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A NOTE ON THE POWELL-EYRING FLUID MODEL

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ABSTRACT

The sensitivity of the fluid time scale obtained from the Powell-Eyring fluid model to the variations in the zero and infinite shear rate viscosities is investigated. It is concluded that this model is extremely sensitive to the small variations in the zero shear rate viscosity and moderately sensitive to the variations in the infinite shear rate viscosity. Recommended values for these shear rate viscosities are given.

Introduction

In addition to the concentration of polymer, there are several other factors which affect the characteristics of momentum and heat transfer in drag-reducing turbulent pipe flows, such as pipe diameter, solvent chemistry, and mechanical degradation. According to Kwack [1], the polymer concentration, the level of mechanical degradation, and the effect of solvent chemistry, influence the fluid time scale (i.e. elasticity), while the flow rate and pipe diameter determine the flow time scale. The ratio of the fluid time scale (λ) to the flow time scale (D/U) forms a dimensionless parameter called Weissenberg number (Ws). This parameter has been found to adequately account for the influence of polymer concentration, mechanical degradation, pipe diameter, and solvent chemistry on the behavior of friction and heat transfer of viscoelastic fluids [1-8]. These studies indicate that Weissenberg number is a major independent parameter controlling the friction

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and heat transfer behavior in viscoelastic turbulent pipe flows. In order to evaluate the Weissenberg number, a fluid time scale must be calculated. This note is intended to help the reader in calculation of an accurate fluid time scale from a simple fluid model (Powell-Eyring), using experimental shear viscosity measurements.

Fluid Time Scale

Successful correlation of polymer characteristics in terms of the Weissenberg number depends on the accurate estimation of the fluid time scale. Several investigators [9-11] have tried to estimate the fluid time scale from the measurements of intrinsic viscosity. The determination of intrinsic viscosity requires the extrapolation of polymer viscosity data to zero shear rate and zero concentration. It was pointed out by Skelland [12] that such extrapolation for non-Newtonian fluids might result in large uncertainties, especially at very low shear rates. Other estimations of the fluid time scale can be obtained from the measurements of the first normal stress difference or the shear rate dependent viscosity. According to Yoo [13], while it is possible to determine the first normal stress difference with good accuracy for the high concentration polymer solutions, it is not so for the low concentration or highly degraded solutions. The third approach in determination of the fluid time scale is the use of shear rate dependent viscosity data. According to the works of Bird [14] and Elbirli and Shaw [15], a generalized Newtonian fluid model in conjunction with steady shear viscosity data can produce the fluid time scale of a viscoelastic fluid quite accurately. The shear viscosity is a rheological property which can be measured with good accuracy for various concentrations of polymer solutions and well-represent their characteristics. Typical shear viscosity-shear rate relationship for a polymer solution is shown in Figure 1. Note that this figure indicates that the zero shear rate viscosity increases with increasing polymer concentration, which implies an increase in the elasticity of the fluid. So the fluid model which is used to calculate the fluid time scale must well-characterize the pronounced change in the low shear rate viscosity as observed in Figure 1. Among generalized Newtonian fluid models published, the Carreau model A, Ellis, Eyring, and the Powell-Eyring models (see Table 1) have been widely used for study of viscoelastic fluids. Several studies [1, 16, 17] have suggested that the Powell-Eyring fluid model appears to be quite accurate and consistent in the calculation of the fluid time scale at various polymer concentrations. In this study, the Powell-Eyring fluid model will be employed to estimate the fluid time scale because of its well-representation of fluid elasticity and simplicity of application to experimental and analytical studies.

TABLE 1
Generalized Newtonian Fluid Models

Model	Fluid Time Scale
Carreau Model A	
$\eta_a = \eta_\infty + (\eta_o - \eta_\infty)[1 + (\lambda\dot{\gamma})^2]^{(n-2)}$	λ
Ellis model	
$\frac{1}{\eta_a} = \frac{1}{\eta_o}\left[1 + \left(\frac{\tau}{\tau_{1/2}}\right)^{\frac{1-n}{n}}\right]$	$\lambda = \eta_o/\tau_{1/2}^{*}$
Eyring model	
$\eta_a = \eta_o\left[\frac{\text{arc sinh } \lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right]$	λ
Powell-Eyring model	
$\eta_a = \eta_\infty + (\eta_o - \eta_\infty)\left[\frac{\text{arc sinh } \lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right]$	λ

*$\tau_{1/2}$ is the shearing stress where the viscosity η_a is equal to the half of η_o .

Powell-Eyring Fluid Model

It was stated earlier that the successful correlation of polymer characteristics in terms of Weissenberg number depends on the accurate estimation of the fluid time scale. In this study, the Powell-Eyring fluid model, which has been known to be superior to other models, will be used to estimate the fluid time scale. The Powell-Eyring fluid model has the following expression:

$$\eta_a = \eta_\infty + (\eta_o - \eta_\infty)\,\frac{\sinh^{-1}(\lambda\dot{\gamma})}{\lambda\dot{\gamma}} \qquad (1)$$

Since this model includes the zero shear rate and the infinite shear rate viscosities as major parameters, it is very important to study the sensitivity of the model to variations in those two parameters. The rheological data of Kwack [1] for aqueous solutions of polyacrylamide (Separan AP-273), which was presented in Figure 1 will be used for this purpose. Kwack's viscosity data were obtained from a Weissenberg rheogoniometer. This type of viscometer is one of the most expensive and sophisticated viscometers commercially available and generally it is believed to produce accurate results.

Sensitivity of the Fluid Time Scale to Variations in the Zero Shear Rate Viscosity

It has been well-known that the elasticity in a viscoelastic fluid is reflected by the pronounced increase of shear viscosity with the decrease of shear rate at the low range of shear rates as shown in Figure 1. The Powell-Eyring fluid model used in this study includes the zero shear rate viscosity as a key parameter. However, the mechanical sensitivity of the viscometer available for the particular use does limit the lowest measurable shear rate to a certain range. Skelland [12] pointed out that the extrapolation of shear viscosity data for non-Newtonian fluids is not reliable at such a low shear rate. So it is valuable to study the sensitivity of the Powell-Eyring fluid model to the variations in the zero shear rate viscosity in the estimation of the fluid time scale. The fluid time scale was determined by a linear regression method with the use of Kwack's [1] viscosity data for Separan AP-273 solutions. Figure 2 shows the dependency of the fluid time scale on the variations in the zero shear viscosity for the representative concentrations of 10, 300, and 1000 ppm. It can be noticed from this figure that the higher concentration solution demands the zero shear rate viscosity to be measured at much lower shear rate in order to maintain the same accuracy in the estimation of the fluid time scale. Furthermore, the change of the fluid time scale is drastic for small variation in the zero shear rate viscosity for the high concentration solution. Figure 2 suggests that for the reliable estimation of the fluid time scales for 10, 300 and 1000 ppm concentrations the viscosity data should be measured at shear rates of order of 10 sec^{-1}, 1 sec^{-1} and 10^{-1} sec^{-1}, respectively. This implies that with the increase of polymer concentration, the zero shear rate viscosity should be measured at much lower shear rates.

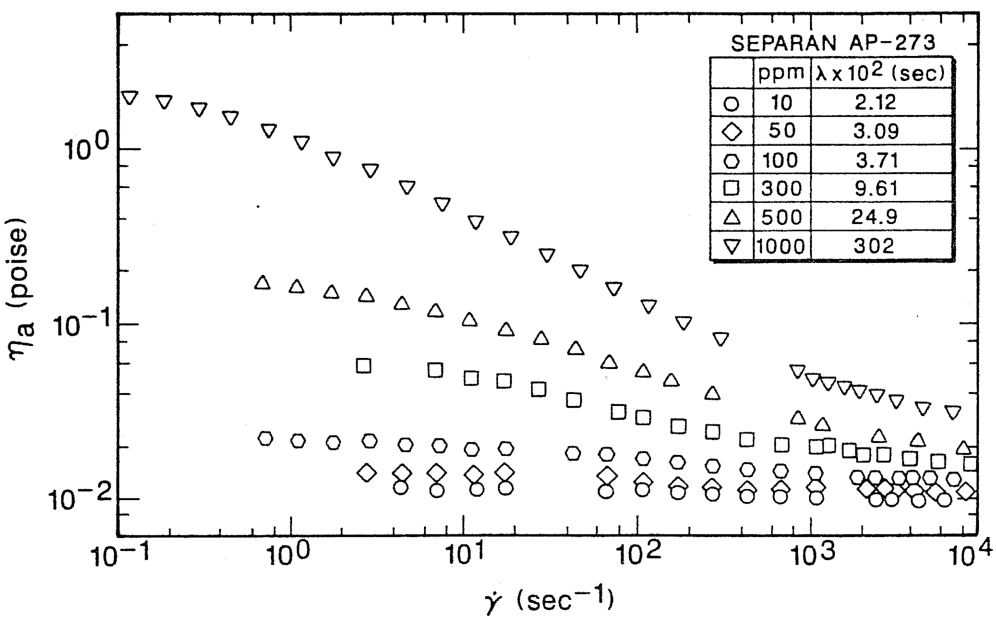

FIG. 1
Shear rate dependent viscosity of
Separan solutions (taken from Kwack [1])

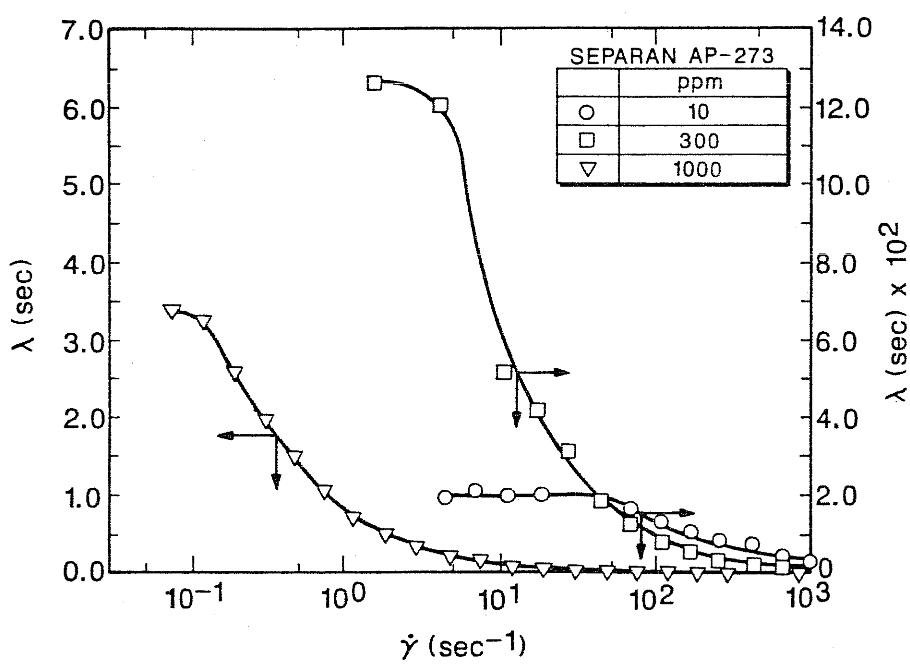

FIG. 2
Sensitivity of the fluid time scale to variations in the
zero shear rate viscosity using Powell-Eyring fluid model

Sensitivity of the Fluid Time Scale to Variations in the Infinite Shear Rate Viscosity

The other parameter of the Powell-Eyring fluid model is the infinite shear rate viscosity. Figure 1 indicates that as compared with the pronounced change in the zero shear rate viscosity, the infinite shear rate viscosity shows a modest increase with the increase of polymer concentration. Since due to constraints on the instrumentation it is not feasible to measure the viscosity at the infinite shear rate, the viscosity obtained at the possible highest shear rate should be used as the infinite shear rate viscosity. Consequently, it is essential to study the dependency of the fluid time scale on the variations in the infinite shear rate viscosity. Figure 3 shows the sensitivity of the Powell-Eyring fluid model to the variations in the infinite shear rate viscosity in the determination of the fluid time scale for the representative concentrations of 10, 300 and 1000 ppm. It can be noticed from this figure that the characteristic curves of the fluid time scale tend to flatten out at the high shear rate range, especially at the shear rates greater than 10^3 sec^{-1}. Kwack [1] suggested that the infinite shear rate viscosity should be evaluated at a shear rate greater than 10^5 sec^{-1} for reliable estimation of the fluid time scale. However, the viscosity at such a high shear rate of 10^5 sec^{-1} is obtainable not by a direct measurement, but by an extrapolation. Unfortunately, the uncertainty from the extrapolation of viscosity data at such high shear rates has not been fully known. The current study indicates that the uncertainty in the estimation of the fluid time scale, due to variations in the infinite shear rate viscosity, is negligible if the viscosity is measured at a shear rate greater than 10^4 sec^{-1}. This shear rate can be readily obtained in most laboratories with the use of a capillary tube viscometer.

Performance of the Powell-Eyring Fluid Model

The fluid time scale was estimated by a linear regression method using all the viscosity data. So it is expected that the fluid model should be able to reproduce accurately all the viscosity data with the back-substitution of the fluid time scale into the expression with the measured values of the zero shear rate viscosity and the infinite shear rate viscosity. In this study, the performance of a model is defined as the capability of the model to

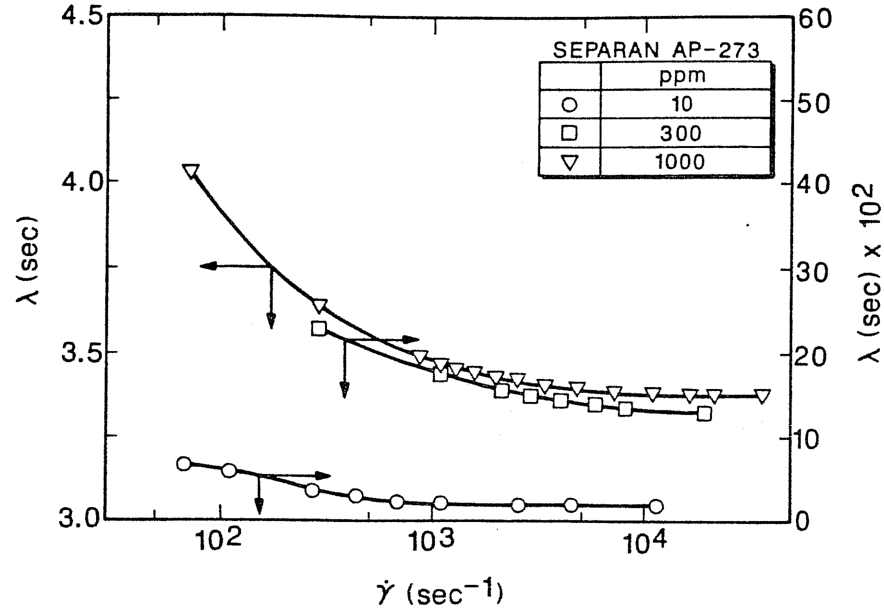

FIG. 3
Sensitivity of the fluid time scale to variations in the infinite shear rate viscosity using Powell-Eyring fluid model

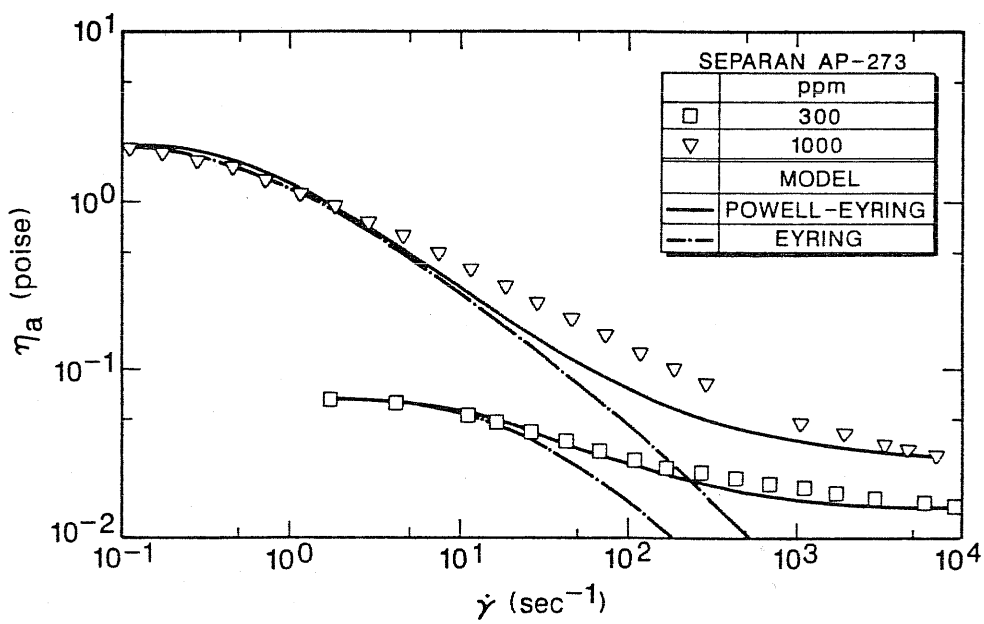

FIG. 4
Performance of the fluid models as compared with measurements of Kwack [1]

generate the original viscosity data. Figure 4 shows the characteristic curves generated by the Powell-Eyring fluid model and the Eyring fluid model (see Table 1) together with the original viscosity data (see Figure 1). It is clearly shown in Figure 4 that the performance of the Eyring fluid model is extremely poor at the high shear rate regions. The Powell-Eyring fluid model is the improved version of the Eyring fluid model with the addition of the infinite shear rate viscosity. As compared with the Eyring fluid model, the Powell-Eyring fluid model performs relatively well at both low and high shear rates. However, considerable gap still exists between the calculated data and the measured ones at the intermediate shear rates. This difference is more pronounced for the higher concentration. The case of 10 ppm is excluded from Figure 4, because there is virtually no difference between the predictions and the measurements. It can be noticed in Figure 4 that there are two regions showing gaps between the predictions and the measurements. The first region, where the predictions are greater than the measurements, is located at the low shear rate region. The second region, where the predictions are less than the measurements, is located at the relatively high shear rate region. It was found that in order to bring the predictions close to the measurements, a fluid time scale greater than the estimated value is needed at the first region, and smaller one at the second region. This implies that an arbitrary increase or decrease of the fluid time scale can not close the gap between the predictions and the measurements. It is not fully known that how much this gap can affect the estimation of the fluid time scale. However, since the elasticity of a fluid is mainly characterized by the zero shear rate viscosity, this effect should be of little significance. Even though there are a few fluid models available in the published literature [16] which are known to have slight edge in performance as compared with the Powell-Eyring fluid model, the use of those models in this study was precluded because of their complexity and arbitrariness in the determination of the fluid time scale.

Conclusions

The Powell-Eyring fluid model used to estimate the fluid time scale was investigated. It was found that this model is extremely sensitive to the small variations in the zero shear rate viscosity, especially for the high concentration solutions. It was shown that for accurate estimation of the fluid time scale for the high concentration solutions the zero shear rate viscosity data should be measured at shear rates of order of 10^{-1} sec^{-1}. On the other hand, the Powell-Eyring fluid model was found to be moderately sensitive to variations in the infinite shear rate viscosity. It was concluded that the uncertainty in the estimation of the fluid time scale using the Powell-Eyring fluid model, due to variations in the infinite shear rate viscosity, is negligible if the viscosity is measured at a shear rate greater than 10^4 sec^{-1}. As compared with other fluid models, the Powell-Eyring fluid model is known to produce quite consistent results in the calculation of the fluid time scale at various polymer concentrations. However, this model still has room for improvement.

Acknowledgment

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Nomenclature

D	inside diameter of test section
n	power law index defined in Table 1
U	mass average velocity
Ws	Weissenberg number, $Ws = \lambda/(D/U)$
η_a	apparent viscosity
η_o	zero shear rate apparent viscosity
η_∞	infinite shear rate apparent viscosity
$\dot{\gamma}$	shear rate
λ	fluid time scale
τ	shear stress

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