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HEAT EDDY DIFFUSIVITY FOR VISCOELASTIC TURBULENT PIPE FLOWS

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ABSTRACT

A new semi-empirical equation for eddy diffusivity of heat in terms of friction drag reduction ratio and Weissenberg number was earlier reported. The assumptions made in the evaluation of the constants of the proposed equation are verified and its general applicability to various experimental data is further established. For this purpose experiments have been performed for Separan AP-273 and Polyox WSR-301 solutions with concentrations ranging from 10 to 1000 ppm and Separan AP-30 with concentration of 3000 ppm in thermally fully developed turbulent flow in pipes with diameters of 1.11 and 1.88 cm I.D. under constant wall heat flux. From the experiments it is concluded that the minimum asymptotes for friction and heat transfer and the critical Weissenberg number for heat transfer are universal and independent of the type of polymer used. The prediction of heat transfer coefficients with the use of the proposed equation for all of the experimental data is within a maximum deviation of 30%.

Introduction

It has been established that small addition of certain polymers to turbulent pipe flows could result in a drastic reduction in friction drag and heat transfer [1,2]. This finding initiated a number of studies in the possible use of polymer additives in practical engineering systems. However, recent reviews of the relevant works [1,3] suggested that most of the previous experimental and analytical studies have been carried out under inadequate experimental conditions and inaccurate experimental results. Particularly, Reynolds analogy, which has been used to correlate momentum and heat transfer

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phenomena in most analytical studies, was verified to be invalid for viscoelastic turbulent pipe flows [4-6].

To remedy the inadequacy of the existing heat transfer models, in a pervious paper [7] the authors presented a new heat eddy diffusivity equation for viscoelastic turbulent pipe flows which was developed with the aid of Kwack's [6] experimental data and was verified with limited experimental data. The aim of this study is to conduct new experiments in order to verify the assumptions made in the evalutaion of the adjustable constants in the proposed heat eddy diffusivity equation and to establish its general applicalibility to various experimental data.

Experiments

The present experiments were conducted in the fluid dynamics laboratory at Oklahoma State University. A schematic diagram of the flow circulation system is shown in Figure 1. The test sections used have inside diameters of 1.88 cm (L/D = 617) and 1.11 cm (L/D = 1046). These test sections ensure the thermally fully developed condition for viscoelastic fluids which require 400 to 500 diameters for the minimum heat transfer asymptote [8]. To minimize mechanical degradation of polymer solutions, the overall flow system was

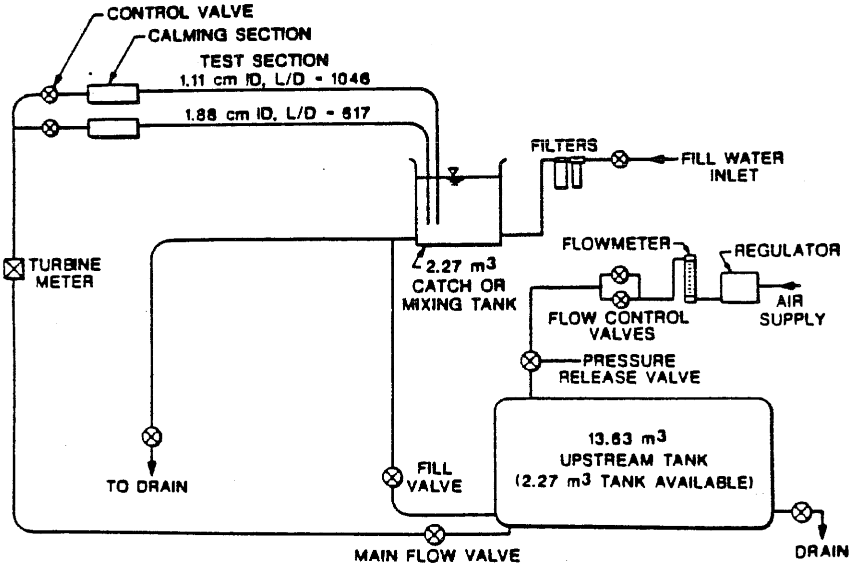

FIG. 1
Schematic diagram of the flow circulation system

operated with pressurized air (up to 80 psig) using the once-through mode. The constant heat flux boundary condition was maintained by a Lincoln DC-600 welder. It can operate in the constant voltage or constant current mode, and has a 100% duty cycle rating at 600 amps and 44 volts. In the present flow system, either the hydrodynamic and thermal entrance regions can develop simultaneously from the beginning of the test section, or the velocity profile can be fully developed before heat transfer starts. The measurements of pressure drop and heat transfer were taken at the same time in the thermally fully developed region with the use of one U-tube mercury manometer and # 30 gauge copper-constantan thermocouples. The flow rate was measured by a one-inch turbine meter located upstream from the test section. This turbine meter monitored by a Hewlett-Packard frequency counter can produce instant or time-averaged readings so that it enables one not only to obtain the average flow rate but also to check the flow stability. Apparent viscosities of solutions were measured at wide range of shear rates (0.36 to 2 x 10^4sec^{-1}) with the use of two Couette viscometers (Brookfield Synchro-Electric Model LVT with UL adaptor and a Fann Model VG) and a capillary tube viscometer (0.94 mm I.D. and $l/d = 325$). The reliability of the flow circulation system and the experimental procedures were checked with several calibration runs for measurements of friction factors and heat transfer coefficients for a Newtonian fluid (tap water) by comparing the experimental results with well-established Newtonian correlations [9-12]. The uncertainty analyses of the overall experimental procedures for water showed that there is 5-6 precent uncertainty for friction factors and 8-10 percent uncertainty for heat transfer coefficients. More detailed description of the experimental apparatus and procedures are presented elsewhere [13].

The viscoelastic fluids used were the well-mixed homogeneous aqueous solutions of polyacrylamide (Separan AP-273) with concentrations of 10, 50, 100, 300, 500, and 1000 ppm and polyethylene oxide (Polyox WSR-301) with concentrations of 100, 300, 500, and 1000 ppm. The apparent viscosity of each polymer solution at wide range of shear rates was measured and the results for Separan AP-273 solutions are presented in Figures 2 and 3. Similar results for Polyox WSR-301 solutions may be found elsewhere [13]. The measured viscosity data were used to estimate the fluid time scale by the Powell-Eyring model, which has the following expression

$$\eta_a = \eta_\infty + (\eta_0 - \eta_\infty) \left[\frac{\sinh^{-1} \lambda\dot{\gamma}}{\lambda\dot{\gamma}}\right] \qquad (1)$$

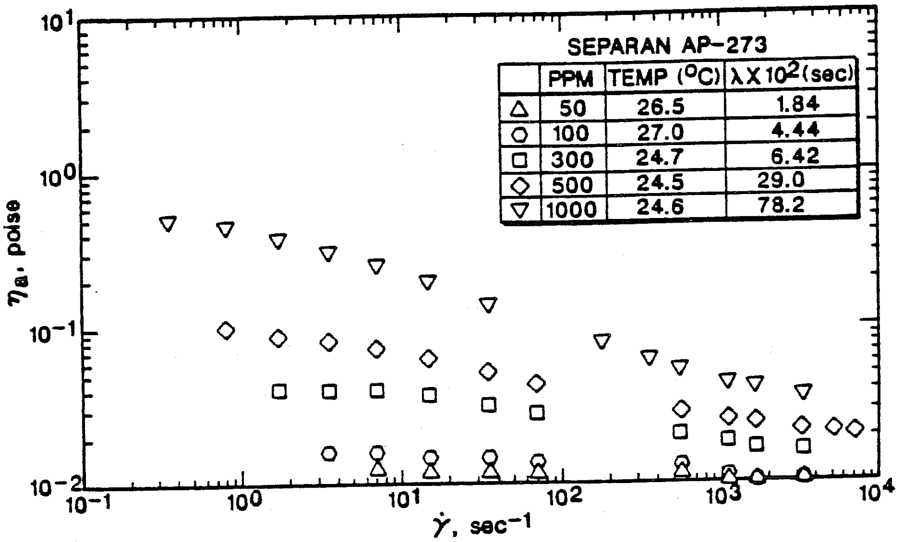

FIG. 2
Apparent viscosity vs. shear rate for Separan AP-273 solutions in the 1.88 cm test section

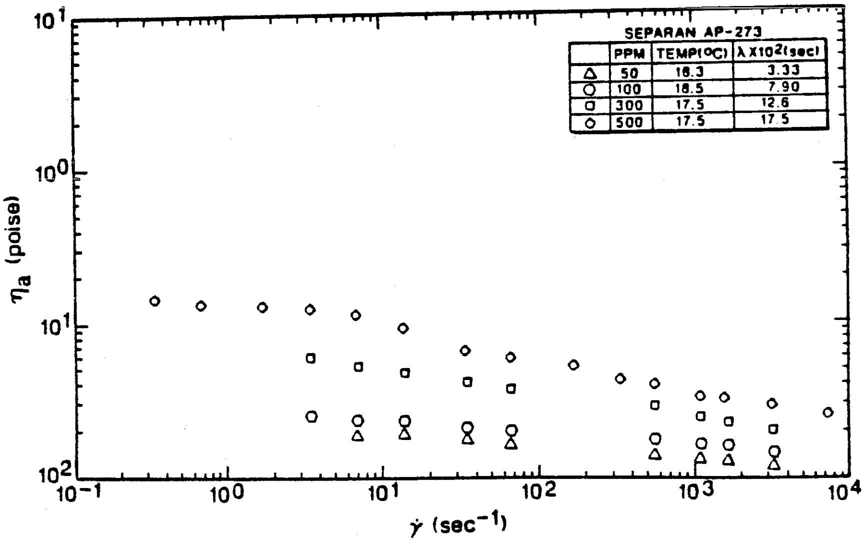

FIG. 3
Apparent viscosity vs. shear rate for Separan AP-273 solutions in the 1.11 cm test section

The fluid time scale was determined by a linear regression method with the use of all the viscosity data for each solution.

The measurements of pressure drop and heat transfer are presented in terms of Fanning friction factor and Colburn j-factor in Figures 4 and 5 for Separan AP-273 and Figures 6 and 7 for Polyox WSR-301, respectively. It is observed from these figures that the reduction in friction factors and heat transfer coefficients for the smaller pipe (1.11 cm test section) is more pronounced than that for the larger one (1.88 cm test section). This can be explained by the following interpretation: the polymer molecules are considered to influence the boundary layer close to the pipe wall. This influence should be seen in the smaller pipe before the larger one since the boundary layer would form a larger portion of the total flow in the small pipe [2]. However, the minimum asymptotes for friction factors and heat transfer coefficients remain the same, independent of pipe diameter. Comparison of Figures 4-7 also indicates that Polyox WSR-301 solution is much less effective in reducing friction drag and heat transfer than the Separan AP-273 solution. This is due to the fact that unlike Separan AP-273 solution, Polyox WSR-301 solution is weakly shear dependent. In addition, the average molecular weight of Separan AP-273 (M = 6 million) is much greater than that of Polyox WSR-301 (M = 4 million).

Heat Eddy Diffusivity Equation

To remedy the inadequacy of the existing analytical studies for heat transfer in viscoelastic turbulent pipe flows, the previous work of the authors [7] formulated a semi-empirical equation for eddy diffusivity of heat in terms of friction drag reduction ratio (FR) and Weissenberg number (Ws). These two important dimensionless parameters for viscoelastic fluids can be determined from the experimental measurements of pressure drop and fluid rheological properties. The use of friction drag reduction ratio and Weissenberg number plays an important role in correlating friction factor with heat transfer coefficient and can account for several important factors influencing the friction and heat transfer behavior of viscoelastic turbulent pipe flows, such as pipe diameter, solvent chemistry, degradation, as well as the type and the concentration of the polymer. The proposed equation has the following form:

$$\varepsilon_h/\varepsilon_m = a(1-FR)^b e^{[1-(Ws/Ws_{ch})]^c} \qquad (2)$$

where $a = 0.37$, $b = 0.75$, $c = 3.0$, and $Ws_{ch} = 200$. Further details on the

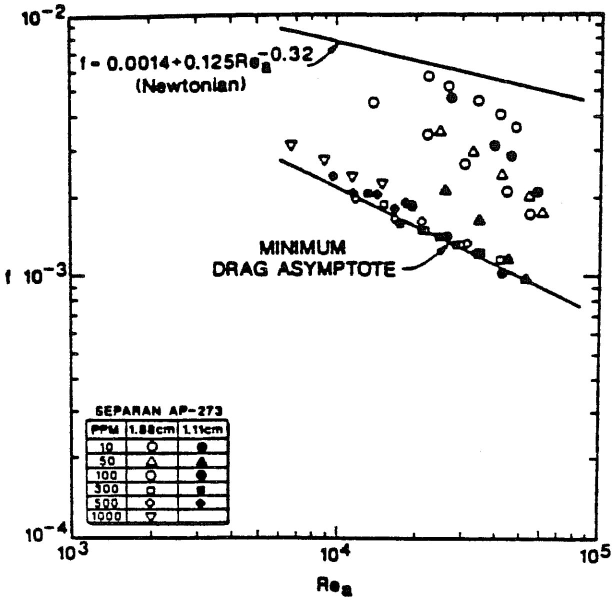

FIG. 4
Fanning friction factor vs. apparent Reynolds number for Separan AP-273 solutions in the 1.88 and 1.11 cm test sections

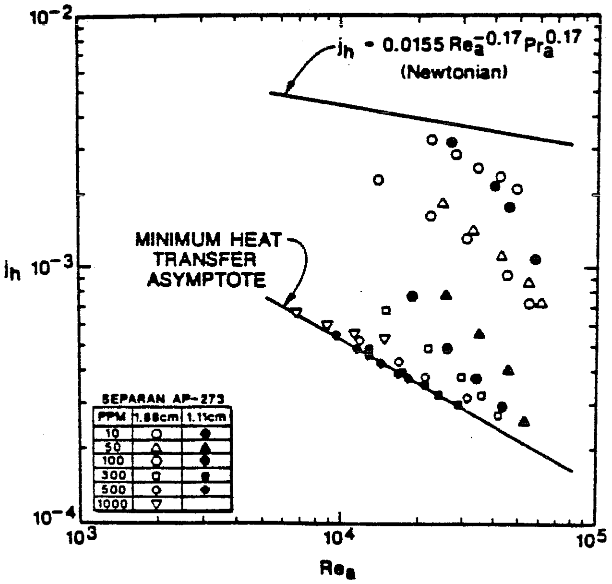

FIG. 5
Colburn j-factor vs. apparent Reynolds number for Separan AP-273 solutions in the 1.88 and 1.11 cm test sections

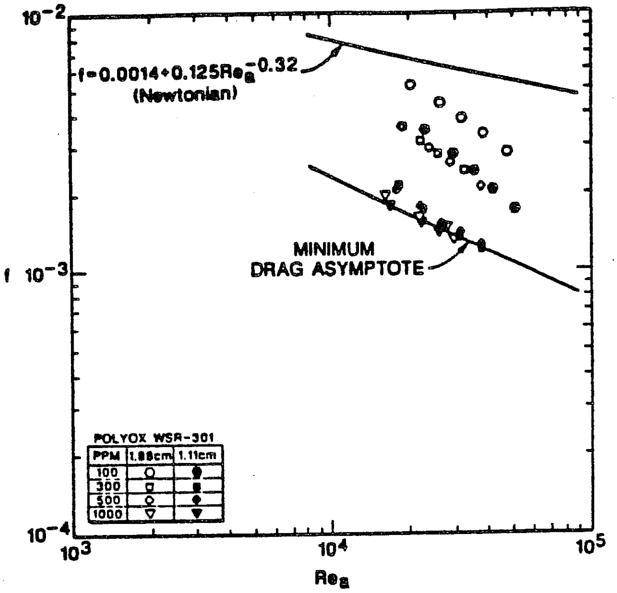

FIG. 6
Fanning friction factor vs. apparent Reynolds number for Polyox WSR-301 solutions in the 1.88 and 1.11 cm test sections

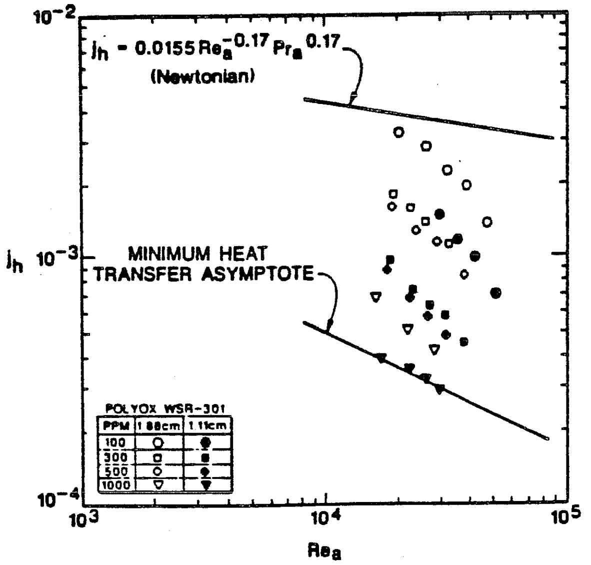

FIG. 7
Colburn j-factor vs. apparent Reynolds number for Polyox WSR-301 solutions in the 1.88 and 1.11 cm test sections

development of equation (2) and the computational scheme used for calculation of heat transfer coefficients are given in [7].

In our previous work [7], the constant **a** was determined from the Newtonian fluid behavior, and the constants **b** and **c** from the minimum asymptotic conditions for heat transfer [8] and friction [14], respectively. It was assumed that these maximum reduction asymptotes are general and independent of the experimental apparatus, procedures employed, and the types of polymer used. In order to establish the generality of the constants in equation (2), this assumption should be verified. As compared with the well-established Newtonian fluid behavior, considerably different values have been suggested for the minimum asymptotic cases, especially for heat transfer [8]. Since this difference is considered to be caused by the inadequate experimental conditions such as short test section, severe mechanical degradation and insufficient study of polymer rheology, it is quite valuable to investigate the minimum asymptotic cases for friction and heat transfer based on the experimental data of previous studies [8,14] and this study. These studies were designed to minimize the above mentioned experimental deficiencies. Figure 8 shows that the minimum asymptotes of this study are in agreement with those of [8,14] within the uncertainty range of the experimental apparatus of both works. This confirms that the minimum asymptotic conditions for heat transfer and friction are general, independent of the experimental apparatus, procedures employed, and the polymer types used.

One of the key parameters in equation (2) is the critical Weissenberg number for heat transfer (Ws_{ch}). Based on the experimental results of Kwack [6] the value of Ws_{ch} was taken to be approximately equal to 200. In our previous work [7] it was assumed that this value is universal. To verify the generality of equation (2), this assumption should be verified. In addition, the critical Weissenberg number can suggest the optimum concentration compromising the performance and the economics of polymer addition. Therefore, it is very important to make an accurate determination of this value. Several studies [6,15,16] reported the critical Weissenberg number for heat transfer to be of the order of 200 - 250 over Reynolds number range of 20,000 to 30,000 for aqueous polyacrylamide solutions (Separan AP-273). The current study is planned to confirm the previous results and to verify the general applicability of them to other solutions. In this study, a highly concentrated polymer solution was diluted to investigate the flow

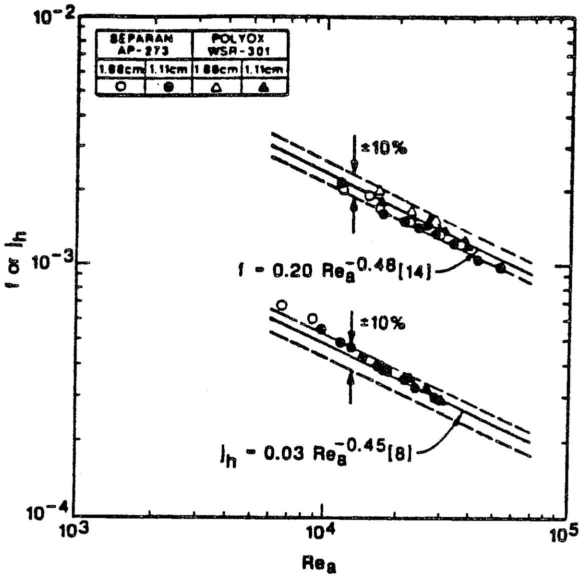

FIG. 8
Comparison of the maximum reduction asymptotes of this study with correlations of [8,14]

characteristics over wide range of concentrations. For this purpose, an aqueous Separan AP-30 solution of 3000 ppm was prepared for dilution. For each concentration, the heat transfer data were taken at various flow rates together with the apparent viscosities at wide range of shear rates. For dilution, the proper amount of tap water was added to obtain the desired concentration. This process of dilution continued until the heat transfer data covered a wide range of Weissenberg numbers. The apparent viscosity data and the fluid time scale for each concentration are presented in Figure 9. This figure shows that the viscosity data for Separan AP-30 solution follow the similar trend to that for Separan AP-273 (see Figures 2 and 3). This solution is also strongly shear dependent. The dimensionless heat transfer coefficients at an apparent Reynolds number of approximately 10,000 as a function of Weissenberg number are presented in Figure 10. The critical Weissenberg number for heat transfer is defined at a value of Weissenberg number for which dimensionless heat transfer coefficient increases from the minimum heat transfer asymptote with decreasing Weissenberg number. From Figure 10, the critical Weissenberg number for heat transfer is estimated to be of the order of 200. This result is in agreement with those of the

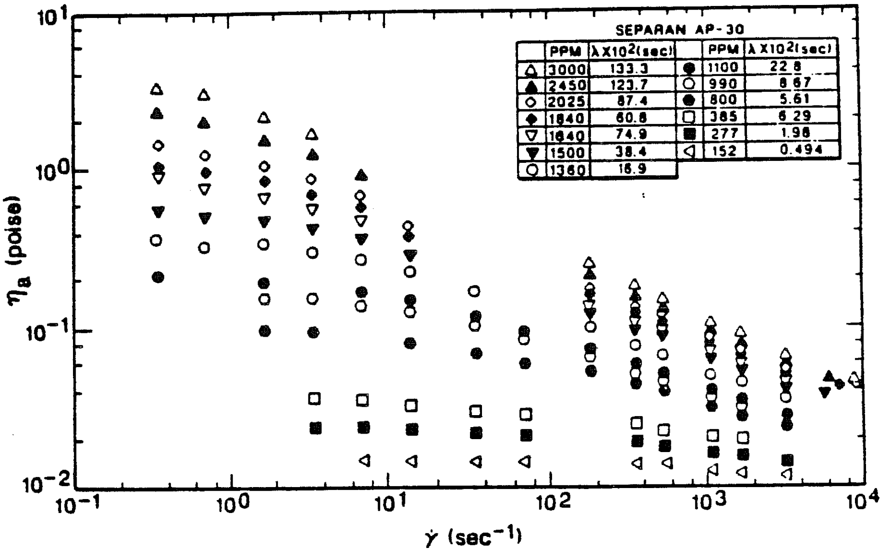

FIG. 9
Apparent viscosity vs. shear rate for
Separan AP-30 in the 1.88 cm test section

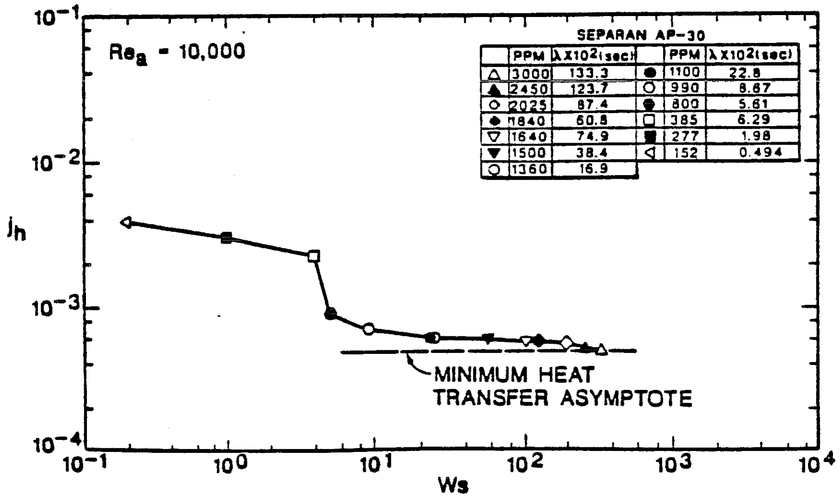

FIG. 10
Colburn j-factor vs. Weissenberg number for
Separan AP-30 in the 1.88 cm test section

previous works conducted for Separan AP-273 solutions [6,15,16]. This indicates that the critical Weissenberg number for heat transfer is general, independent of the experimental apparatus, procedures employed, and the polymer types used.

Now that the assumptions made in the evaluation of the adjustable constants in the proposed heat eddy diffusivity equation have been verified, the predictability of the equation should be further validated with experimental data of this study. Experiments were performed for Separan AP-273 and Polyox WSR-301 solutions with wide range of concentrations in the thermally fully developed region of two different pipe diameters under constant wall heat flux. Figure 11 indicates that the prediction of dimensionless heat transfer coefficients with the use of the proposed equation for all of the experimental data of this study and Kwack [6] are within a maximum deviation of 30%.

Conclusions

The general applicability of the proposed heat eddy diffusivity equation [7] was further verified with our recent experiments for Separan AP-273 and Polyox WSR-301 solutions and Kwack's [6] experiments for Separan AP-273 solution. The generality of the assumptions made in the evaluation of the constants in the proposed equation were established based on the findings that the minimum asymptotes for friction and heat transfer and the critical Weissenberg number for heat transfer are universal and independent of the experimental apparatus, procedures employed, and the types of polymer used. The findings of this study confirmed that the single equation proposed for heat eddy diffusivity can be used to predict with good accuracy the heat transfer coefficients for the two polymer solutions with wide range of concentration.

Acknowledgment

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Nomenclature

a,b,c	constants used in equation (2)
c	specific heat of fluid
D	inside diameter of test section
f	Fanning friction factor, $f = \tau_w/(\rho U^2/2)$
f_p	Fanning friction factor for polymer solution

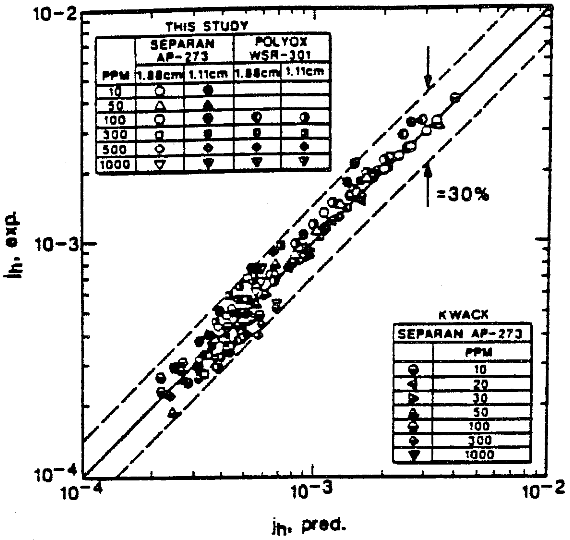

FIG. 11
Comparison of the predicted Colburn j-factors using the proposed heat eddy diffusivity equation with measurements

f_s	Fanning friction factor for solvent
FR	friction drag reduction ratio, $FR = (f_s - f_p)/f_s$
h	heat transfer coefficient
j_h	Colburn j-factor, $j_h = St\ Pr_a^{2/3}$
k	thermal conductivity
L	length of test section
Pr_a	apparent Prandtl number, $Pr_a = \eta_a c/k$
Re_a	apparent Reynolds number, $Re_a = \rho UD/\eta_a$
St	Stanton number, $St = h/\rho cU$
U	average velocity
Ws	Weissenberg number, $Ws = \lambda/(D/U)$
Ws_{ch}	critical Weissenberg number for heat transfer

Greek Letters

ε_h	turbulent eddy diffusivity of heat
ε_m	turbulent eddy diffusivity of momentum
η_a	apparent viscosity at the wall
η_0	zero shear rate apparent viscosity
η_∞	infinite shear rate apparent viscosity
$\dot{\gamma}$	shear rate

λ fluid time scale
ρ fluid density
τ_w wall shear stress

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