi,\ \text{at}\ \eta=0 \tag{15}$$

or

$$\frac{Q}{k(T_0-T_\infty)Re_L^{1/2}}=\frac{2}{N_c}\left.\frac{\mathrm{d}\theta_f}{\mathrm{d}\xi}\right|_{\xi=1} \tag{16}$$

The results of the overall rate of heat transfer Q from the fin are shown in Fig. 1 for various values of N_c. Figure 1 indicates that the overall heat-transfer rate of the fin with radiative effect is higher than that of the fin without radiative effect. The agreement of the results from the simple model with those of Ref. 4, and the special case ($q^r=0$) in this paper is good.

The distributions of the modified local heat-transfer coefficient $\bar{h}^*$ for forced convection and radiation along the fin with different N_c are shown in Fig. 2. The modified heat transfer coefficient can be obtained from Eq. (14). For higher values of N_c, the fin is more nonisothermal. Although the local heat-transfer coefficient without radiative effects[4] monotonically decreases in the fluid flow direction, the modified local heat transfer coefficients computed in this paper with fixed radiative effect do not vary monotonically. In the direction from tip to base, those coefficients decrease at first, attain a minimum, and then increase. Figure 2 also shows that the local heat transfer coefficient with radiative effect is higher than that without radiative effect.

Figure 3 presents fin temperature distributions for forced convection flow with radiative effect. In this figure, it is shown that the fin temperature decreases monotonically from the root to tip. The figure also shows that the larger values of N_c give rise to larger fin temperature variations and the fin temperature without radiative effect is always higher than that with radiative effect.

Conclusion

The analysis in this Note has yielded the results of physical fin for forced convection flow with radiative effect under the optically thick limit approximation. The agreement of the results for special case $q^r=0$ with Ref. 4 is remarkable.

References

[1] Sparrow, E. M., Baliga, B. R., and Patankar, S. V., "Forced Convection Heat Transfer from a Shrouded Fin Array with and without Tip Clearance," *ASME Journal of Heat Transfer*, Vol. 100, 1978, pp. 572-579.

[2] Stachiewicz, J. W., "Effect of Variation of Local Film Coefficients on Fin Performance," *ASME Journal of Heat Transfer*, Vol. 91, 1969, pp. 21-26.

[3] Cebeci, T. and Bradshaw, P., *Momentum Transfer in Boundary Layers*, Hemisphere Publishing, Washington, D.C., 1977.

[4] Sparrow, E. M. and Chyu, M. K., "Conjugated Forced Convection-Conduction Analysis of Heat Transfer in a Plate Fin," *ASME Journal of Heat Transfer*, Vol. 104, 1982, pp. 204-206.

Improved Free Convective Heat-Transfer Correlations in the Near-Critical Region

Saeed Ghorbani-Tari* and Afshin J. Ghajar†
Oklahoma State University, Stillwater, Oklahoma

Nomenclature

C_p = specific heat at constant pressure
$\bar{C}_p$ = mean-integral heat capacity, $\bar{C}_p=(i_w-i_\infty)/(T_w-T_\infty)$
D = diameter
g = gravitational constant
Gr = Grashof number, $(gD^3\rho^2/\mu^2)[(\rho_\infty-\rho_w)/\rho_w]$
h = heat-transfer coefficient
i = enthalpy
k = thermal conductivity
L = characteristic length
Nu = Nusselt number, hL/k
Pr = Prandtl number, $\mu C_p/k$
Ra = Rayleigh number, $Gr\cdot Pr$
T = temperature
μ = absolute viscosity
ρ = density

Subscripts

f = evaluated at the film temperature, $T_f=(T_w+T_\infty)/2$
w = evaluated at the wall temperature
∞ = evaluated at the freestream (bulk) temperature

Introduction

A VARIETY of correlations have been developed for prediction of free convective heat-transfer rates[1-7] (see Table 1). Discrepancies exist between these correlations when they are applied in the near-critical region. The most significant sources of discrepancy appear to be the influences of 1) the particular reference temperature used in the evaluation of the physical properties, 2) the particular physical properties selected in reducing the dimensional experimental data to the dimensionless variables, and 3) the differences in values of the physical properties used by the various investigators.

Presented as Paper 84-1705 at the AIAA 19th Thermophysics Conference, Snowmass, CO, June 25-28, 1984; received Oct. 4, 1984; revision received Feb. 8, 1985.

*Graduate Student, School of Mechanical and Aerospace Engineering.

†Associate Professor, School of Mechanical and Aerospace Engineering. Member AIAA.

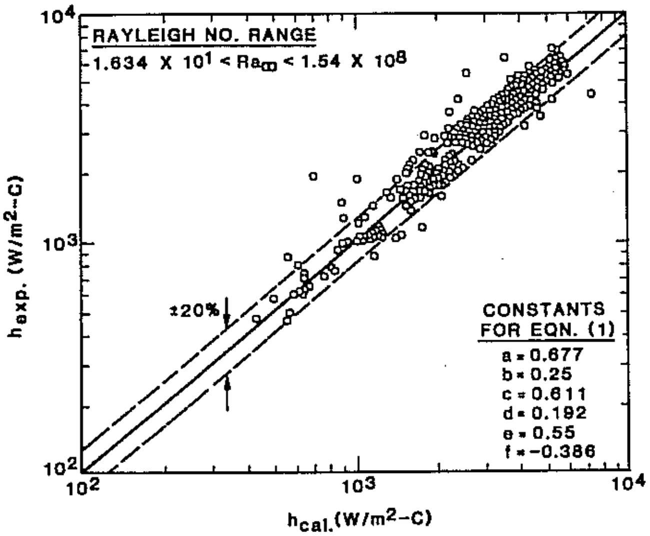

Fig. 1 Comparison of predicted heat-transfer coefficients with carbon dioxide experimental data of Refs. 1, 2, 15, and 19 obtained from three different size wires.

Table 1 Types of free convective heat-transfer correlations

Correlation	Ref.
$Nu=a(Ra)^b$	1
$Nu=a(Ra)^b\left(\frac{Pr_w}{Pr_\infty}\right)^c$	2
$Nu=a(Gr)^b(Pr)^c\left(\frac{T_\infty}{T_w-T_\infty}\right)^d$	3,4,14
$Nu=a(Ra)^b(Pr)^c\left(\frac{\rho_\infty-\rho_w}{\rho_\infty}\right)^d\left(\frac{T_\infty}{T_w-T_\infty}\right)^e$	5
$Nu=a(Ra)^b\left(\frac{\rho_w}{\rho_\infty}\right)^c\left(\frac{\bar{C}_p}{C_{p_\infty}}\right)^d\left(\frac{k_w}{k_\infty}\right)^e\left(\frac{\mu_w}{\mu_\infty}\right)^f$	6,7,17

The objective of the present Note is to identify an appropriate free convective heat-transfer correlation for the near-critical region (similar to the ones listed in Table 1) and to improve its predictive capability by the use of appropriate physical properties and reference temperature.

Method of Approach

Identification of an appropriate heat-transfer correlation was based on how well the correlations given in Table 1 could predict the available near-critical heat-transfer experimental data. The numerical constants in these correlations were based on different sources of the physical property inputs, which in most cases did not properly represent the variations in the near-critical region. Therefore, for a meaningful comparison, it was necessary to compare the correlations based on the same physical property inputs. For this purpose, the constants in the heat-transfer correlations were determined by curve fitting the equations to the experimental data based on correct values of the physical property inputs. Then, the predicted results were compared not only with the experimental data of those authors who developed the specific correlations, but also with the experimental data of others. After sufficient amount of study, it was decided that the last equation in Table 1 predicted all the available experimental data better than the rest of the correlations given in Table 1. Further details may be found in Ref. 8.

The effect of reference temperature on evaluating the physical properties in Nusselt and Rayleigh numbers was examined for film and freestream temperatures. Usually, for free convective heat transfer, the film temperature is used for horizontal wires and the freestream temperature for vertical plates. But, in this study, better results were obtained for both cases when the reference temperature was chosen to be the freestream temperature. Further details may be found in Refs. 8 and 9.

Therefore, based on the findings of this study, the following heat-transfer correlation is proposed for horizontal wires and vertical plates for a wide range of Rayleigh numbers:

$$Nu_\infty=a(Gr_\infty Pr_\infty)^b\left(\frac{\rho_w}{\rho_\infty}\right)^c\left(\frac{\bar{C}_p}{C_{p_\infty}}\right)^d\left(\frac{k_w}{k_\infty}\right)^e\left(\frac{\mu_w}{\mu_\infty}\right)^f \quad (1)$$

where a, b, c, d, e, and f are curve-fitted constants.

Table 2 Comparison between data and correlations

						Avg. deviation, %	
Fluid	Ref.	Geometry	Rayleigh no. range	No. of points	Constants for Eq. (1)	Present study	Previous works
CO_2	16	Horizontal wire, horizontal and vertical ribbon	$0.2<Ra_\infty<2.92\times10^2$	54	$a=1.03$, $b=0.333$, $c=10.07$, $d=0.438$, $e=0.561$, $f=-5.6$	10.7	—
CO_2	15	Horizontal wire, and flat strip	$1.31\times10^1<Ra_\infty<1.26\times10^3$	79	$a=0.717$, $b=0.231$, $c=0.404$, $d=0.320$, $e=0.245$, $f=0.007$	6.7	10
CO_2	1	Horizontal wire	$8.82\times10^1<Ra_\infty<1.02\times10^4$	92	$a=1.153$, $b=0.187$, $c=0.045$, $d=0.132$, $e=0.722$, $f=-0.110$	7.7	—
CO_2	2	Horizontal pipe	$1.09\times10^6<Ra_\infty<1.55\times10^8$	57	$a=0.054$, $b=0.432$, $c=2.02$, $d=0.423$, $e=0.221$, $f=-0.398$	11.9	20
CO_2	6	Vertical plate	$4.66\times10^{11}<Ra_\infty<9.02\times10^{12}$	21	$a=0.024$, $b=0.393$, $c=1.213$, $d=0.394$, $e=-0.316$, $f=-0.314$.	9.8	20
CO_2	17	Vertical plate	$7.97\times10^{11}<Ra_\infty<4.04\times10^{13}$	38	$a=0.103$, $b=0.333$, $c=-2.0$, $d=0.726$, $e=0.52$, $f=1.23$	13.5	25
H_2O	13	Vertical ribbon	$8.88\times10^6<Ra_\infty<4.45\times10^8$	87	$a=0.15$, $b=0.333$, $c=-0.533$, $d=0.268$, $e=0.455$, $f=2.24$	15.6	—
H_2O	3,14	Vertical ribbon	$5.95\times10^{10}<Ra_\infty<2.14\times10^{13}$	85	$a=0.142$, $b=0.333$, $c=-0.702$, $d=0.507$, $e=0.04$, $f=2.81$	14.7	15

The appropriate thermodynamic and transport property inputs required by the proposed heat-transfer correlation were determined from the expressions given in Refs. 8-12. These expressions were exclusively developed for fluids in the near-critical region and are capable of predicting physical properties with very good accuracy. Extensive comparison of these expressions with experimental data can be found in Refs. 10-12.

Experimental Data Used

An intensive literature survey was carried out to obtain the available free convective experimental data in the near-critical region. In this study, water and carbon dioxide were chosen for the development of the heat-transfer correlations. These fluids were chosen because of the availability of reliable experimental heat-transfer data and accurate physical property expressions. Table 2 summarizes the experimental data used in this study. A detailed table of the available experimental free convective heat-transfer data may be found in Ref. 9.

Results and Discussion

The constants in the proposed heat-transfer correlation were obtained by curve fitting the expression to the experimental heat-transfer data of water[3,13,14] and carbon dioxide.[1,2,6,15-17] For this purpose a nonlinear least-squares fit was used. The constants to be used in the proposed heat transfer correlation [Eq. (1)] are given in Table 2.

The fit of the proposed expression to the available experimental data was good for both substances. The results of comparison with eight different experimental works are summarized in Table 2.

In this study, the effects of reference temperature and wire diameter on the heat-transfer correlation were also investigated. Analysis showed that, when the physical properties in the Nusselt, Prandtl, and Grashof numbers were evaluated at the freestream temperature, the proposed heat-transfer correlation produced better results. For example, the average absolute error in correlating the data of Ref. 2 was reduced from 15 to 11.9% when the reference temperature was changed from film to freestream temperature. The same results were obtained for the data of Refs. 1 and 15. For Ref. 1 the error was reduced from 12.5 to 7.7% and for Ref. 15 from 13.3 to 6.7%. The effect of diameter on heat-transfer predictions was reduced when heat-transfer coefficients were expressed as $(h)(D)^{0.25}$, see Refs. 8, 9, and 18. This was accomplished by rearrangement of the expression for Nusselt number to the form $hD^{0.25}=Nu_\infty k_\infty D^{-0.75}$ and substitution of Eq. (1) for Nu_∞. Then, the effect of diameter on the right-hand side of the rearranged expression for Nusselt number was eliminated when the exponent of Rayleigh number was constrained to 0.25 (i.e., set $b=0.25$). In this fashion, it was possible to combine the heat-transfer experimental data obtained from three different size wires (0.1 mm,[1,15] 2 mm,[2] and 0.076 mm[19]) and predict these data with a single heat-transfer correlation. As shown in Fig. 1, the proposed correlation predicts the 266 experimental data for free convective heat transfer from horizontal wires to carbon dioxide with an average absolute error of 13.7%.

Conclusions

The proposed heat-transfer correlation based on correct values for physical property inputs used in this study and an appropriate reference temperature showed that the predicted values are in good agreement with free convective experimental data for water and carbon dioxide. The average absolute errors between the predicted results and the experimental data for all cases were below 16%. Also, this single correlation showed better accuracy when compared to other correlations available in the literature (see Table 2).

References

[1] Hahne, E. W. P., Feurstein, G., and Grigull, U., "Free Convective Heat Transfer in the Supercritical Region," *Proceedings of 5th International Heat Transfer Conference*, Tokyo, Vol. 3, 1974, pp. 5-9.

[2] Beschastnov, S. P. and Petrov, V. P., "Heat Transfer by Free Convection From Horizontal Cylinder to CO_2 under Near-Critical Conditions," *High Temperature*, Vol. 11, No. 3, 1974, pp. 524-528.

[3] Larson, J. R. and Schoenhals, R. J., "Turbulent Free Convection in Near-Critical Water," *Journal of Heat Transfer, Transactions of ASME*, Vol. 88, No. 4, 1966, pp. 407-414.

[4] Ghajar, A. J. and Parker, J. D., "Reference Temperatures for Supercritical Laminar Free Convection on a Vertical Flat Plate," *Journal of Heat Transfer, Transactions of ASME*, Vol. 103, 1981, pp. 613-616.

[5] Hilal, M. A. and Boom, R. W., "An Experimental Investigation of Free Convection Heat Transfer in Supercritical Helium," *International Journal of Heat Mass Transfer*, Vol. 23, 1980, pp. 697-705.

[6] Protopopov, V. S. and Sharma, G. K., "Experimental Investigation of Heat Transfer from a Vertical Surface to Carbon Dioxide with Natural Convection in the Supercritical Region of the Parameter of State," *High Temperature*, Vol. 14, No. 4, 1976, pp. 693-698.

[7] Seetharam, T. R. and Sharma, G. K., "Free Convective Heat Transfer to Fluids in the Near-Critical Region from Vertical Surfaces with Uniform Heat Flux," *International Journal of Heat Mass Transfer*, Vol. 22, 1979, pp. 13-20.

[8] Ghorbani-Tari, S., "Improved Free Convective Heat Transfer Correlations in the Near-Critical Region," M.S. Report, Mechanical and Aerospace Engineering Dept., Oklahoma State University, Stillwater, OK, 1984.

[9] Ghorbani-Tari, S. and Ghajar, A. J., "Improved Free Convective Heat Transfer Correlations in the Near-Critical Region," AIAA Paper 84-1705, June 1984.

[10] Najjar, T. S. and Ghajar, A. J., "Predictions of Density and Constant Pressure Specific Heat for Several Fluids in the Near-Critical Region," AIAA Paper 83-1476, June 1983.

[11] Asgeirsson, L. S. and Ghajar, A. J., "Prediction of Thermal Conductivity and Viscosity for Some Fluids in the Near-Critical Region," AIAA Paper 83-1475, June 1983.

[12] Levett Sengers, J. M. H., Kamgar-Parsi, B., Balfour, F. W., and Sengers, J. V., "Thermodynamic Properties of Steam in the Critical Region," *Journal of Physical and Chemical Reference Data*, Vol. 12, No. 1, 1983.

[13] Fritsch, C. A. and Grosh, R. J., "Free Convective Heat Transfer to Supercritical Water Experimental Measurements," *Journal of Heat Transfer, Transactions of ASME*, Vol. 85, No. 4, 1963, pp. 289-294.

[14] Larson, J. R., "Turbulent Free Convection in Near Critical Water," Ph.D. Thesis, Purdue University, West Lafayette, IN, 1964.

[15] Neumann, R. and Hahne, W. P., "Free Convective Heat Transfer to Supercritical Carbon-Dioxide," *International Journal of Heat Mass Transfer*, Vol. 23, 1980, pp. 1643-1652.

[16] Dubrovina, E. N. and Skripov, V. P., "Convective Heat Transfer to Carbon Dioxide in the Supercritical Region," *Proceedings of the Second All-Soviet Union Conference on Heat and Mass Transfer*, Vol. 1, 1966, pp. 36-45.

[17] Beschastnov, S. P., Kirillov, P. R., and Saikin, A. M., "Heat Transfer by Turbulent Free Convection from a Plate to Carbon Dioxide," *High Temperature*, Vol. 11, No. 2, 1973, pp. 299-304.

[18] Nishikawa, K. and Miyabe, K., "Effect of Diameter of Heated Wire on Free Convective Heat Transfer to a Supercritical Fluid," *Memoirs of the Faculty of Engineering*, Kyushu University, Japan, Vol. 31, No. 3, 1972.

[19] Abadic, S. and Goldstein, R. J., "Film Boiling and Free Convection Heat Transfer to Carbon Dioxide Near the Critical State," *International Journal of Heat Mass Transfer*, Vol. 13, 1970, pp. 1163-1175.