Energy Vol. 15, No. 1, pp. 23–34, 1990
Printed in Great Britain

0360-5442/90 $3.00 + 0.00
Pergamon Press plc

METHODS FOR PRODUCING LINEAR DENSITY GRADIENTS IN LABORATORY TANKS

YOUSEF H. ZURIGAT,† KIYOUNG BANG, and AFSHIN J. GHAJAR‡
School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, OK 74078, U.S.A.

(*Received* 30 *May* 1989)

Abstract—The published methods for producing linear density gradients in laboratory tanks have been surveyed. Four of the five identified methods were tested experimentally in our laboratory using fresh saline water systems. These methods have been compared with respect to reproducibility and time consumption. The transient behavior of density profiles generated by the methods tested is presented. The results show that the most efficient and reproducible methods are those of Oster.

INTRODUCTION

The environment is often stably stratified, i.e., the density decreases with elevation as in the lower atmosphere. It also increases with depth in the hydrosphere. The stratification in these environments is mostly linear, except in restricted regions where a thermocline may be present due to diurnal and seasonal variations in thermal fluxes.

There are a large number of practical applications involving linearly-stratified ambients. The discharge of heated effluents from large power plants into the atmosphere and the release of heated condenser water into oceans and lakes are examples associated with thermal pollution and ecological problems that have become foci of environmental and geophysical research. Laboratory experiments dealing with these and other problems of practical interest require generation of linear density gradients in scale models of stratified fluid body. Several methods for generating linear density gradients in laboratory tanks have been devised and reported in the literature. While all of these methods produce the desired linear density gradients, there have been no general guidelines for the selection of a particular method. Experimentation with these methods is needed to establish the preferred approach for a particular tank size and geometry. The time involved in using any particular method and the density-gradient reproducibility are important aspects that need to be addressed.

DESCRIPTIONS OF THE METHODS

Our literature survey has shown that five methods for producing linear density stratification have been developed and used by various investigators. The earliest method is the layer by layer method (LBL), which involves introducing several layers of liquids of different densities decreasing with height into the test tank and allowing diffusion between the layers to smooth out the boundaries into a density gradient that is linear with height.

In experiments of Schooley and Stewart[1] on the effects of density gradients on mixing in the turbulent wake of a self-propelled body moving in a fluid, the linear density gradient was generated by using 12 layers, each 1 cm thick and made up of different mixtures of water and glycerin. With the bottom layer containing 22% glycerin (by volume), each succeeding layer had 2% less glycerin, thus yielding 100% water at the top layer. The various layers were placed ontop of each other by means of a fountain syringe with the nozzle pointed downward towards a thin balsa-wood sheet floating on the surface. After filling, the tank (which was 17.5 cm wide,

†Present address: Virgina Polytechnic State University, Blacksburg, VA 24061, U.S.A.
‡To whom all correspondence should be addressed.

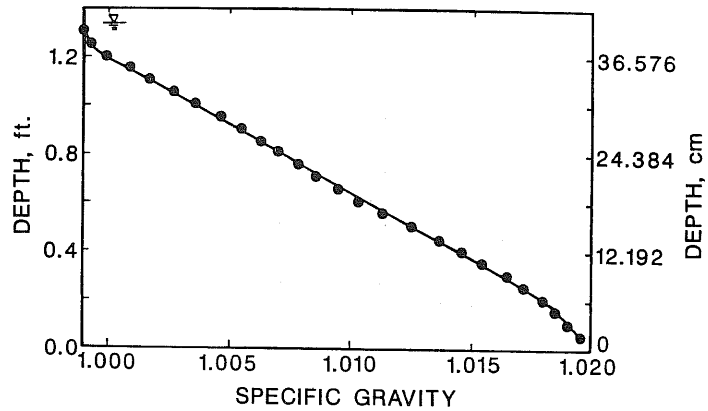

Fig. 1. A typical measured density profile using the LBL method.[2]

33 cm deep, 100 cm long at the bottom, and 150 cm long at the top) was left to settle for a few hours to allow diffusion and blending of the layers to smooth the initially nonlinear density profile into a linear profile. The density gradient generated in this manner[1] was about 5.2×10^{-3} g/(ml-cm).

The LBL method was used later by Koh[2] in experiments on growth of the withdrawal-layer thickness in a stratified flow toward a sink. The test tank (45 cm high and 250 cm long) was first filled with layers of unreported thickness of salt concentrations that decreased from the bottom to top. A linear density gradient was achieved after 15 h of settling. Figure 1 shows the measured[2] density gradient as determined indirectly by measuring the electric conductivity at locations in the tank.

The most obvious drawback of the LBL method is the long settling time needed (15 h in the experiments of Koh[2]) for the density gradient to become linear. A method was developed by Oster[3] to overcome this problem. In this method (Oster method), a linear density gradient is produced rapidly by using two miscible liquids of different densities at controlled rates as they are introduced into the test tank (see Fig. 2). Two tanks of the same shape containing equal depths of miscible liquids are joined by a pipe. The denser liquid (tank 1) is introduced into the tank together with the lighter fluid (tank 2) and the two liquids are vigorously stirred while the resulting mixture flows into the bottom of the test tank. As is shown in the next section, a linear density gradient can be obtained only if the rate at which the denser fluid flows into the

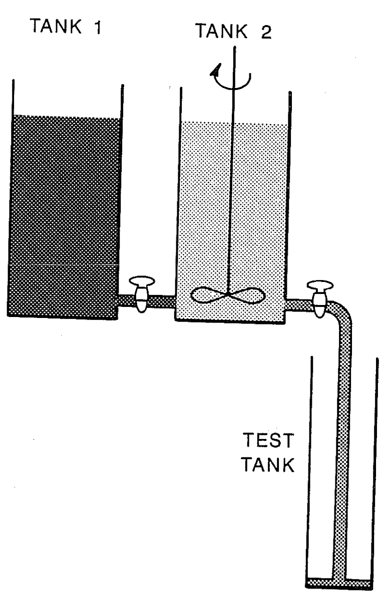

Fig. 2. The two-tank arrangement used in the Oster method.[3]

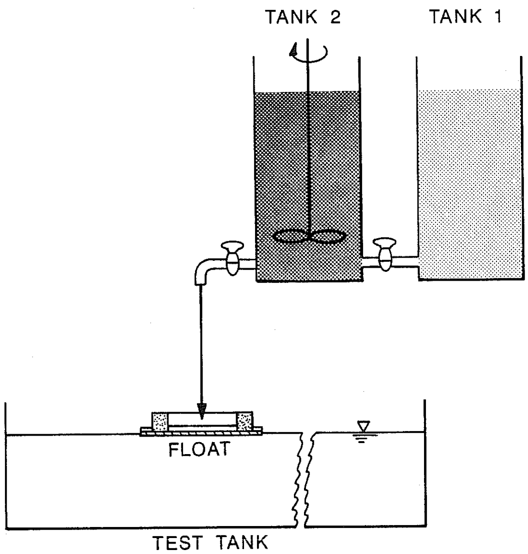

Fig. 3. Schematic of the test set-up with floating raft used in the modified Oster method.[6]

stirred tank is equal to half of that flowing out of the stirred tank. The density gradients obtained by the Oster method will be compared by us with those calculated using the equations developed in the next section.

The Oster method has been used by several investigators, for example, Turner[4] (circular cylinder tank, 29 cm dia and 25 cm high) and Fox[5] (rectangular tank, 44.5 cm high and 45.6 × 45.6 cm base). The linearity of the density gradient was confirmed by withdrawing samples at different heights and weighing[4] and measuring[5] the density by using a calibrated specific gravity hydrometer.

Maxworthy[6] used the Oster method with a slight modification, which will be referred to as the modified Oster method. The lighter fluid was allowed to flow into the tank containing the heavier fluid; they were mixed and fed at twice the rate of inflow to a floating raft in the rectangular test tank (3.66 m long, 45.7 cm high, and 30.4 cm wide). Figure 3 shows the set-up

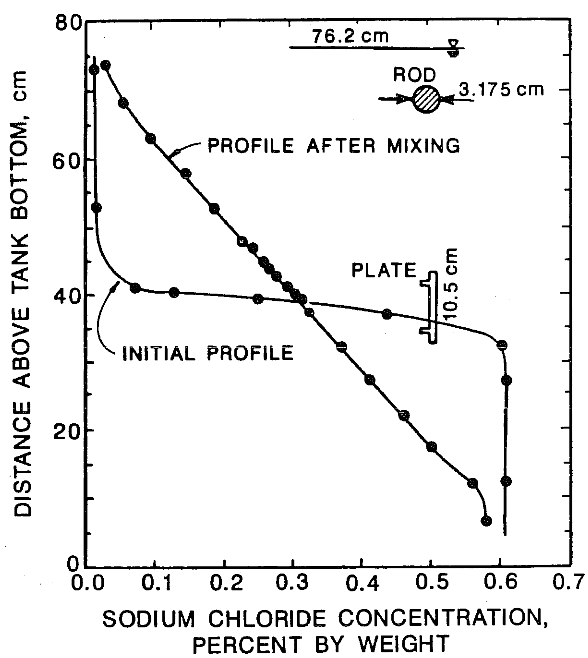

Fig. 4. Profiles generated by the plate–rod assembly used by the method of Clark et al. The positions of the plate and rod are also shown.[7]

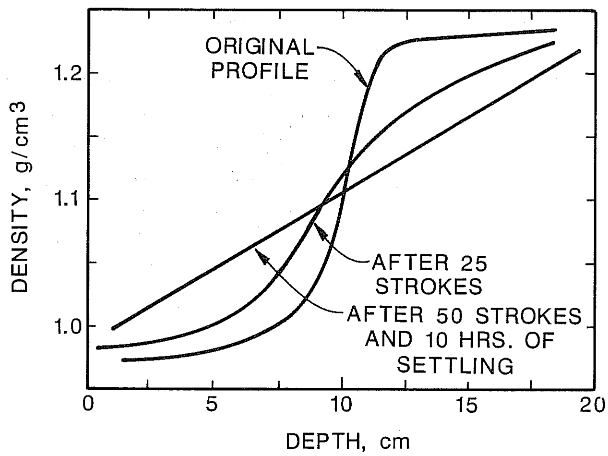

Fig. 5. Density gradient profiles as they develop using the Oster II method.[3]

required for this method. Fluid samples were slowly withdrawn at selected elevations and the specific gravity of the samples was measured with a hydrometer.

The Oster method was tried by Clark et al[7] and found to be impractical for the tank used (15.24 m long, 1.37 m wide, and 1.07 m deep). They developed a method (Clark et al method), which consists of passing a flat plate (oriented normal to the direction of travel and spanning the width of the tank) and a circular rod (parallel to the plate) through an initially two-layered fluid. By properly adjusting the speed, dimensions, and positions of the plate-rod assembly, a linear density gradient was achieved with one pass of the assembly through the fluid. However, a settling time of over 24 h was necessary to achieve a linear density profile. Figure 4 shows a typical result using this method as reported by Clark et al.[7]

The method of Clark et al,[7] although hardware-wise different, in principle is not different from an earlier method described by Oster.[3] In that method, which will be referred to as Oster II method, the boundary region between an initially two-layered fluid of different densities is agitated with smooth up-and-down stirring motion by means of a stirring loop. Figure 5 shows the density profile obtained using this method as it develops after 25 strokes, 50 strokes, and 10 h of settling thereafter.

Two modifications on the Clark et al method were introduced by Imberger[8] and used later by Darden et al.[9] The first modification was in the manner the two layers were obtained. The whole tank (13 m long, 1 m high, and 45 cm wide) was filled with fresh water and a gated partition was introduced vertically into the center of the tank, preventing any transfer of water

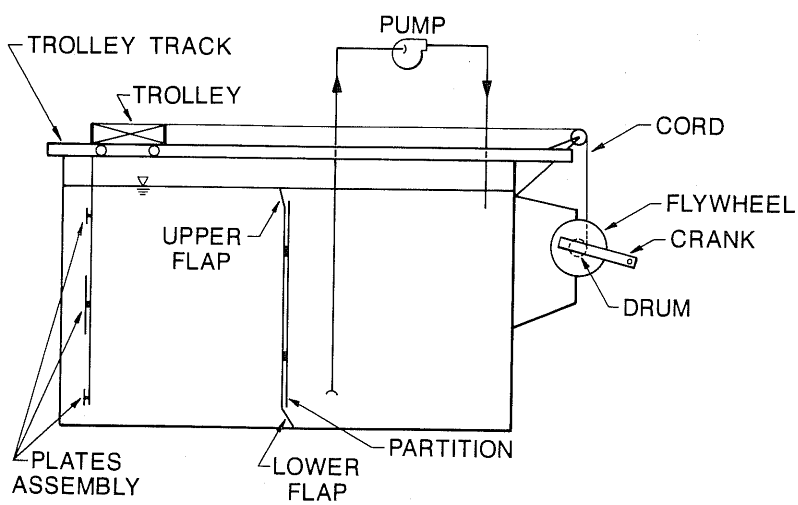

Fig. 6. Schematic of the test set-up used by Imberger[8] and Darden et al.[9]

from one side to the other (see Fig. 6). The appropriate amount of salt was then added to one-half of the partitioned tank where complete mixing took place by pumping from the bottom to the top. After having a uniform mixture, it was allowed to come to rest. Then the flaps on the upper and lower sides of the partition were opened slightly to allow the saline water to flow beneath the fresh water and the fresh water to flow over the salt water. The partition was then carefully removed after the flow between the two parts had ceased and the two layers had formed. The second modification was the replacement of the rod (see Fig. 4) by a plate and the addition of another plate at the lower part of the plate assembly (see Fig. 6).

In this study the LBL, Oster, modified Oster, and Clark et al methods were tested and compared with respect to reproducibility and time consumption.

THEORETICAL DENSITY GRADIENTS GENERATED FOR THE OSTER AND MODIFIED OSTER METHODS

Figure 7 shows the arrangement required for the Oster and modified Oster methods. These methods produce rapid linear density gradients if $q_2 = 2q_1$ as is easily shown from a material balance on tank 2 as follows:

$$\mathrm{d}Vc/\mathrm{d}t = q_1c_1 - q_2c, \tag{1}$$

or

$$V(\mathrm{d}c/\mathrm{d}t) + c(\mathrm{d}V/\mathrm{d}t) = q_1c_1 - q_2c. \tag{2}$$

Since

$$\mathrm{d}V/\mathrm{d}t = q_1 - q_2, \tag{3}$$

integration of Eq. (3) from an initial volume V_0 to V yields

$$V = V_0 + (q_1 - q_2)t. \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2) yields

$$[V_0 + (q_1 - q_2)t](\mathrm{d}c/\mathrm{d}t) + q_1(c - c_1) = 0. \tag{5}$$

With the initial condition $c = c_0$ at t_0, integration of Eq. (5) yields

$$c = c_1 - (c_1 - c_0)[1 + (q_1 - q_2)t/V_0]^{q_1/(q_2-q_1)}. \tag{6}$$

Equation (6) shows that concentration is linear with time only if the exponent $q_1/(q_2 - q_1)$ is equal to unity, i.e., $q_2 = 2q_1$.

The time t may be expressed in terms of the test tank height H, volume-flow rate q_2, and test-tank cross sectional area A. For the Oster method,

$$t = A(H - x)/q_2; \tag{7}$$

for the modified Oster method,

$$t = Ax/q_2. \tag{8}$$

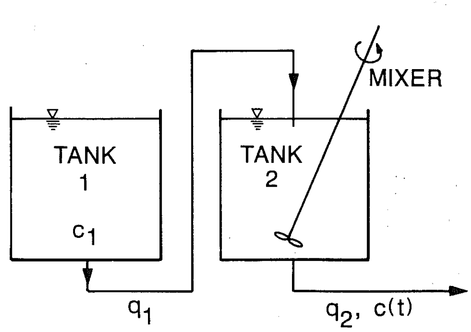

Fig. 7. The two-tank general arrangement used in the Oster and the modified Oster methods.

Setting $q_2 = 2q_1$ in Eq. (6) and substituting for t from Eq. (7), we get for the Oster method

$$c = c_0 + [A(H - x)(c_1 - c_0)/2V_0]. \tag{9}$$

Similarly, for the modified Oster method

$$c = c_0 + [Ax(c_1 - c_0)/2V_0]. \tag{10}$$

Equations (9) and (10) show that concentration (or density) is linear with height only if $q_2 = 2q_1$ as evidenced by Eq. (6).

It should be noted that concentration c, in the above equations is based on volume fraction. In the experiments conducted in this study, the saline solution with the desired density was prepared based on standard solutions with known mass concentrations, c_m. In order to compare the experimental data with those calculated by Eqs. (9) and (10), the data were converted to concentrations based on volume fraction by the relationship

$$c = 1/[1 + (\rho_s/\rho_w)(1/c_m - 1)]. \tag{11}$$

The density can be obtained from the measured c_m by the relationship

$$\rho = \rho_w/[1 + c_m(\rho_w/\rho_s - 1)]. \tag{12}$$

EXPERIMENTAL STUDIES

The experimental set-up used in this investigation is shown schematically in Fig. 8. It consists of two supply tanks (1100 l. each), a Plexiglas test-tank (1.43 m long, 81.3 cm high, and 61.0 cm wide), a metered flow system, and a data-acquisition system[10] (a conductivity probe interfaced with a Texas Instruments Professional Computer through a Monitor Labs model 9302 data logger). Additional hardware needed for flow distribution and mixing by different methods was constructed, i.e., a plate–rod assembly attached to a movable carriage which slides on two rails mounted on the long side rims of the tank, and a floating raft for filling the test-tank that is required for use with the LBL, modified Oster, and Clark et al methods. The floating raft was prevented from touching the sides of the test-tank by using a fixed guide mounted on the top flanges of the tank. The plate–rod assembly resembles that used by Clark et al.[7] A second rod similar to the upper one was added in the lower part of the assembly.

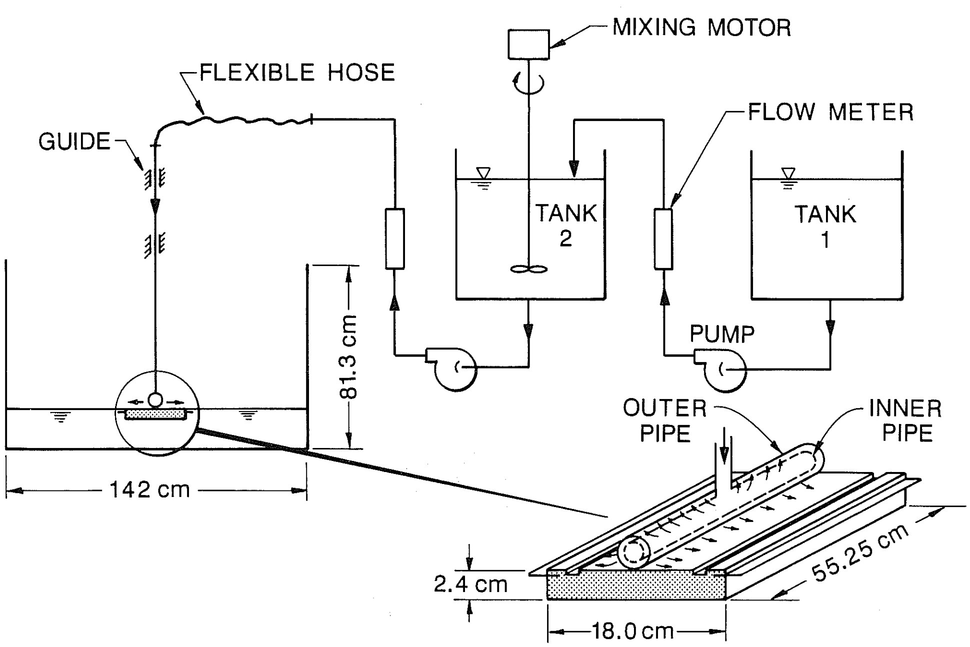

Fig. 8. Schematic of the experimental set-up used in the present work.

For the Oster method, a perforated pipe assembly located at the center of the tank bottom was used. A single pipe pointing downward at the center of the tank bottom (see Fig. 2) was also used in some of the experiments.

The conductivity probe was mounted at the center of the tank and was allowed to slide vertically in the test-tank. The position of the probe was monitored by means of a graduated scale fixed to the probe extension. The probe was calibrated frequently to ensure the reliability of the measurements. Conductivity measurements were taken at selected stations (2.54 cm intervals) up the tank. The experiments with different methods closely followed the procedures described by their originators.

RESULTS AND DISCUSSION

The experiments showed that linear density stratification is obtainable with all of the methods tested. The concentration profiles obtained by the methods of LBL, Oster, modified Oster, and Clark et al are shown in Figs. 9–12, respectively.

The LBL method (see Fig. 9) was found to be the most time consuming. For the tank size used in the experiments, it required about 6 man-hours to fill the tank with 12 layers (each 2 in. thick) at concentrations decreasing with height. The layers with the required concentrations were prepared by successively diluting the initally high-concentration fluid in tank 2 (see Fig. 7) with pumped fresh water from tank 1 and mixing until the required concentration was obtained.

Although the LBL method was tedious and time consuming to use, the desired density gradient could be obtained after few hours of settling with high degree of reproducibility. This was not always possible with the other methods since the resulting density gradient is a function of several variables, i.e., type of distribution device and flow rates in the Oster method, floating raft construction and positioning and the flow rates in the modified Oster method, and the dimensions of the plate-rod assembly and its travel speed in the Clark et al method.

The concentration profile obtained using the Oster method is shown in Fig. 10 along with the theoretical profile generated by Eq. (9). It can be seen that the experimental profile agrees well with the theoretical profile over most of the tank height. A fully mixed region is observed to

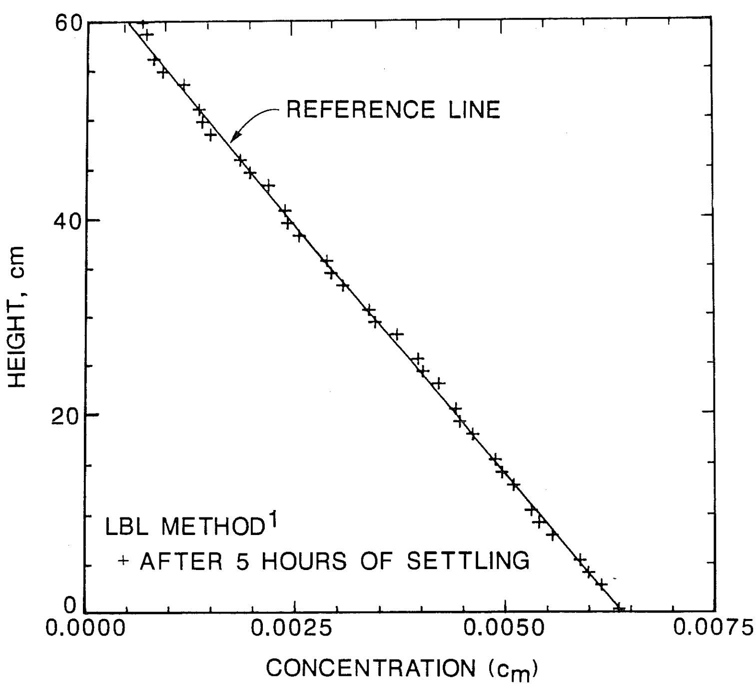

Fig. 9. Concentration profile using the LBL method.

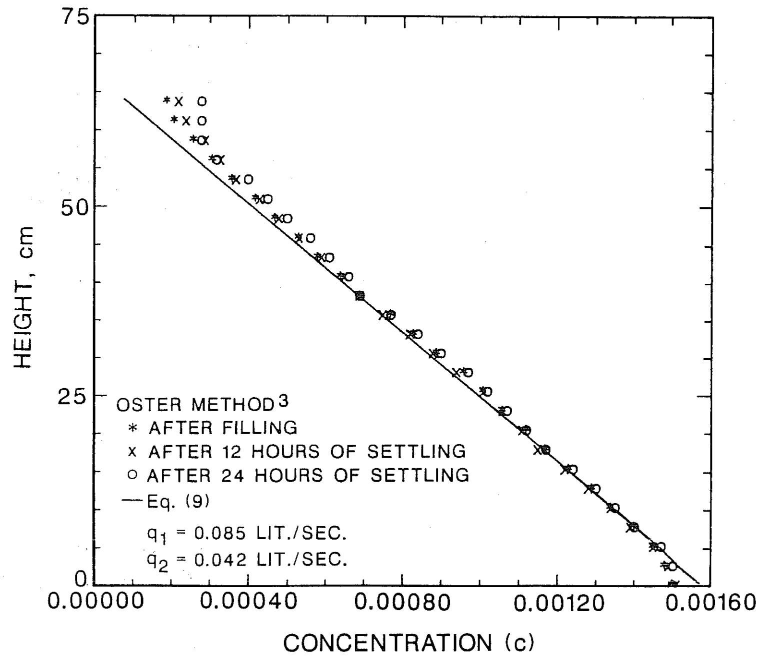

Fig. 10. Concentration profiles using the Oster method.

form in the upper part of the tank after long settling time. Measurements after 12 and 24 h show that the concentration profile experiences a negligible change (except at the upper layer), indicating a stable profile. The effect of the flow distribution device is shown in Fig. 13. This figure indicates that extensive mixing results when a single pipe is used (see Fig. 2 for the schematic of the test set-up). With the perforated pipe assembly spanning the tank bottom, the flow is nearly two-dimensional which causes less mixing as opposed to the single pipe case which gives rise to nonuniform velocities across the bottom of the tank and three-dimensional effects which enhance mixing appreciably. The profile generated by the Oster method is shown

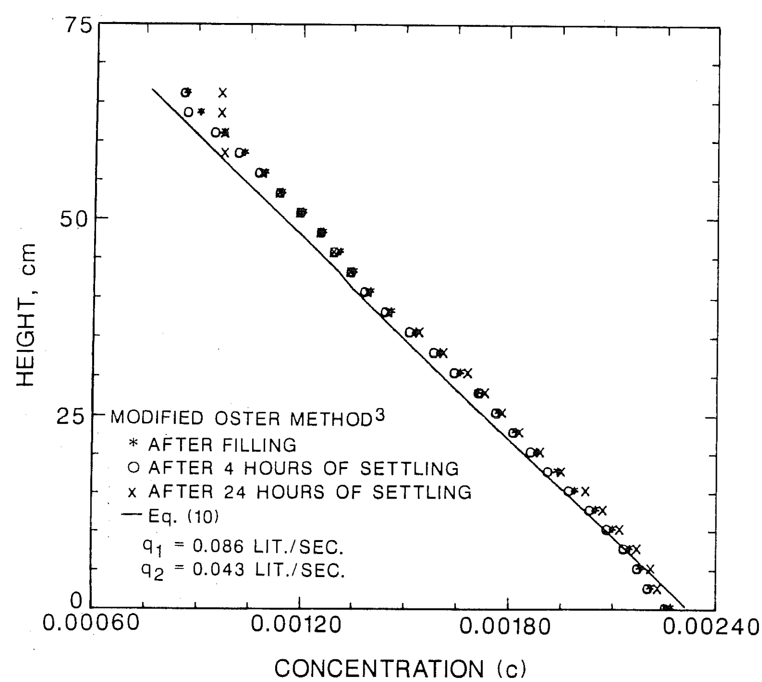

Fig. 11. Concentration profiles using the modified Oster method.

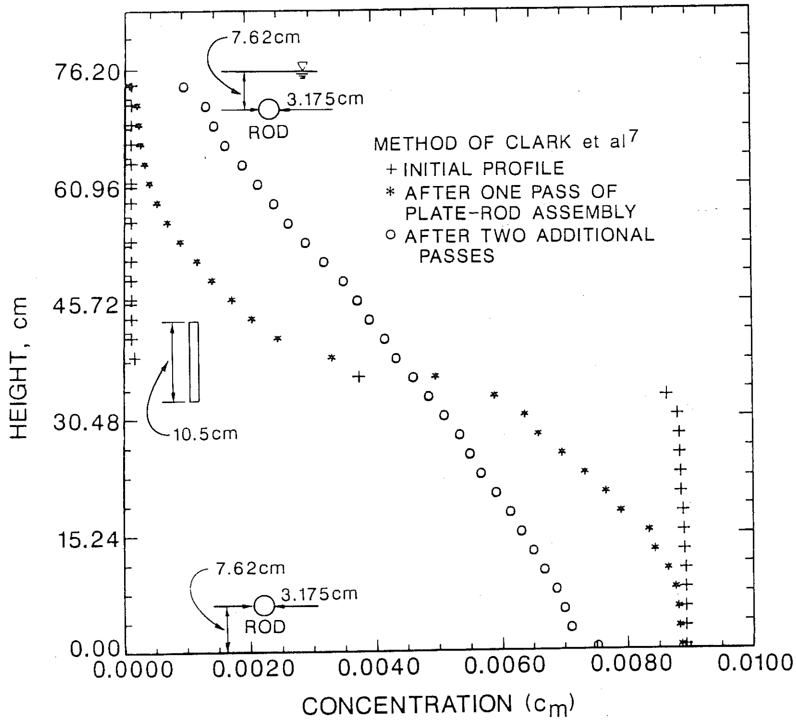

Fig. 12. Concentration profiles using the method of Clark et al.

to be reproducible (see Fig. 14) by repeating the experiments under the same conditions. This is a desirable feature since it eliminates the need for repeated measurements each time the test-tank is filled.

The experimental results obtained using the modified Oster method demonstrated the same features as those of the Oster method. Figure 11 shows the experimental concentration profile as it develops with time along with the theoretical profile as predicted by Eq. (10). Good

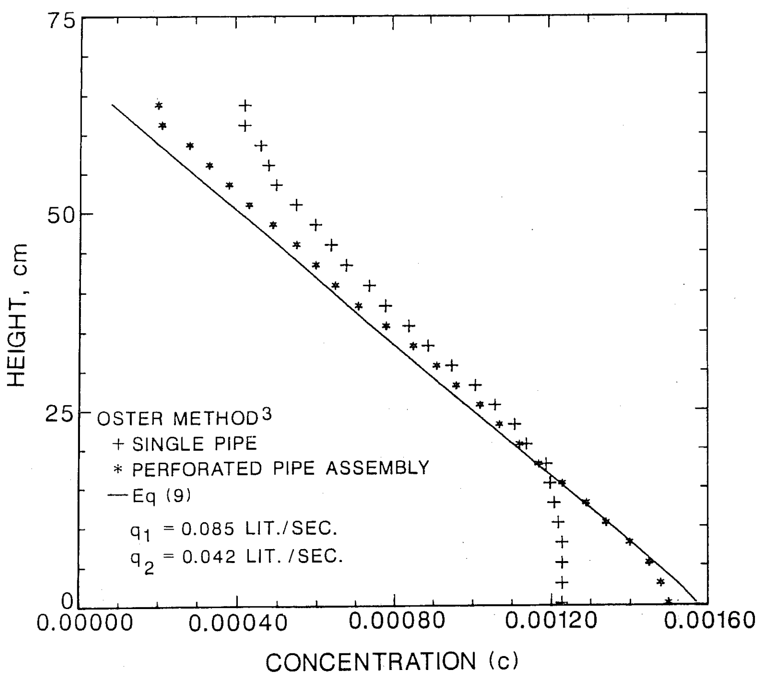

Fig. 13. The effect of the flow distribution device on the concentration profiles (Oster method).

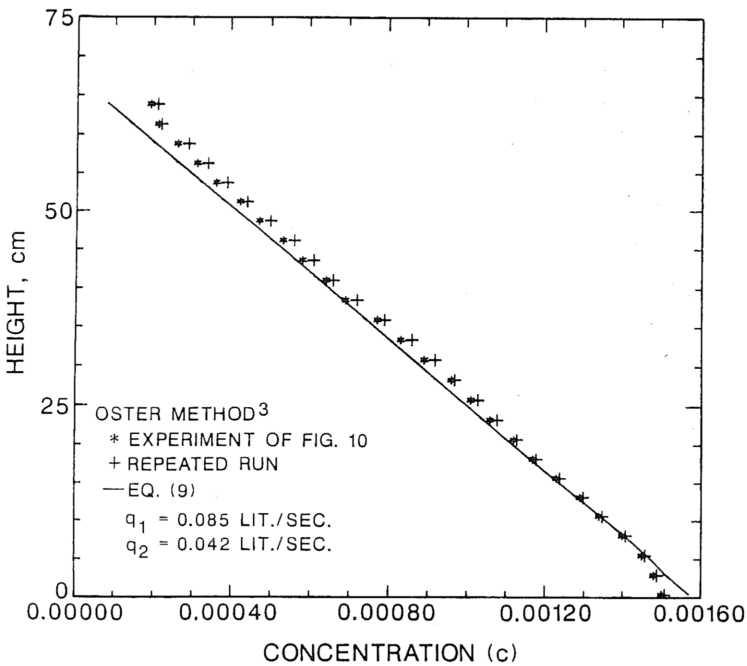

Fig. 14. Comparison of the experimental data obtained from repeated experiments (Oster method).

agreement with the theoretical predictions is seen to exist. Figure 15 shows an example wherein thick mixed regions are observed to form after extensive settling time. At high flow rates (see Fig. 16) the mixed region is thicker in the bottom of the tank as compared with the low flow rate case (see Fig. 11).

The agreement between the experimental and theoretical concentration profiles for all of the tests indicate that the Oster and modified Oster methods are reproducible and the equations developed [Eqs. (9) and (10)] can be used with confidence as a guide for determing the conditions under which a desired density gradient is obtained. However, due to mixing, which

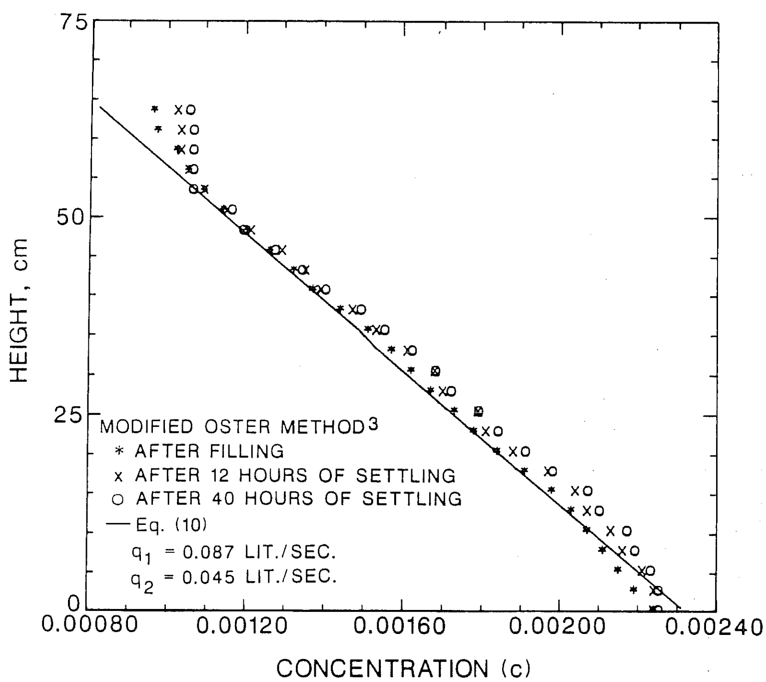

Fig. 15. The effect of settling time on the concentration gradients (modified Oster method).

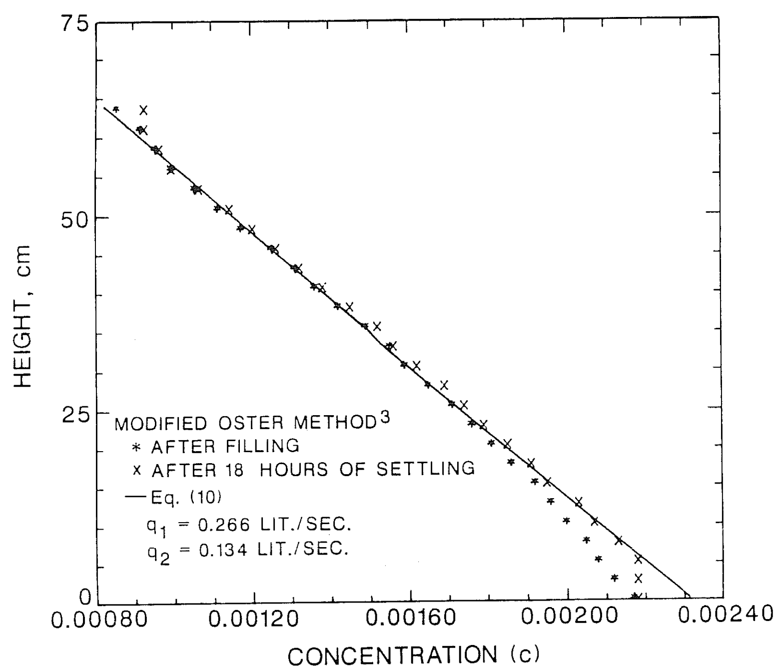

Fig. 16. The effect of the flow rate on the concentration gradients (modified Oster method).

was not allowed for in obtaining Eqs. (9) and (10), the experimentally observed concentration (density) gradients were generally less than the theoretically predicted gradients.

While the desired concentration profile can be generated with some certainty in the previous three methods, this was not the case with the fourth method, i.e., Clark et al method. Figure 12 shows the concentration profile as it develops, after one pass of the plate–rod assembly, and after two more passes. Although the final resulting profile is nearly linear, it differs from the intended profile which should have the same concentration extremes of the initially two-layered fluid. When applying the recommended procedure[7] as pertains to plate–rod assembly

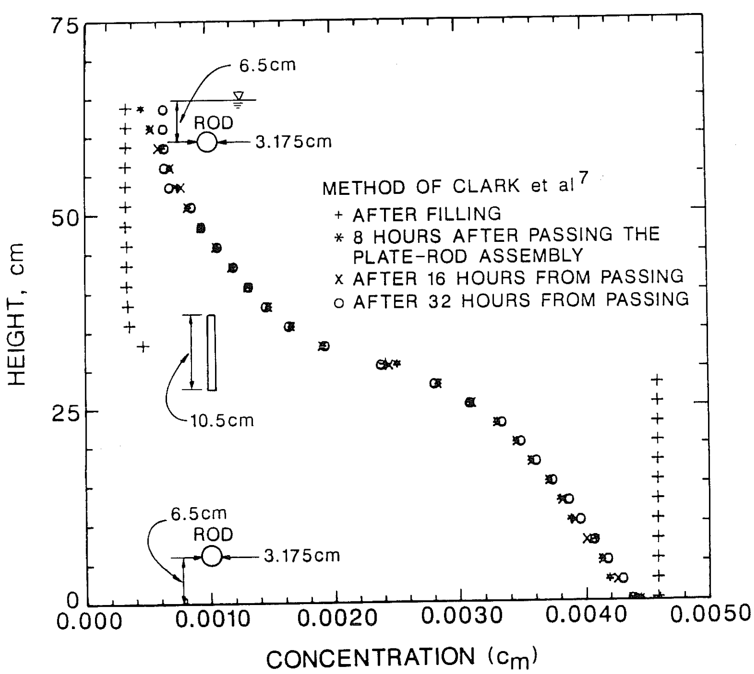

Fig. 17. Concentration profiles using the method of Clark et al.

dimensions and its speed of travel, the resulting concentration profile is shown to be far from linear even after extensive settling time (see Fig. 17).

To quantify the effect of different parameters on the density gradient generated by the Clark et al method, more experiments are needed. This includes tests with different tank sizes, initial salt concentration of the lower fluid layer, dimension of the plate- or plate–rod assemblies, and speed of travel.

SUMMMARY AND CONCLUSIONS

With the exception of the Oster II method, the methods were tested in our laboratory using fresh–saline water systems. The tests showed that the Oster and modified Oster methods showed high degrees of reproducibility. While the flow distribution device in the Oster method is simpler, it may cause some distortion of the density profile if it has to be removed from the test tank after filling. This is not a problem with the modified Oster method since the floating raft can be removed with negligible disturbance.

The equations derived for prediction of the concentration (density) gradients generated with the Oster and modified Oster methods have been shown to be reliable tools for producing the desired concentration gradients. Based on the experimental results, it is recommended that the predicted density gradients be reduced by about 10% on the average to account for mixing effects.

More experiments are planned with the method of Clark et al in order to quantify the effects of different parameters on the linearity of density stratification.

Acknowledgement—This work was supported by the University Center for Energy Research (UCER) at Oklahoma State University.

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NOMENCLATURE

A = Cross-sectional area of test-tank
c = Salt concentration
H = Tank height
q = Volume-flow rate
t = Time
V = Volume
x = Distance up the tank
ρ = Density

Subscripts

m = Mass
0 = Initial
s = Salt
w = Pure water