A Comparison Study of One-Dimensional Models for Stratified Thermal Storage Tanks

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A survey of the stratified thermal storage tank one-dimensional models available in the literature has been conducted. Six of these models were tested and compared against the experimental data obtained at our laboratories and from the literature. Although various factors affecting the performance of a stratified tank can be accounted for by the higher order models, i.e., two- and three-dimensional models, the introduction of empirically-based mixing parameters into the one-dimensional models renders them widely applicable and practical in the simulation of energy systems incorporating thermal storage tanks.

Introduction

Thermal storage tanks which keep the warm and cold water separated by means of gravitational stratification (single stratified tanks) have been used for load management and energy conservation in an increasing number of installations. The single stratified tank is attractive in low- to medium-temperature thermal storage applications due to its simplicity and low cost. Its performance can be as high as other types employing physical barriers [1] and has superior reliability [2]. In these storage tanks, water is the most widely used storage medium for its abundance, low cost, high specific heat, and benign character. There are four primary mechanisms which contribute to the loss and/or degradation of stored energy. These are: (1) the heat loss to the ambient, (2) thermal diffusion from the hot layer to the cold layer, (3) vertical conduction in the tank wall which along with (1) induce convective currents (mixing) in the fluid body, and (4) mixing introduced during charge and discharge cycles. While the first three mechanisms can be accounted for and/or their effect can be minimized, the fourth mechanism remains difficult to evaluate, since it is inlet design dependent among other factors.

In the models described next, the emphasis is placed on comparison of these models with respect to the fourth mechanism. This is due to the experimental data being obtained from experiments conducted in scaled models (insulated and with thin wall) in which the first and third mechanisms are essentially absent. Also, the second mechanism is usually not insignificant in short duration tests (on the order of one hour).

Two thermal cases in a single stratified tank are distinguished [3]. In the first case the temperature of the incoming fluid is constant or experiences negligible variation. This is normally the case encountered in chilled water storage. In the second case the temperature is allowed to vary. This case is characteristic of solar thermal energy storage in which the liquid heated in the collectors may have a temperature that varies with each passing cloud.

The single stratified tank has been investigated experimentally and analytically. Experimental studies [1], [3] to [13], [16], [18] established the feasibility of using the natural thermocline to achieve separation of hot and cold liquid inside a single container and identified the various parameters affecting the performance. Analytical studies have dealt with modeling the flow phenomenon in the storage tank to predict the thermal performance under a variety of flow conditions and geometric configurations. For maximum thermodynamic availability, it is desirable to achieve a plug-type flow by placing the inlet hot (cold) liquid at the uppermost (lowermost) portion of the tank with minimum mixing with the existing fluid in the tank. In this manner, modeling efforts using a one-dimensional flow assumption are somewhat justified. In the case where the inlet temperature varies, in addition to the one-dimensional flow assumption, the assumption that the flow seeks its temperature level in the tank without [14] to [16] and with [17] mixing is imposed.

Several one-dimensional models have been reported in the literature. The models include the fully-stratified storage tank models of Close [14], Duffie and Beckman [15] and its modified version by Sharp [16], and the viscous entrainment model by Han and Wu [17]. In these models the inlet temperature is always allowed to vary. The models in which constant inlet temperature is assumed include those with mixing effects; Cole and Bellinger [11], Oppel et al. [12] and its upgraded version by Zurigat et al. [18], Wildin and Truman [1], and the model of Cabelli [19] with no mixing effects.

More complex models have appeared in the literature [19] to [22]. The two-dimensional model of Cabelli [19], has incorporated two flow circuits and two geometric configurations with horizontal and vertical entries into the tank. Comparison of the predicted temperature profiles with the results from

their one-dimensional model [19] showed a discrepancy of less than 12 percent. Nevertheless, the values of Reynolds numbers used were limited by the mesh size to magnitudes smaller than those expected in practical situations. This discrepancy was treated later by Guo and Wu [20] who developed a two-dimensional model applicable for high Reynolds numbers. The Richardson number, Ri, was identified [20] as the important parameter for characterization of the physical conditions of the flow pattern and temperature stratification inside the storage tank. At Ri < 1, the forced convection becomes important, and leads to a complete mixing case [20]. In the two-dimensional model of Chan et al. [21] different inlet and outlet locations were tested and compared with respect to thermal efficiency.

From the foregoing review, it can be noted that most of the modeling efforts were one-dimensional, and in a few cases two-dimensional. For thermal storage design assessments under various constraints, it is important that three-dimensional effects be included. A three-dimensional model has been developed by Sha et al. [22] to study improved storage tank designs. The model incorporates a simple eddy diffusivity turbulence model and is restricted to a cylindrical tank geometry.

While the two and three-dimensional models are more capable in accounting for different factors affecting the thermal storage tank performance, they are not suitable for use in large energy systems load management programs. Simpler one-dimensional models may be advantageous since they are computationally more efficient. Moreover, the object of any design of a single stratified tank should be the achievement of one-dimensional flow as any motion in the second or third direction would enhance mixing and degrade stratification. For this purpose, a variety of diffusers have been investigated and/or used (11), (7), (9), (11) to (13), (18).

The development of one-dimensional models has been crucial to the successful implementation of load management programs. However, when choosing a model, one is confronted with a series of questions concerning its accuracy, computational efficiency, and simplicity. The present effort was undertaken to answer these questions.

Selected One-Dimensional Models

In the study of Kuhn et al. [23], several one-dimensional models of a single stratified tank were identified and compared by validation with experimental data. The model of Sharp [16] was recommended as the best among the ones compared (see [23]). However, some models developed during the period when the study of Kuhn et al. [23] was in progress were not compared (i.e., (17), (19)). In addition, since the submission of their final report [23], several one-dimensional models have appeared in the literature ([11], [11], (12), (18)]. In this paper the following single stratified tank one-dimensional models were selected for comparison with experimental data obtained at our laboratory and from the report by Kuhn et al. [23], both conducted under constant inlet temperature condition:

1. Sharp [16]
2. Han and Wu [17] (Viscous Entrainment model)
4. Zuriqat et al. [18] (Effective Diffusivity model)
5. Cabelli [19]
6. Wildin and Truman [1]

It should be noted that although a one-dimensional assumption may be justified in properly designed tanks, the two and possibly three-dimensional effects are still present, especially in the inlet region. This was recognized by some investigators [1], (11), (17), (18) and mixing effects were subsequently included explicitly in their models in different ways.

Models' Description

In this section a brief description of the selected models is presented. The references cited should give the interested reader the complete description.

The model of Sharp [16] is a one-dimensional, finite difference model. The tank is divided into a number (N) of isothermal, constant volume segments, and energy, and mass balance equations are written for each segment. Thermal losses to the environment and vertical conduction through the tank walls are accounted for. The model incorporates two flow loops; the collector and the load loops. Using control functions, the inlet fluid is directed to the segment whose temperature most closely matches the inlet temperature. Vertical mixing due to the introduction of the fluid is neglected. However, the number of segments used in the model has an effect on the calculated temperature profile, similar to that due to mixing. Figure 1 shows the dimensionless transient temperature profiles (T* versus t*), at a fixed location in the tank, calculated by the model of Sharp using different values of N. Higher values of N result in closer prediction to ideal stratification or, equivalently, less mixing.

The model of Han and Wu (Viscous Entrainment model) [17] is a one-dimensional, finite difference model that includes

Nomenclature

\[ a = \text{heat capacity ratio used in the model of Cole and Bellinger [11]} \]
\[ C = \text{mixing parameter used in the model of Cole and Bellinger [11]} \]
\[ D = \text{characteristic length} \]
\[ g = \text{acceleration of gravity} \]
\[ H = \text{normalized film heat-transfer coefficient used in the model of Cole and Bellinger [11]} \]
\[ L = \text{vertical length between tank inlet and outlet} \]
\[ N = \text{number of finite difference segments used in the model of Sharp [16]} \]
\[ NM = \text{number of mixed segments near the inlet used in the model of Wildin and Truman [1]} \]
\[ \text{Re} = \text{Reynolds number, Re} = \frac{VD}{\nu} \]
\[ \text{Ri} = \text{Richardson number, Ri} = \frac{\Delta \rho g L}{\rho m \nu^2} \]
\[ T = \text{average temperature} \]
\[ T* = \text{dimensionless average temperature, } T* = \frac{(T - T_B)}{(T_{in} - T_B)} \]
\[ t = \text{time} \]
\[ t* = \text{dimensionless time, } t* = \frac{t V_m}{L} \]
\[ V = \text{characteristic velocity} \]
\[ V_m = \text{average vertical velocity based on tank cross-sectional area} \]
\[ X = \text{vertical distance measured from the top of the tank} \]
\[ \Delta \rho = \text{absolute density difference between initial and inlet conditions} \]
\[ \text{\epsilon}_{\text{eff}} = \text{effective diffusivity factor used in the model of Zuriqat et al. [18]} \]
\[ \text{\epsilon}_{\text{eff}} = \text{the value of } \epsilon_{\text{eff}} \text{ taken at the inlet of the tank} \]
\[ \gamma = \text{boundary condition mixing parameter used in the model of Han and Wu [17]} \]
\[ \nu = \text{kinematic viscosity} \]

Subscripts

\[ \text{in} = \text{inlet} \]
\[ m = \text{mean} \]
\[ o = \text{initial} \]
the mixing effects due to viscous entrainment by the incoming stream of the tank fluid and heat loss to the environment. The model incorporates collector and load circuits. Mass and energy balance equations are derived for both circuits and solved using an implicit finite difference technique. An additional equation describing the rate of viscous entrainment is provided.

A boundary condition parameter, $\gamma$, is introduced to account for mixing in the upper and lower regions of the tank due to the introduction of collector and load flows, respectively. Figure 2 shows the calculated transient temperature profiles with different values of $\gamma$ at a fixed location in the tank. As $\gamma$ exceeds 20, it is apparent that an unrealistic mixing takes place from one side of the thermocline, that is, the thermocline fronts for different values of $\gamma$ start at approximately the same location irrespective of the severity of mixing experienced from the inlet side.

The model of Cole and Bellinger [11] is a one-dimensional analytical model with empirically derived parameters. These are the mixing parameter, $C$, that accounts for mixing due to the introduction of fluid into the tank, the normalized film heat-transfer coefficient, $H$, which accounts for the fluid-wall thermal interaction, and the capacity ratio, $a$, which accounts for the effect of wall heat capacity on stratification. Heat loss to the surroundings was neglected and a constant flow rate assumption was imposed. Also, constant inlet temperature and uniform initial temperature assumptions were imposed.

The parameter $C$ has a significant influence on the thermocline (the region of steepest temperature gradient in the fluid body) shape as can be seen in Fig. 3. However, the parameter $H$ was found to have an insignificant effect on the temperature profile which is in agreement with the findings of Cole and Bellinger [11]. The capacity ratio, $a$, is equal to unity for thin walls.

The model of Zurigat et al. [18] (Effective Diffusivity model) is a one-dimensional, finite difference model which accounts for turbulent mixing in the tank and heat loss to the ambient surroundings. The energy equation for turbulent flow is solved by splitting it into two equations representing the conduction and convection cases, and handling them with different computational time step. This technique was shown [12], [18] to completely eliminate the numerical diffusion in the finite difference solution. By introducing an effective diffusivity factor, $\epsilon_{eff}$ [18], the turbulent energy equation is reduced to the laminar one, magnifying the molecular diffusivity by that factor.

The effective diffusivity factor was modeled based on experimental data. Correlations were developed for this factor in terms of flow parameters and inlet geometry. The spatial variation of $\epsilon_{eff}$ was expressed in a decreasing hyperbolic function. The maximum value at the inlet, $\epsilon_{eff}$, is different for different flow conditions and inlet configurations. Figure 4 shows the thermocline calculated using different values of $\epsilon_{eff}$. For the variable flow-rate case, the time step was selected using a fictitious “buffer tank” concept [12]. The model, in its present form, incorporates three different inlet geometries; a side inlet, an impingement inlet, and a perforated inlet (see Fig. 7).

The model of Cabelli [19] is a closed-form solution of the one-dimensional energy equation with heat loss to the environment (cylindrical tank). No mixing effects were included.
Therefore, poor agreement with the experiments is expected. However, in this work the model was mainly used to ascertain the accuracy of the experimental data, since the data plotted in normalized coordinates (T' versus t') should intersect the analytical solution of Cabelli.

The model of Wildin and Truman [1] is a one-dimensional, finite difference model that accounts for mixing in the inlet region, vertical conduction through the storage medium, thermal capacitance and resistance of the tank walls and floor, and convective heat transfer between these elements and the storage medium. Mixing in the inlet region was modeled by averaging the temperatures of a specified number of liquid segments, NM, near the inlet. Figure 5 shows the effect of the variation of NM on the thermocline. It was concluded [1] that the mixing layer thickness obtained this way for well-designed inlet diffusers was no more than seven percent of the water depth, up to a maximum of one foot. However, the mixing introduced in this manner gave a varying agreement throughout the storage tank. In certain cases, good agreement was found at the start of charging and poor agreement after the thermocline has formed. This was attributed to flow-rate measurement errors and heat transfer between the fluid and the tank [1]. The model, however, gave generally satisfactory agreement with the experiments. A compiled version of the computer program for this model was obtained from the authors.

The thermocline shape predicted by different models (see Figs. 1 to 5) gives some indication about their predictive capability as compared with the experiments. Except at low flow rates, the experiments showed the thermocline being asymmetrical around some inflection point. In this respect the models of Han and Wu [17], Zurigat et al. [18], and Wildin and Truman [1] show thermocline predictions closer to the experimental evidence than those of Sharp [16] and Cole and Bellinger [11]. This is attributed to the use of localized [17], [1] or nearly localized [18] mixing parameters. This agrees with the physical reality of the mixing being significant in the inlet region and affecting the thermocline extent asymptomatically after its formation close to the inlet.

**Experiments**

Experiments with a hot-cold water system were conducted at our laboratory for the development of the Effective Diffusivity model reported by Zurigat et al. [18] and to validate the models selected in the previous section.

The experimental setup is shown schematically in Fig. 6. It consists of a hot water supply tank, an insulated steel test tank (0.1 in. thick wall wrapped with 3 in. of fiberglass insulation of 3.7 R-value), a metered flow system, temperature sensor arrays, and a data acquisition system.

The hot water supply tank is capable of supplying hot water at any desired temperature up to 200°F. The test tank (16 in. dia and 56.95 in. high) is equipped with an inlet adapter to facilitate the installation of different inlet diffusers. Three different inlet geometries were tested (see Fig. 7):

1. Impingement inlet: 0.709 in. inside diameter pipe entering the side of the tank, turning upwards approximately 0.4 in. from the top surface of the tank with the flow impinging on the center of the top side of the tank.
2. Side inlet: 0.634 in. inside diameter pipe located 2.17 in. from the pipe center to the top of the tank and extending 0.76 in. into the tank.
3. Side inlet with perforated baffle (perforated inlet): a perforated circular baffle 15.75 in. diameter, with 482 holes (0.201 in. diameter each) was installed 0.5 in. below the side inlet previously described.

The experiments (constant inlet fluid temperature) were carried out for the charging mode of operation (hot water fed to the top of the tank and cold water withdrawn from the bottom) for flow rates ranging from 0.5 to 3.0 gpm and temperature differences of 40 to 120°F. The flow rate was
measured using a calibrated rotameter with full-scale accuracy of ±2 percent and full-scale repeatability of ±1 percent. These measurements were verified by using a calibrated catch tank to measure the total volume of the fluid forced out of the test tank and then dividing this volume by the total time of the experiment.

Transient temperature profiles inside the test tank were measured using 36 J-type thermocouples mounted in nine levels with four thermocouples at each level (see Fig. 8). At each level, the two sets of opposing thermocouples were extended 2 in. and 3 in., respectively. The average temperature at each level was obtained by averaging the four temperature readings at that level. The first level is located at 4.9 in. below the top of the tank (just below the inlet adapter flange). The rest of the levels are located at 5.4 in. intervals down the tank (see Fig. 8). Four more thermocouples were used to measure the hot water supply tank, test tank inlet and outlet, and the ambient temperatures.

The data acquisition system used consists of a 40-channel data logger interfaced with a TI computer. Temperature readings were taken at 20 to 40-second intervals. Calibration of thermocouples showed that the measurements were accurate within ±1°F. The data reduction software written in C-language and the data acquisition system are described in Rao et al. [24].

The experimental data of Kuhn et al. [23], which were also used for validation of the models, were obtained using a cylindrical test tank 84 in. high, 45 5/8 in. inside diameter, and 3 1/4 in. thick steel wall. The side of the tank was insulated with 3.5 in. thick fiberglass blanket (R-11) and the top covered with a rigid foam block (R-27). Two inlet geometries were employed; a vertical inlet and a perforated cup diffuser. The vertical inlet was a schedule 40 PVC pipe extending from the top of the tank to a point 2 in. below the water surface. The cup diffuser is shown in Fig. 9. The tests were of single cycle type, that is, charge or discharge. The tests with the cup diffuser were used for validation of the models.

**Results and Discussion**

The models [11], [16], [17], [19] were coded based on formulations by their originators. Initial runs established the working condition of these models with the exception of Cabelli model [19] where a factor of one-half was found omitted in the published analytical solution.

In the validation of these models with our experimental data and those from Kuhn et al. [23], the variables N, γ, C, and NM in the models of references [16], [17], [11], and [1] were optimized to fit the experimental data as close as possible. Some of these comparisons with our experimental data are presented in Figs. 10 and 11 and with data from Kuhn et al.
Table 1: Computer CPU processing time of different models for a typical run on IBM 3081K

<table>
<thead>
<tr>
<th>Model</th>
<th>CPU Processing Time (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cabelli [19]</td>
<td>1.8</td>
</tr>
<tr>
<td>Han and Wu [17]</td>
<td>1.84</td>
</tr>
<tr>
<td>Zurigat et al. [18]</td>
<td>2.38</td>
</tr>
<tr>
<td>Sharp [16]</td>
<td>2.7</td>
</tr>
<tr>
<td>Wildin and Truman* [1]</td>
<td>6.01</td>
</tr>
</tbody>
</table>

*Based on estimated IBM 3081 CPU time from reference [1].

In Figs. 12 and 13. Note that in Figs. 10 and 11, for better visual comparisons of different models with the experimental data, only every other temperature level (starting with the first level) in the tank is presented (see Fig. 8). Figure 10 shows the predictions of thermocline in the storage tank by the models as compared to our experimental data for low flow rate with an impingement inlet. With the exception of model of Han and Wu [17], the models considered predict the passage of thermocline throughout the tank quite well. However, they differ in their predictions of the extent of the thermocline. The best model, in this respect, is the model of Wildin and Truman [1] with which the thermocline was predicted well throughout the tank. The value of the parameter \( NM \) used to obtain the best prediction is 3, which corresponds to five percent of the tank height. The model by Cole and Bellinger [11] also showed very good results. The mixing parameter, \( C \), used to obtain the best fit (\( C = 2 \)) falls in the lower range of accuracy (\( 1.7 < C < 21 \)) suggested by the correlation for \( C \) as opposed to the calculated value of 6.

As can be seen from our experimental data (see Fig. 10) for low flow rate with a relatively good inlet (impingement inlet), the mixing was not severe as evidenced by the thermocline approaching the ideal case. In this case the model of Cabelli [19], without a mixing parameter, is seen to give good predictions. Also the parameters accounting for mixing in different models were low, as in the case of Wildin and Truman [1] (\( NM = 3 \), or 5 percent of tank height), Cole and Bellinger [11] (\( C = 2 \)), and Han and Wu [17] (\( \gamma = 4 \)), and the number of segments needed in the model of Sharp [16] was high (\( N = 100 \)). The model of Han and Wu [17] showed an unrealistic mixing taking place in the front of the thermocline contrary to the physical expectation of the tail of the thermocline normally experiencing such mixing due to the inlet disturbances. The model of Zurigat et al. [18] needed no adjustment as the only adjustable parameter, \( e_{ir} \), was correlated with the flow parameters and inlet conditions.

The performance of different models, when applied to the cases where mixing is significant (see Fig. 11, side inlet), showed a varying degree of agreement with our experiments. As expected, all the models fail to predict the thermocline passage and shape in the upper region of the tank where mixing effects were severe. However, the predictions in the outlet region were much better. The best predictions were obtained using the models of Zurigat et al. [18] and Wildin and Truman [1] (see Fig. 11). The model of Wildin and Truman [1] predicted the first two levels very well when high value of \( NM (NM = 10 \), or 16.6 percent of the tank height) was used. However, this value produced a poor agreement with the experiment in the rest of the levels. For better prediction, \( NM \) value of 6 (or 10 percent) was used, which is still higher than the recommended value [1] of 4 (7 percent).

As in the previous cases, adjustment of the parameters, \( N \), \( \gamma \), \( C \), and \( NM \) in the corresponding models was necessary. Higher values of \( \gamma = 18 \), \( C = 20 \), and \( NM = 6 \) and a lower value for \( N = 45 \) in the corresponding models were chosen for better predictions. The value of \( C \) used in this case (\( C = 20 \)) agrees closely with the calculated value of 17 and falls in the lower range of accuracy [11] (\( 5 < C < 61 \)).

Figure 12 shows a comparison between the predictions of the models and the experimental data of Khun et al. [23] for the discharge mode of operation. The best predictions are obtained with Cole and Bellinger [11] and Zurigat et al. [18]. Due to the low mixing, the mixing parameters used in the models were low (\( C = 0.2 \), \( \gamma = 1 \), \( NM = 1 \) or 2 percent of tank height and \( N = 90 \)). Figure 13 shows another case (charging) where the predictions were made with the same values of mixing parameters used in the case of Fig. 12. The models of Cole and Bellinger [11], Zurigat et al. [18], and Wildin and Truman [1] show good agreement with the experiment. The model of Cabelli showed good predictions, indicating a very low mixing is taking place.

The results of further comparisons of the selected models with our experimental data (charge mode with impingement, side and perforated inlets), and those reported by Kuhn et al. [23] (charge and discharge modes with a cup diffuser inlet) with different flow rates. And temperature differences, may be found in Maloney [25]. From these comparisons it was concluded that the most accurate models, consistent with the aforementioned findings, are those of Cole and Bellinger [11], Zurigat et al. [18], and Wildin and Truman [1].

The selected models were also compared with respect to computational efficiency. Table 1 shows the processing time in...
Summary and Conclusions

The one-dimensional single stratified tank models available in the literature were surveyed and six models were validated with the experimental data obtained at our laboratory and from the literature. The models showed varying degree of agreement with thermocline test data. Generally, the best agreement was obtained with the models of Wildin and Truman [1], Cole and Bellinger [11], and Zurigat et al. [18]. However, while some adjustment of the mixing parameters was needed in the first two models [1], [11], no adjustment is required in the latter [18]. On the other hand, the analytical model of reference [11] is computationally more efficient than those of references [1] and [18] and the simplest to implement. It should be noted that the models of references [16] and [17] are more general since they incorporate two flow circuits and can be applied in both the constant and variable inlet fluid temperature cases. However, based on the results presented for the model of Han and Wu [17], its accuracy may be questionable, since it failed to give good predictions in the limiting case; the constant inlet temperature case.

The parameter $N$ in reference [16] should be correlated with the flow variables so that predictions can be made with less adjustment of this parameter. Although this may not accurately represent the mixing processes in the storage tank, the empiricism involved in the one-dimensional models has the advantage of rendering them simple to use and computationally more efficient in energy systems simulation programs as compared to two- or three-dimensional models. While computational efficiency is not an issue in simulations over a short period of time, for example, diurnal cycle, it may become a restrictive factor in simulations over seasonal or yearly cycles.

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References