of elliptical ducts with uniform wall heat flux is solved in closed form by the simple integral method. The resulting solution is extremely compact and of a usable form. Moreover, its accuracy in predicting the Nusselt numbers is on par with that of the complex variable solution. A unique feature of the present analysis is generation of the universal temperature profile applicable to elliptical ducts of all aspect ratios. A generalization of this temperature profile can be extremely valuable in developing a manageable solution to hitherto unsolved problem of simultaneous development of velocity and temperature fields in elliptical ducts.

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An Analysis of the Heat Transfer to Drag Reducing Turbulent Pipe Flows

H. K. Yoon[1] and A. J. Ghajar[2]

Nomenclature

A^+ = constant that characterizes thickness of wall layer in Van Driest's law
c = specific heat
E = nondimensional momentum eddy diffusivity, $E = \epsilon_m/\nu$
F = Fanning friction factor, $F = 2\tau_0/\rho U_m^2$
h = heat transfer coefficient
j = Colburn j factor, $j = \mathrm{St}\,\mathrm{Pr}^{2/3}$
K = von Karman constant, $K = 0.4$
k = thermal conductivity
$\overline{p^*}$ = mean pressure
$\bar{p}$ = nondimensional mean pressure, $\bar{p} = \overline{p^*}/\rho U_m^2$
Pr = apparent Prandtl number, $\mathrm{Pr} = \nu/\alpha = \mu c/k$
$\dot{q}_0$ = heat flux at surface
Re = Reynolds number, $\mathrm{Re} = (2r_0)U_m/\nu$
R = Reynolds number based on pipe radius, $\mathrm{R} = r_0 U_m/\nu$
r_0 = radius of pipe
r_0^+ = nondimensional pipe radius, $r_0^+ = r_0 U_\tau/\nu$
r = radial coordinate direction
$\bar{r}$ = nondimensional radial coordinate, $\bar{r} = r/r_0$
St = Stanton number, $\mathrm{St} = h/\rho c U_m$
$\overline{T^*}$ = mean temperature
T_m = mixed mean temperature
T_0 = fluid temperature at surface
$\overline{U^*}$ = streamwise mean velocity
$\bar{U}$ = nondimensional streamwise mean velocity, $\bar{U} = \overline{U^*}/U_m$
U_m = mass average velocity
U_τ = shear velocity, $U_\tau = (\tau_0/\rho)^{1/2}$
x = streamwise coordinate direction
$\bar{x}$ = nondimensional streamwise coordinate direction, $\bar{x} = x/r_0$
y = coordinate direction normal to wall, $y = r_0 - r$
$\bar{y}$ = nondimensional direction normal to wall, $\bar{y} = y/r_0$
y^+ = nondimensional distance normal to wall, $y^+ = y\,U_\tau/\nu$
ν = apparent kinematic viscosity, $\nu = \mu/\rho$
μ = apparent absolute viscosity (solution viscosity at the wall)
ρ = fluid density
ϵ_m = turbulent eddy diffusivity of momentum
ϵ_h = turbulent eddy diffusivity of heat
τ_0 = wall shear stress
α = molecular thermal diffusivity, $\alpha = k/\rho c$

1 Introduction

The fascinating effects of long-chain polymer molecules on the friction drag and heat transfer coefficient of turbulent flows have stimulated numerous studies of the phenomenon known as drag reduction or Toms effects. The better understanding of friction factor and heat transfer coefficient reduction phenomena has direct applications in designs of more efficient heat exchangers for food processing, chemical and biochemical industires, of increased capacity pipelines, and of faster ships for shipping industries.

Most heat transfer models take advantage of the Reynolds analogy to correlate heat and momentum transfer phenomena. However, recent studies [1, 2] have shown that Reynolds analogy is not applicable to drag reducing turbulent pipe flows. These studies have revealed that eddy diffusivity of heat is smaller than that of momentum. Objectives of the current study were to clarify the validity and limitations of Reynolds analogy for viscoelastic fluids and to investigate various eddy diffusivity models of heat by comparing the predicted results with heat transfer experiments of Kwack et al. [3]. Their experimental data are the only available data that are reliable and well documented. Their experiments take into account such important effects as thermal entrance length, polymer degradation, and solvent chemistry.

2 Mathematical Background

The analysis begins with the Navier-Stokes equation in the x-direction for an axisymmetic flow in a circular tube written in terms of mean velocity and fluctuations from the mean. After averaging with respect to time, this equation becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu+\epsilon_m)\frac{d\overline{U^*}}{dr}\right] = \frac{1}{\rho}\frac{\partial \overline{P^*}}{\partial x} \qquad (1)$$

Before integrating equation (1), first nondimensionalize such that

$$\bar{U} = \frac{\overline{U^*}}{U_m}, \quad \bar{P} = \frac{\overline{P^*}}{\rho U_m^2}$$

$$E = \frac{\epsilon_m}{\nu}, \quad \bar{x} = \frac{x}{r_0}, \quad \bar{r} = \frac{r}{r_0} \qquad (2)$$

where

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$$U_m = \frac{2}{r_0^2}\int_0^{r_0} \overline{U^*} r\, dr \quad (3)$$

Thus equation (1) becomes

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left[\bar{r}(1+E)\frac{d\bar{U}}{d\bar{r}}\right] = \mathrm{R}F \quad (4)$$

where

$$R = \frac{r_0 U_m}{\nu} \quad \text{and} \quad F = \frac{d\bar{P}}{d\bar{x}} \quad (5)$$

Equation (4) may be integrated twice with the aid of boundary conditions

$$\frac{d\bar{U}}{d\bar{r}} = 0 \quad \text{at} \quad \bar{r} = 0$$

and

$$\bar{U} = 0 \quad \text{at} \quad \bar{r} = 1 \quad (6)$$

Finally, we can obtain the nondimensionalized form of the mean velocity profile from equations (3) and (4)

$$1 = \mathrm{R}F\int_0^1 \int_1^{\bar{r}} \left[\frac{r_1}{1+E(r_1)} dr_1\right] \bar{r}\, d\bar{r} \quad (7)$$

The eddy diffusivity expression in equation (7) must be chosen so that this equation is satisfied. Consequently, the eddy diffusivity must be determined such that the product of RF determined from equation (7) agrees with experimental conditions. Details concerning the eddy diffusivity expression for momentum will be given in section 3.

With the obtained time-mean velocity profiles, we are ready to solve the time-mean energy equation in the x-direction

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha+\epsilon_h)\frac{\partial \overline{T^*}}{\partial r}\right] = \overline{U^*}\frac{dT_m}{dx} \quad (8)$$

The heat transfer coefficient is defined by

$$\dot{q}_0'' = h(T_0 - T_m) \quad (9)$$

where

$$\dot{q}_0'' = +k\left(\frac{\partial \overline{T^*}}{\partial r}\right)_{r=r_0} \quad (10)$$

Some mathematical manipulations of equations (8), (9), and (10), with well-defined boundary conditions result in a Stanton number, which can be expressed by

$$\mathrm{St} = \frac{\int_1^0 \bar{U}(1-\bar{y})\,d\bar{y}}{\mathrm{Re}\int_0^1 \bar{U}(1-\bar{y})\left\{\int_0^{\bar{y}}\left[\frac{\int_1^{\bar{y}_1} \bar{U}(1-\bar{y}_2)d\bar{y}_2}{\left(\frac{\epsilon_h}{\nu}(\bar{y}_1)+\frac{1}{\mathrm{Pr}}\right)(1-\bar{y}_1)}\right]d\bar{y}_1\right\}d\bar{y}} \quad (11)$$

Additional details of mathematical derivations may be found in [4, 5]. Details concerning an eddy diffusivity expression for heat will be discussed in section 4.

3 Predictions of Mean Velocity Profile

In the proposed heat transfer analysis, the Cess eddy diffusivity model was used to predict the mean velocity profile. Cess [6] combined the wall region eddy diffusivity of Van Driest with Reichardt's expression for the diffusivity in the center portion of a pipe flow to obtain a single, continuous expression for eddy diffusivity of momentum ϵ_m. The eddy diffusivity expression proposed by Cess is

$$\frac{\epsilon_m}{\nu} = \frac{1}{2}\left\{1+\frac{K^2 r_0^{+2}}{9}\left[1-\left(\frac{r}{r_0}\right)^2\right]^2\left[1+2\left(\frac{r}{r_0}\right)^2\right]^2 \times\left[1-\exp\left\{\frac{-(1-r/r_0)}{A^+/r_0^+}\right\}\right]^2\right\}^{1/2} - \frac{1}{2} \quad (12)$$

The unique feature of this model is that in drag reducing flows, it is not possible to determine one of the constants in the eddy diffusivity model a priori. An iterative scheme proposed by Tiederman and Reischman [7] was used to determine the constant A^+ that characterizes the thickness of the near-wall region of the flow. This iterative scheme requires as inputs an experimental value for the von Karman contant K and some initial guess for A^+. The procedure then determines an exact value for A^+ and integrates the equations to yield velocity profiles.

The Cess model has proved to be able to predict the mean velocity profile excellently when compared with experimentally measured profiles in both Newtonian and drag reducing flows. The simplicity of using the Cess model should not be overlooked. It is an explicit technique and the distribution is continuous. Consequently, there is no "patching" of diffusivity expressions at arbitrary locations. Finally, it combines the best features of two good diffusivity models so that the final diffusivity has the correct behavior both at the wall and in the center portion of the pipe.

4 Prediction of Heat Transfer Coefficients

In order to predict heat transfer coefficients, the differential energy equation should be solved based on an accurate and general eddy diffusivity model for heat ϵ_h. It is common to assume that Reynolds analogy is valid for viscoelastic fluids. In this section, an attempt will be made to define the validity and limitations of Reynolds analogy. Additionally, to close the gap between analytical predictions and experimental data, depending on the Prandtl number of polymer solutions, three different eddy diffusivities of heat were employed. For dilute (Pr $<$ 6.5) polymer solutions, Reynolds analogy was used and for intermediate ($6.5 \le \mathrm{Pr} < 8.2$) and high concentration ($\mathrm{Pr} \ge 8.2$) polymer solutions, heat eddy diffusivity models proposed by Mizushina and Usui [2] were adopted. The proposed models are

$$\epsilon_h/\epsilon_m = 1 \quad \text{for} \quad \mathrm{Pr} < 6.5 \quad (13)$$

$$\epsilon_h/\epsilon_m = 1.5[1-\exp[-y^+(42+120/\mathrm{Pr}^{1/2})^{-1}]] \times[1-\exp(-y^+/26)]^{-1} \quad \text{for } 6.5 \le \mathrm{Pr} < 8.2 \quad (14)$$

$$\epsilon_h/\nu = \mathrm{C}y^{+3} \quad \text{for} \quad \mathrm{Pr} \ge 8.2 \quad (15)$$

where $C = 1.7\times10^{-6} - 2.5\times10^{-6}$

These models were used to predict Colburn j-factors, using experimental data of Kwack et al. [3]. The results are summarized in Figs. 1 and 2. In these calculations, the Cess eddy diffusivity model for momentum was used to predict mean velocity profiles. As shown in Fig. 1, Reynolds analogy is valid only for Pr $<$ 6.5, and considerable error (as much as 150 percent) can be introduced in the calculations for Pr $\ge$ 6.5 if Reynolds analogy is assumed. However, as shown in Fig. 2, the employment of different eddy diffusivities of heat depending on the Prandtl number of polymer solutions, ushered the predicted heat transfer results into the $\pm$ 20 percent error bound, with the exception of three points, which is believed to be due to the inconsistency in the reported experimental data.

5 Closure

Based on the observations made in the previous section, the following heat eddy diffusivity models are proposed:

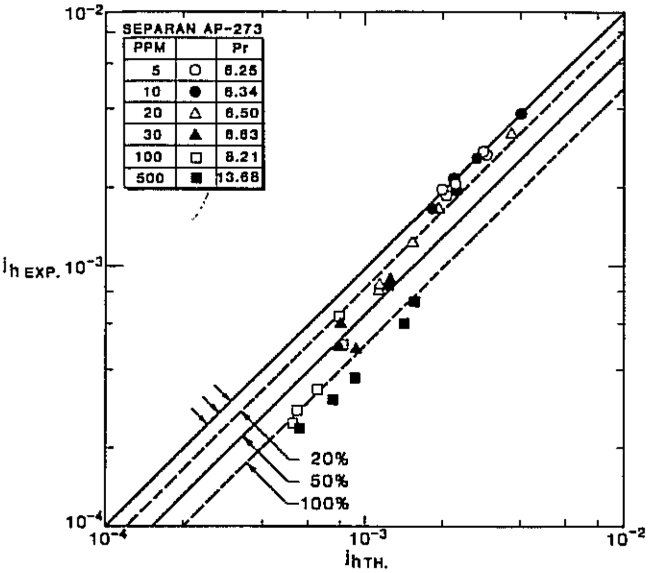

Fig. 1 Comparison of the predicted Colburn *j*-factors based on Reynolds analogy with measurements [3]

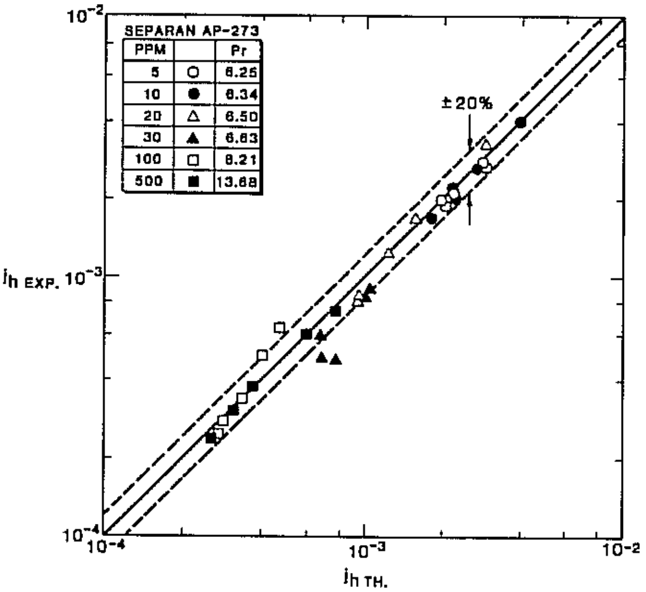

Fig. 2 Comparison of the predicted Colburn *j*-factors using three different heat eddy diffusivities, depening on polymer concentration, with measurements [3]

1 for $\Pr < 6.5$, the model proposed in equation (13).
2 for $6.5 \leq \Pr < 8.2$, the model proposed in equation (14).
3 for $\Pr \geq 8.2$, the model proposed in equation (15).

However, there are shortcomings associated with the use of these heat eddy diffusivity models for future work. The validity and limitations of the diffusivity models are not fully known. The models should be evaluated further with more reliable experimental data that cover wide ranges of polymer concentrations and different types of polymers. Unfortunately, presently this type of data is not available. In addition, no relationship exists that would relate the models to one another. To remedy these shortcomings and increase the predictive capability of the proposed analytical model, the eddy diffusivities of momentum and heat must be expressed as a function of Deborah number (ratio of the elastic forces to the viscous forces). The Deborah number must be determined experimentally for different types of polymers at different concentrations. Presently, this type of information is not available. Efforts are now underway to generate this type of data.

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Stability of Steam–Water Countercurrent Flow in an Inclined Channel: Part II—Condensation-Induced Waterhammer[1]

S. C. Lee[2] and S. G. Bankoff[2,3]

Introduction

Two important instabilities have been observed in inclined countercurrent flow of steam and cold water [1]: condensation-induced waterhammer and flooding. Flooding has been discussed in detail in a preceding paper [2]. Condensation-induced waterhammer, considered to be a combined thermal and hydraulic instability of countercurrent stratified flow, is characterized by significant pressure drop oscillations, probably due to unsteady collapse or near-collapse of local vapor pockets. The initiating mechanism of this condensation-induced instability is not clear yet, although work has been done on particular situations, such as waterhammer in PWR steam generators [3] or in horizontal pipes containing steam and subcooled water [4].

The present study deals with condensation-induced waterhammer in stratified steam–water flow in rectangular ducts inclined at two different angles, approximately 30 and 4.5 deg, to the horizontal. A stability map is constructed, with distinct regions of waterhammer, flooding, and stable operation. The necessary conditions for waterhammer are established and analyses of these conditions are presented on a basis of interfacial wave stability and heat transfer considerations.

Experimental System and Procedure

The experimental apparatus includes a test section, two water storage tanks, heat exchanger, and two circulating pumps. The test section is a rectangular channel approximately 2.13 m long and 0.38 m wide with adjustable depth. The distance between the water inlet and outlet is 1.27 m. The test section has its own support system which permits any inclination between 0 and 90 deg. Two channel depths (H = 0.076 m and 0.038 m) and two inclination angles were employed. Measurements of temperatures, pressure drops,

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