INT. COMM. HEAT MASS TRANSFER 0735-1933/86 $3.00 + .00
Vol. 13, pp. 449-464, 1986 ©Pergamon Journals Ltd. Printed in the United States

A NEW HEAT EDDY DIFFUSIVITY EQUATION FOR CALCULATION OF HEAT TRANSFER TO DRAG REDUCING TURBULENT PIPE FLOWS

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(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

A new semi-empirical equation of heat eddy diffusivity in terms of friction drag reduction ratio and Weissenberg number is presented. The proposed equation was validated with heat transfer experimental results of Kwack [1] and our recent experimental results, both for aqueous solutions of polyacrylamide (Separan AP-273) for concentrations ranging from 10 to 1000 ppm in turbulent flow through pipes under constant wall heat flux condition. The predictions of heat transfer coefficients with the use of the proposed equation are in good agreement with both sets of independent experimental results. The results of this study indicate that the proposed equation for eddy diffusivity of heat has predictive capability provided experimental measurements of pressure drop and the fluid time scale are available. The fluid time scale for these predictions was estimated using the Powell-Eyring fluid model and apparent viscosity measurements.

Introduction

It is well-known that small addition of certain polymers to turbulent pipe flows causes a drastic reduction in the friction drag and heat transfer. This finding initiated a great deal of interest in the possible use of polymer additives in practical engineering systems. Viscoelastic fluids play a role in the chemical, biochemical and food processing industries since many of the industrial chemicals and fluids are viscoelastic in nature. Furthermore, they often undergo heat exchange processes during preparation or in their application. The ability to accurately predict the heat transfer characteristics of such fluids is of prime importance for engineering design purposes.

Previous studies [2,3] have indicated that the reduction in heat transfer is much more drastic than that in momentum transfer for viscoelastic turbulent pipe flows. However, few investigators have accounted for this fact in their analytical heat transfer studies and the present paper is directed to this end. The aim of this study is to develop a general semi-empirical equation for eddy diffusivity of heat in viscoelastic turbulent pipe flows. The two key dimensionless parameters in the proposed equation are the friction drag reduction ratio and the Weissenberg number.

Experiments

The present experiments were conducted in the fluid dynamics laboratory at Oklahoma State University. A schematic diagram of the flow circulation system is shown in Figure 1. The test section used has an inside diameter of 1.88 cm (L/D = 617). This test section ensures the thermally fully developed condition for viscoelastic fluids which requires 400 to 500 diameters for the minimum heat transfer asymptote [4]. To minimize mechanical degradation of polymer solutions, the overall flow system was operated with pressurized air (up to 80 psig) using the once-through mode. The constant heat flux boundary condition was maintained by a Lincoln DC-600 welder. It can operate in the constant voltage or constant current mode, and has a 100% duty cycle rating at 600 amps and 44 volts. In the present flow system, either the hydrodynamic and thermal entrance regions can develop simultaneously from the beginning of the test section, or the velocity profile can be fully developed before heat transfer starts. The measurements of pressure drop and heat transfer were taken at the same time in the thermally fully developed region with the use of one U-tube mercury manometer and # 30 gauge copper-constantan thermocouples. The flow rate was measured by a one-inch turbine meter located upstream from the test section. This turbine meter monitored by a Hewlett-Packard frequency counter can produce instant or time-averaged readings so that it enables one not only to obtain average flow rate but also to check the stability of the flow. Apparent viscosities of solutions were measured at wide ranges of shear rates (0.36 to 2 x 10^4sec-1) with the use of two Couette viscometers (Brookfield Synchro-Electric Model LVT with UL adaptor and a Fann Model VG) and a capillary tube viscometer (0.9398 mm I.D. and l/d = 325). The

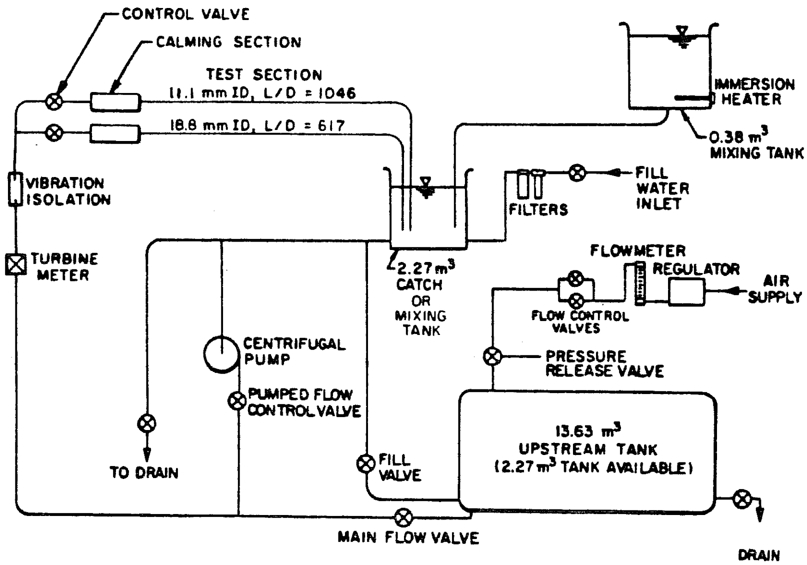

FIG. 1
Schematic diagram of the flow circulation system

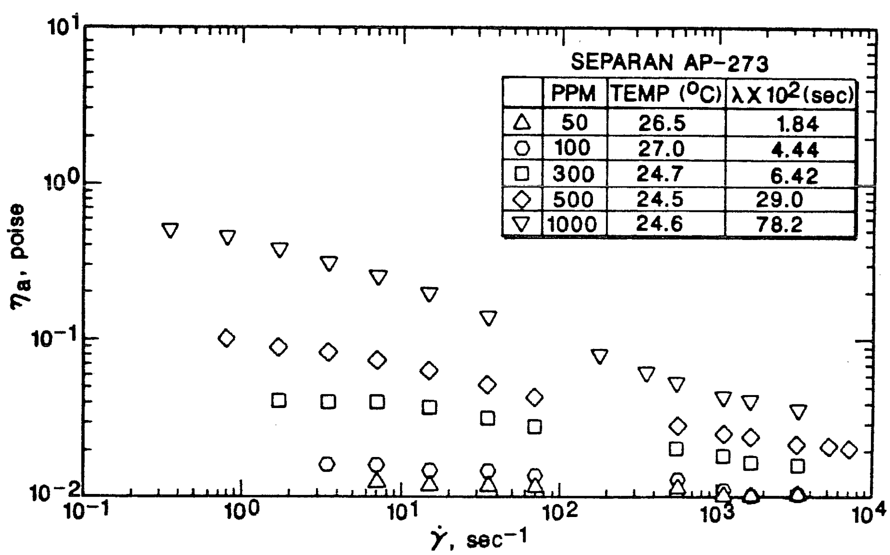

FIG. 2
Apparent viscosity vs. shear rate

reliability of the flow circulation system and the experimental procedures were checked with several calibration runs for measurements of friction factors and heat transfer coefficients for a Newtonian fluid (tap water) by comparing the experimental results with well-established Newtonian correlations [5-8]. More detailed description of the experimental apparatus and procedures are presented elsewhere [9]. The uncertainty analyses of the overall experimental procedures for water showed that there is 5-6 percent uncertainty for friction factors and 8-10 percent uncertainty for heat transfer coefficients.

The viscoelastic fluids used in the current study were the well-mixed homogeneous aqueous solutions of polyacrylamide (Separan AP-273) with concentrations of 10, 50, 100, 300, 500, and 1000 ppm. The apparent viscosity of each polymer solution was measured at wide ranges of shear rates (see Figure 2). The viscosity data presented in Figure 2 were used to estimate the fluid time scale by the Powell-Eyring model, which has the following expression

$$\eta_a = \eta_\infty + (\eta_o - \eta_\infty)\left[\frac{\sinh^{-1} \lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right] \quad (1)$$

The fluid time scale was determined by a linear regression method with the use of all the viscosity data for each solution.

The measurements of pressure drop and heat transfer are presented in terms of Fanning friction factor and Colburn j-factor in Figures 3 and 4. These experimental data together with those of Kwack [1] will be used to validate the proposed equation for eddy diffusivity of heat.

Eddy Diffusivity of Heat

In order to solve the time-mean energy equation, an expression for eddy diffusivity of heat, ε_h, which takes into account particular characteristics of viscoelastic fluids is required. Most analytical studies use either an expression for eddy diffusivity of heat which is valid for a particular polymer concentration, mostly for the maximum heat transfer reduction asymptotic case, or a direct analogy between eddy diffusivities of heat and momentum. Neither one of these schemes have general predictive capability for wide ranges of polymer concentrations throughout the flow field [1, 10]. The latter case is even less desirable since several studies [1-3] have shown that

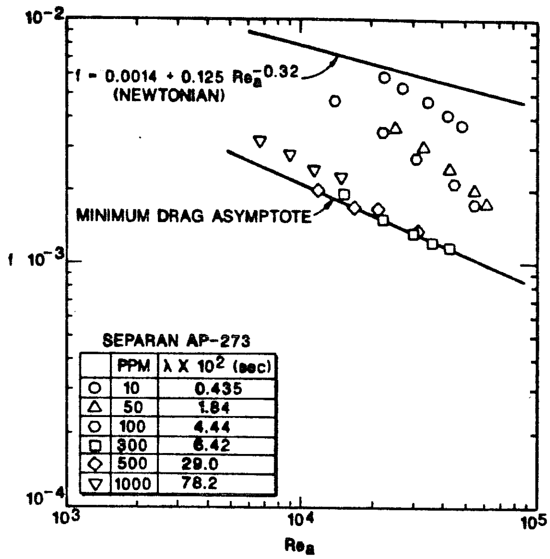

FIG. 3
Fanning friction factor vs. apparent Reynolds number

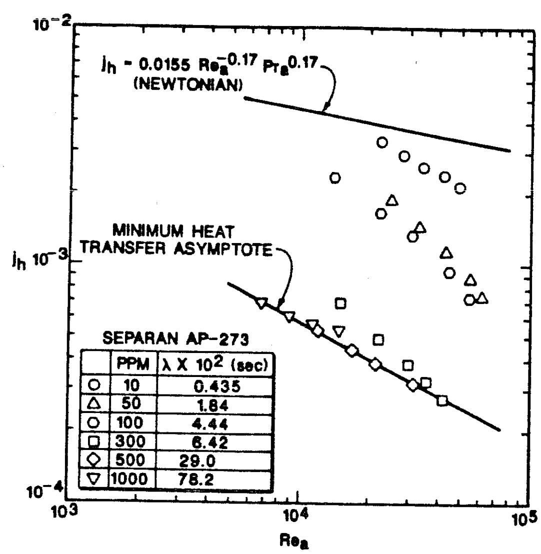

FIG. 4
Colburn j-factor vs. apparent Reynolds number

the turbulent Prandtl number of concentrated viscoelastic fluids is not unity, especially near the wall, where it is important to have accurate values of heat eddy diffusivity for heat transfer calculations.

Since the characteristics of viscoelastic turbulent flows can be affected by several factors such as pipe diameter, purity of the water, aging, type and concentration of the polymer, as well as method of formulating the solution, predictions based solely on first principles are not likely to exist in the near future. The objective of this study is to develop a general semi-empirical equation for eddy diffusivity of heat in terms of pertinent dimensionless parameters for viscoelastic fluids, which can be determined directly from experimental measurements of pressure drop and rheological properties.

The fluid time scale (λ) is the most useful and readily measurable material property that can be used to measure the elasticity of a fluid. It can be accurately determined from the rheological properties of the fluid with the use of an appropriate constitutive equation. Several studies have used the fluid time scale to interpret the influence of the elasticity of the solution on the friction and/or heat transfer behavior of viscoelastic turbulent flows. Corman [11] included the fluid time scale in the expression for the dimensionless thickness of the laminar layer to account for the thickening of this layer due to the addition of polymer. Poreh and Paz [12] explained the drag reduction phenomenon as the upward shift of velocity profile in the log portion of the universal law. The velocity shift was expressed in terms of the fluid time scale. Mizushina and Usui [2] suggested that the drag reduction phenomenon is due to the increased effect of damping near the wall region. They modified Van Driest's damping factor model for momentum with the use of the fluid time scale. Darby and Chang [13] developed a generalized correlation for friction loss in drag reducing polymer solutions, which includes a fluid time scale. Kale [14] used the fluid time scale to correlate heat transfer coefficient with friction factor for viscoelastic turbulent pipe flows. Recently, Kwack [1] suggested that for the fully developed turbulent pipe flows, friction factor and heat transfer coefficient can be expressed as a function of Reynolds number and Weissenberg number (the ratio of the fluid time scale to the flow time scale). The above studies indicate that the fluid time scale is one of the key properties for proper representation of the elasticity in viscoelastic turbulent pipe flows. Since the friction and heat transfer behavior of viscoelastic turbulent pipe flows are affected by not only the fluid conditions but also the flow conditions, such as the pipe diameter and the flow rate, the fluid time scale should be combined with the flow time scale (D/U) to form a complete dimensionless parameter. In this study, Weissenberg number (Ws) will be used as one of the key dimensionless parameters in the eddy diffusivity expression for heat.

Another important parameter in viscoelastic fluids is the friction drag reduction ratio (FR), which can be determined from the experimental measurements of pressure drop. Since the energy equation is directly related to the equation of motion, in heat transfer calculations involving viscoelastic fluids, friction drag reduction ratio plays a very important role. Pruitt et al. [15] used the friction drag reduction ratio in correlating friction factor with heat transfer coefficient in viscoelastic turbulent pipe flows. Mizushina and Usui [2] showed that Colburn's analogy in general does not hold for viscoelastic fluids except in the high Reynolds number range of low concentrations. They suggested that the correction factor for the deviation from Colburn's analogy depends only on the Reynolds number and the friction drag reduction ratio. Hughmark [16,17] in the development of the expressions for friction factor and heat transfer coefficient using the three region resistance model, related the wall region frequency to the ratio of friction factor for the polymer solution to that for the solvent. The above studies indicate that the friction drag reduction ratio is another important dimensionless parameter relating friction and heat transfer behavior in viscoelastic turbulent pipe flows.

Based on the above observations, a new semi-empirical equation for eddy diffusivity of heat will be formulated in terms of Weissenberg number and friction drag reduction ratio. The basic assumptions made in the formulation of the equation are: a) the ratio of eddy diffusivity for heat to that for momentum is constant across the test section; b) eddy diffusivity of heat decreases exponentially with the increase of Weissenberg number up to the critical value of Weissenberg number for heat transfer (Ws_{ch}),which corresponds to the minimum heat transfer asymptote. This is analogous to the observations made by Kwack [1] for the behavior of heat transfer coefficient with respect to Weissenberg number; and c) the influence of friction drag reduction ratio on the eddy diffusivity of heat is of the form (1-FR), which is consistent with the correlation of Pruitt et al. [15]. Based on these assumptions the following expression for eddy diffusivity of heat is proposed:

$$\varepsilon_h/\varepsilon_m = a(1-FR)^b e^{[1-(Ws/Ws_{ch})]^c} \quad (2)$$

The proposed equation is in qualitative agreement with the experimental observations, and is presented in a generalized form which is applicable to non-Newtonian viscoelastic as well as Newtonian fluids. For Newtonian fluids (FR=0 and Ws = 0) the ratio of eddy diffusivities is equal to a constant which is in agreement with Eq. (2). For viscoelastic fluids the degree of reduction in heat transfer is even more drastic than that in friction. This behavior is accounted for through the use of the two dimensionless parameters FR and Ws. Particularly, even after the friction factor of a solution reaches the minimum drag reduction asymptote, there is still a decrease in the heat transfer coefficient up to the critical Weissenberg number for heat transfer. This behavior is accounted for by the second term in Eq. (2). Even though the reduction in heat transfer is limited by the minimum heat transfer asymptote, further polymer addition results in an increased Weissenberg number beyond the critical value. The proposed equation can not explain this phenomenon. For the case of $Ws>Ws_{ch}$, the term (Ws/Ws_{ch}) in Eq. (2) is set to unity.

The three constants in the proposed equation should be determined such that the equation is applicable to non-Newtonian viscoelastic as well as Newtonian fluids. The constant a has a fixed value which is determined from the Newtonian fluid behavior. It is known that for Newtonian fluids, FR and Ws are equal to zero and $\varepsilon_h/\varepsilon_m$ is close to unity. The substitution of FR = 0, Ws = 0 and $\varepsilon_h/\varepsilon_m = 1$ into Eq. (2) determines the constant a to be 0.37. The constants b and c will be evaluated with the experimental data of Kwack [1] for Separan AP-273. Kwack's data are considered reliable and well-documented. His experiments reported enough information to start the calculations. For the minimum heat transfer asymptote ($Ws \geq Ws_{ch}$), the proposed equation becomes a function of FR only. Kwack [1] showed that for this case the ratio of eddy diffusivities, $\varepsilon_h/\varepsilon_m$, has the limiting values of 0.134 for $Re_a = 20000$ and 0.124 for $Re_a = 30000$. The calculations of FR with the use of the minimum drag asymptotic correlation ($f = 0.20Re_a^{-0.48}$) of Kwack [1] and the well-established Newtonian correlation ($f = 0.0014 + 0.125Re_a^{-0.32}$) of McAdams [6] for friction factor show that the friction drag reduction ratios are 0.74 for $Re_a = 20000$ and 0.76 for $Re_a = 30000$. The substitution of $\varepsilon_h/\varepsilon_m$ and FR values into Eq. (2) with a = 0.37 determines the constant **b** to be 0.75. Before the constant **c** can be determined, Ws_{ch} should be known. The critical Weissenberg number for heat transfer was evaluated between 200 and 250 by Ng and Hartnett [18] and confirmed by Kwack [1] using polyacrylamide (Separan AP-273). A careful examination of their experimental results shows that Ws_{ch} is closer to 200. In this study, Ws_{ch} was taken to be 200. To determine the constant **c**, the experimental data on the minimum drag asymptote (50 ppm) but not on the minimum heat transfer asymptote were used. The preferential choice of experimental data for the minimum drag asymptote was because this asymptotic condition was consistent with the results of several other independent studies [2, 3, 19]. Through a linear regression method, the constant **c** was evaluated to be 3. The substitution of the above determined values in Eq. (2) results in

$$\varepsilon_h/\varepsilon_m = 0.37\ (1-FR)^{0.75} e^{[1-(Ws/200)]^3} \qquad (3)$$

where FR and Ws in Eq. (3) can be determined from the measurements of pressure drop and apparent viscosity of the solution.

It should be remarked that the constant **a** was determined from the Newtonian fluid behavior, and the constants **b** and **c** from the minimum asymptotic conditions for heat transfer and friction, respectively. Since the conditions used for the determination of the constants should be consistent, independent of the experimental procedures, it is expected that Eq. (3) can be used to predict the heat transfer behavior of all viscoelastic turbulent pipe flows without any adjustment of the constants. In the next section, the predictive capability of the proposed equation will be verified with the experimental data of Kwack [1] and our recent study for aqueous solutions of polyacrylamide (Separan AP-273).

Predictions

An accurate prediction of velocity profile is essential not only to investigate the mechanism of momentum transfer but also to predict the phenomenon of heat transfer. In this study, the momentum eddy diffusivity model proposed by Cess [20] was used to solve the time-averaged Navier-Stokes equation. For a fully developed pipe flow, the expression is

$$\frac{\varepsilon_m}{\nu_a} = \frac{1}{2}\{1 + \frac{K^2 r_o^{+2}}{9} [1 - (\frac{r}{r_o})^2]^2 [1 + 2 (\frac{r}{r_o})^2]^2$$

$$\times[1 - \exp\{\frac{-(1 - r/r_o)}{A^+/r_o^+}\}]^2\}^{1/2} - \frac{1}{2} \qquad (4)$$

When the Cess model is applied to the viscoelastic flows, the parameter A^+ that characterizes the thickness of the near-wall region (viscous sublayer and buffer region) should be determined such that it can properly account for the variations of the laminar layer thickness with the addition of polymer. In the current study, the parameter A^+ was determined using the iterative computational scheme proposed by Tiederman and Reischman [21] for calculation of velocity profile in viscoelastic turbulent flows. The procedure requires only pressure drop and flow rate information. The predictions based on this scheme have been compared with several experimental velocity profiles for channel flows [21] and limited pipe flow experiments [22]. In all cases the predictions show excellent agreement with experimentally measured profiles in turbulent viscoelastic flows. In order to further verify the computational scheme, this technique was used to predict the velocity profiles and the momentum eddy diffusivities in both Newtonian and viscoelastic pipe flows. As for comparison data, the measurements of Mizushina and Usui [2] were adopted. They used a laser-Doppler anemometer technique, which can produce accurate measurements of the flow field with the least amount of disturbance to the flow. As shown in Figure 5, the predictions show excellent agreement with experimentally measured velocity profiles in both Newtonian and drag-reducing flows. Figure 6 illustrates that the Cess model can be used successfully in predicting the eddy diffusivities of momentum for the maximum drag reduction case as well as the Newtonian case.

With the known velocity profile, the heat transfer coefficient can be determined by solving the time-mean energy equation with the use of an eddy diffusivity expression for heat. As pointed out earlier, most previous studies have used a direct analogy between momentum and heat transfer for viscoelastic pipe flows. This could introduce considerable error in heat

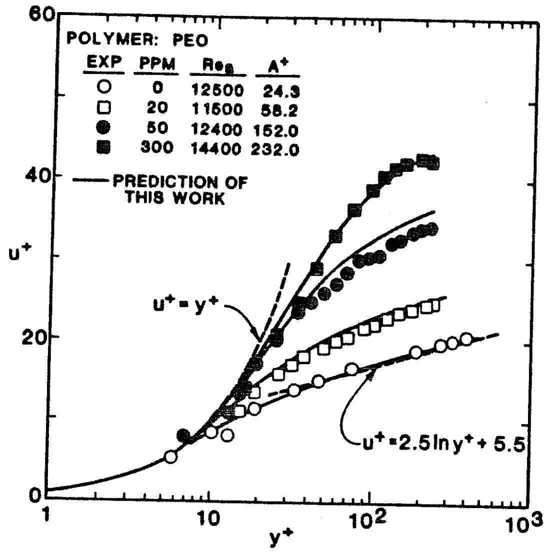

FIG. 5
Newtonian and drag reducing mean velocity profiles: predicted and experimental [2]

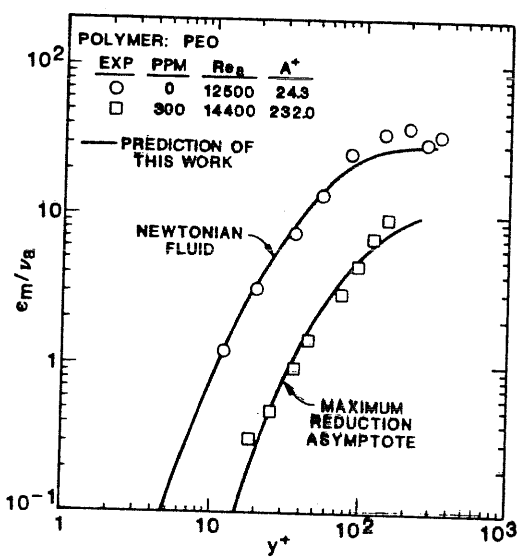

FIG. 6
Newtonian and drag reducing momentum eddy diffusivities: predicted and experimental [2]

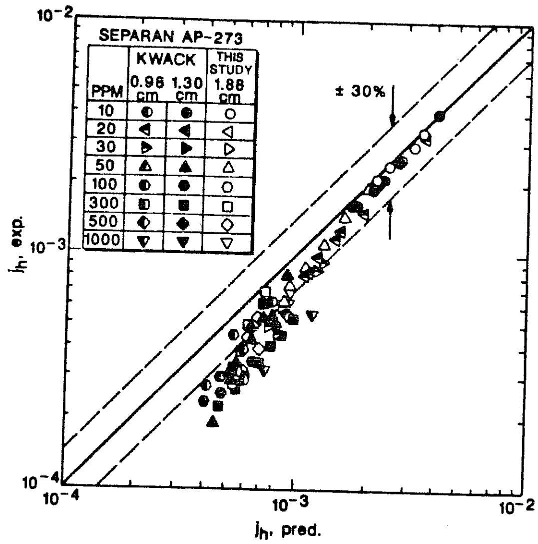

FIG. 7
Comparison of the predicted Colburn j-factors using
a direct analogy between heat and momentum transfer with measurements

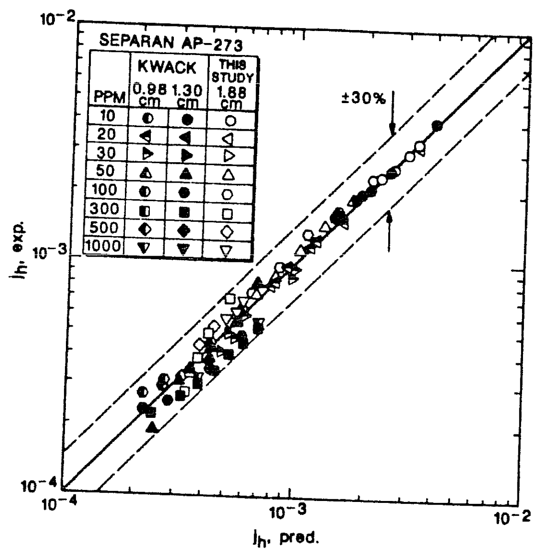

FIG.8
Comparison of the predicted Colburn j-factors using
the proposed heat eddy diffusivity equation with measurements

relatively consistent experimental results for these cases. The predictability of the proposed equation was validated with two independent sets of experimental data, both for aqueous solutions of Separan AP-273, for concentrations ranging from 10 to 1000 ppm in turbulent flow through pipes under constant wall heat flux condition. It is expected that the proposed equation will be applicable to different types of polymers with wide ranges of concentrations. This is due to the fact that the fluid time scale and the flow time scale in the proposed equation can account for several important factors influencing the friction and heat transfer behavior of viscoelastic turbulent pipe flows, such as pipe diameter, solvent chemistry, degradation, as well as the type and the concentration of the polymer. However, since the proof of the predictive capability of the proposed equation was limited to one type of polymer (Separan AP-273), further evaluation of the equation should be done with more reliable experimental data obtained for different types of polymers. Efforts are now underway to produce this type of data.

Acknowledgment

This work was partially sponsored by the University Center for Energy Research (UCER) at Oklahoma State University.

Nomenclature

A^+ constant that characterizes thickness of wall layer

c specific heat of fluid

D inside diameter of test section

f Fanning friction factor, $f = \tau_w/(\rho U^2/2)$

f_p Fanning friction factor for polymer solution

f_s Fanning friction factor for solvent

FR friction drag reduction ratio, $FR = (f_s - f_p)/f_s$

h heat transfer coefficient

j_h Colburn j-factor, $j_h = St\ Pr_a^{2/3}$

k thermal conductivity

K von Karman constant, K = 0.4

L length of test section

Pr_a apparent Prandtl number, $Pr_a = \eta_a c/k$

r radial coordinate direction

r_o radius of test section

r_o^+ dimensionless radius of test section, $r_o^+ = r_o u_\tau/\nu_a$

Re_a apparent Reynolds number, $Re_a = \rho UD/\eta_a$

St Stanton number, $St = h/\rho cU$

u streamwise mean velocity, $u = u(y)$

U mass average velocity

u_τ shear velocity, $u_\tau = (\tau_w/\rho)^{1/2}$

u^+ nondimensional mean velocity, $u^+ = u/u_\tau$

Ws Weissenberg number, $Ws = \lambda/(D/U)$

Ws_{ch} critical Weissenberg number for heat transfer

y coordinate direction normal to wall, $y = r_o - r$

y^+ nondimensional distance normal to wall, $y^+ = yu_\tau/\nu_a$

Greek Letters

ε_h turbulent eddy diffusivity of heat

ε_m turbulent eddy diffusivity of momentum

η_a apparent viscosity, $\eta_a = \tau_w/(du/dy)_w$

η_o zero shear rate apparent viscosity

η_∞ infinite shear rate apparent viscosity

$\dot{\gamma}$ shear rate, $\dot{\gamma} = (du/dy)_w$

λ fluid time scale

ν_a apparent kinematic viscosity, $\nu_a = \eta_a/\rho$

ρ fluid density

τ_w wall shear stress

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